The rotation rate and its evolution derived from improved mode fitting and inversion methodology

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Abstract. We present inferences of the internal solar rotation rate and its evolution during solar cycle 23. A full solar cycle of MDI observations have been analyzed using an improved fitting methodology and using time series of various lengths, up to a single 4,608 day long epoch (64 times 72 days or 12.6 yr). We used time series of spherical harmonic coefficients computed by the MDI group, including those resulting from using their improved spatial decomposition. This decomposition includes our best estimate of the image plate scale and of the MDI instrumental image distortion. The leakage matrix used in the fitting includes the effect of the distortion of the eigenfunctions by the solar differential rotation, while the undistorted leakage matrix was itself carefully reviewed and independently recomputed. Rotation inversions were carried out for all available mode sets, fitted for that epoch, including the MDI and GONG "pipe-line" values. The improved inversion method uses an iterative methodology based on a least-squares regularization, but with an optimal model grid determined by the actual information in the input set. This method also allows us to use an optimized irregular grid, with a variable number of latitudes at different depths.

1. Introduction

Since the deployment of the Global Oscillation Network Group (GONG) instruments and the start of operations of the Michelson Doppler Imager (MDI), soon after the launch of the SOHO spacecraft, we have accumulated some 13 years worth of data. Nevertheless, *one* still uses 20 and 18 year old fitting methodologies, respectively, to derive the solar p-mode characteristics. The GONG *pipe-line* frequencies are produced, using the method described in [4], by fitting *overlapping* 108-day long segments, not using any leakage matrix information and fitting a symmetric profile to individually but independently fitted peaks. Some 147 *GONG months* (each 36 days long) were available, spanning 1995.06.29 to 2009.09.07.

The MDI *pipe-line* frequencies are produced using the methodology described in [5], that includes information about the leakage matrix but still fits a symmetric profile, and parametrizes the frequency splitting with a polynomial expansion in m. Some 67 non-overlapping epochs (each 72-days long) were available, spanning 1996.05.01 to 2009.12.07.

More recently, that approach was improved, see [6]. The spatial decomposition was reprocessed, having gained from our insight based on high-degree mode characterization [3]. Specifically, the actual plate scale was used and our best model of the instrumental image distortion was included. Moreover the modeling of the leakage matrix was improved by including the distortion of the eigenvalues by differential rotation, as introduced by [8], and by allowing, or not, for an asymmetric peak profile (57 & 62 epochs were done so far respectively).

1.1. An improved fitting methodology

Motivated by the embarrassing aging of these methodologies, while prompted by the availability of long time-series combined with high performance computing, I have developed a new fitting methodology. The salient characteristics of this approach are: (1) the simultaneous fit, for all m, of individual modes, combined with a sanity rejection to avoid fitting non-significant peaks; (2) using an *optimal* multi-tapered spectral estimator; (3) fit an asymmetric profile; (4) include our best estimate of the leakage matrix (including distortion by differential rotation); and (5) use time-series of varying lengths to optimize the trade-off between SNR and temporal resolution; see [1, 2] for additional details. This method has been applied to some degree to GONG data, but mostly to MDI times series. At first, time-series of *standard* spherical harmonic coefficients were analyzed using 2088-day long & 728-, 364-, 182-day long, overlapping, epochs, spanning 1996.05.01 to 2002.01.17. More recently, *improved* time-series of spherical harmonic – *i.e.*, resulting from the new spatial decomposition mentioned earlier – combined with the inclusion of the effect of the distortion by differential rotation on the leakage have been fitted for 64×72, 32×72, 16×72-day long, overlapping, time-series, covering the 1996.05.01 to 2009.07.16 time span.

1.2. Potential issue & inverse theory

The archetypal inverse problem can be cast as follow:

$$y_i = \int K_i \, x(p) \, dp \tag{1}$$

where the set of observables, y_i, combined with known kernels, K_i, is used to derive the underlying model, $x(p)$. Since such inverse problems are singular, they require some form of regularization to lift this singularity (like a smoothness constraint). This produces an *estimate* of the solution, $\hat{x}$, that can be related to the *actual* solution through resolution kernels, R, namely:

$$\hat{x} = x \otimes R \tag{2}$$

These resolution kernels, that optimally would be Dirac distributions, depend on the input set, *i.e.* $\{i\}$: the extent of the available observables.

In practice, for inversion of the solar internal rotation, the inverse problem is

$$\delta\nu_{n,\ell,m} = \iint K_{n,\ell,m}(r,\theta) \, \Omega(r,\theta) \, dr \, d\theta \tag{3}$$

where $\delta\nu_{n,\ell,m}$ are the rotational splittings, (r,θ) the radius and co-latitude, $K_{n,\ell,m}$ the known rotational kernels and $\Omega(r,\theta)$ the solar rotation rate. The inversion input set is defined by the available modes (*i.e.*, $\{n,\ell,m\}$ or $\{n,\ell,a_i\}$), whose temporal changes will affect the resolution kernels, R. In order to decouple this and be assured that the changes seen in Ω can be attributed to changes in the sun, we chose to invert a *constant* input set. Such a constraint leads to mode attrition, since the exact same mode set is not fitted for each epoch.

The optimal latitudinal resolution can only be obtained by letting the inversion do it, and is thus achieved when using rotational splittings resulting from fitting individual frequencies rather than a polynomial expansion. While the GONG pipeline produces such individual frequencies, our *constant* input set constraint produces an unacceptable mode attrition. Figure 1 illustrates this for the GONG and MDI pipe-lines, as well as the attrition when fitting 182-day long time

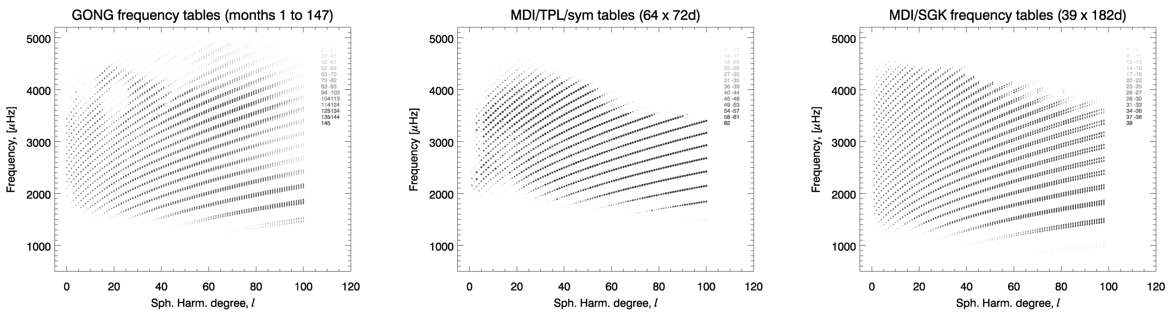

Figure 1. Mode attrition for GONG pipe-line (left) MDI improved pipe-line (middle) and MDI with my fitting (right) frequencies. Size and darkness are indicative the occurence of that mode.

series with my methodology. Also worth pointing out is that fitting using a polynomial expansion ignores the effect of mode visibility. This visibility –the observed amplitude of a mode– has a strong m/ℓ dependence, resulting in a similar dependence of the SNR, since the background noise is nearly independent of m. Using the polynomial expansion coefficients –during or after the fitting– effectively ignores the fact that some peaks are not significant, and is bound to inject *a priori* bias into the resulting splittings.

1.3. Other problems

Since individual modes are *rarely* resolved, inaccuracies in the leakage matrix will result in biased estimates. The closest (in frequency) spatial leak is only $\Delta\nu = \Delta\nu_{\delta m=2,\delta\ell=0}$ away. It is thus resolved if $\Delta\nu \gg \Gamma$, since $\Delta\nu \simeq 2 \times \frac{\Omega}{2\pi} \simeq 0.8\,\mu$Hz.

Error in the plate scale, image distortions and image orientation errors not accounted for will produce inaccurate leakage matrices. Fortunately, the new MDI spherical harmonic coefficients account for plate scale and image distortion. The distortion by differential rotation is, for intermediate degrees, a 1 to 6% effect, but $B_0 = B_0(t)$ is a 3 to 15% effect, while other geometric variations were found to be negligible.

Using a very long time series gives us the unique opportunity to check the leakage matrix, since some of the low order modes are resolved. We found that, despite developing an independent leakage matrix computation, there remains a clear but unaccounted for mismatch for the f-mode – implying a potential bias in those frequency estimates (see animations in the supplementary material.)

1.4. Inversion method

The inversion methodology used for the profiles presented here is a modified regularized least-squares method, described in [7]. Its main unique features are an iterative approach that leads to an *optimal* model grid, based on input set, while using a *non-uniform* model grid. The resulting trade-off curve and model grid are show in Fig. 2

2. Results

The mean solar rotation, derived from 12.6 years of MDI observations (the concatenation of 64×72-day long epochs) is shown in Fig. 3 as well as its associated uncertainty. Note how the unprecedented precision of the obtained rotation splittings, combined with an optimized RLS, allows us to infer a significant solution closer to the core and to the rotational axis than ever before. Note how the uncertainty increases as one gets closer to the rotation axis, and thus the dip around $(0.4, 63^{\circ})$ is barely a one-sigma feature.

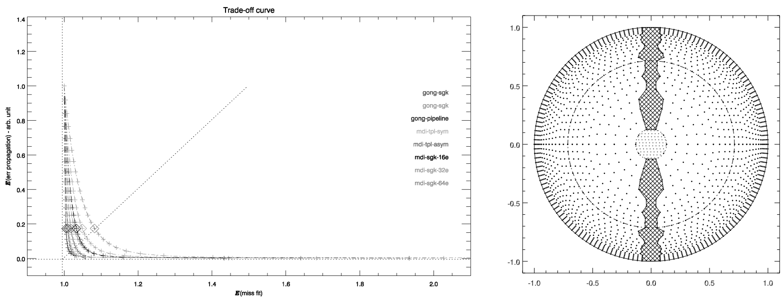

Figure 2. Trade-off curve (left) and model grid (right)

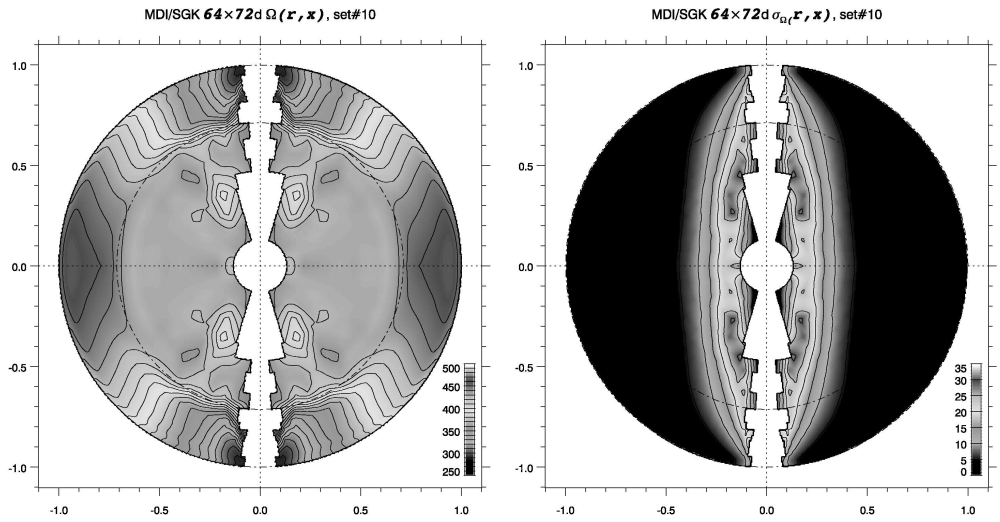

Figure 3. Rotation rate, as a function of depth and latitudes, inferred from 12.6 yr of MDI observations (left) and its associated uncertainty (right).

Figure 4 shows the rotation rate changes at the surface, using MDI and GONG splittings. The signature of the torsional oscillations is clearly visible for all cases, although the latitudinal and temporal resolution change with the data set used.

Figure 5 presents the rotation rate changes at three depths for a selection of data sets. The latitudinal-averaged change and its Fourier transform are shown, showing inconsistencies between data sets except at the surface (see also the animations in the supplementary material.)

3. Conclusions

Despite some 13 years of data, mode fitting remains an *issue*. The standard pipe-lines produce unacceptable mode attrition, while the leakage matrix does not yet appear to be accurate, as seen in the f-mode. This might be due to our ignorance of the MDI instrumental point spread function, or some residual error in our estimate of the horizontal component.

We have been able to derive what is likely to be the best estimate of the solar mean rotation rate, by fitting a very long time-series. It allows us to push it closer to the core and to the rotation axis. There is a barley significant dip at $(0.4, 63^{o})$, that is predominantly seen during the rising branch of the cycle. Inferring the rotation changes is *easy* at the surface, but remains challenging down to the base of the convection zone and difficult below it.

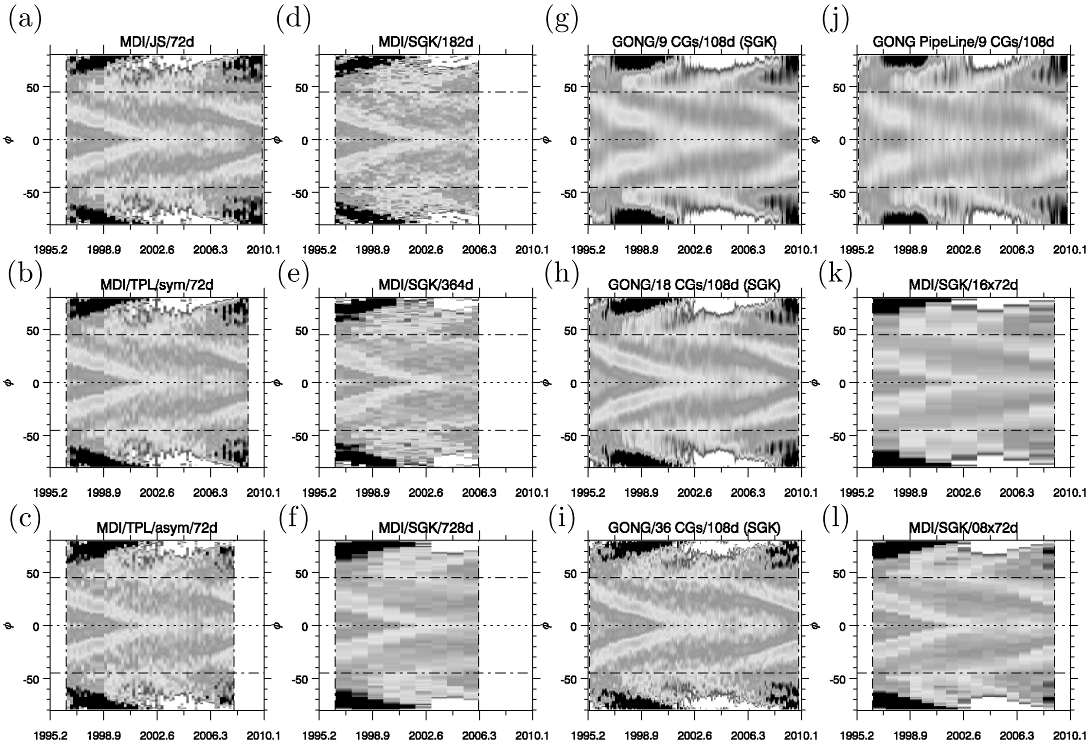

Figure 4. Changes in the rotation rate, as a function of time and latitudes and at the surface, inferred from the available data sets. (a) MDI standard pipe-line, (b) MDI improved pipe-line symmetric fit, (c) MDI improved pipe-line asymmetric fit, (d) MDI my fit to 182-day long series, (e) MDI my fit to 364-day long series, (f) MDI my fit to 728-day long series, (g) GONG pipe-line with my polynomial fit, 9 coefficients only, (h) GONG pipe-line with my polynomial fit, 18 coefficients, (i) GONG pipe-line with my polynomial fit, 36 coefficients, (j) GONG pipe-line with their polynomial fit, 9 coefficients, (k) MDI my fit to 1152-day long series (16×72), and (l) 576-day long series (8×72).

Acknowledgments

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(a)
(b)
(c)

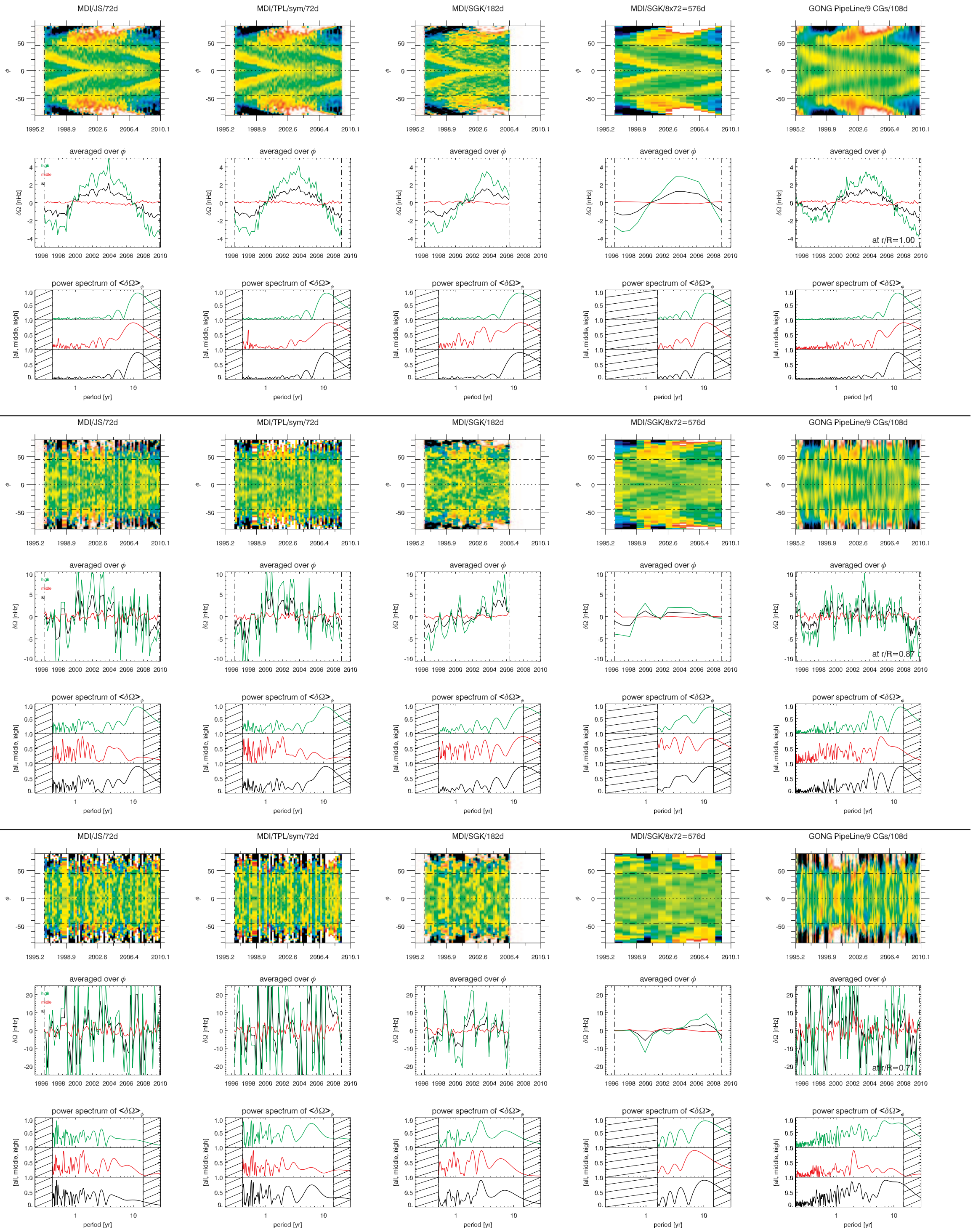

Figure 5. Changes in the rotation rate, for selected data sets (columns), at three depths: (a) $r/R = 1.$, (b) $r/R = 0.87$ and (c) $r/R = 0.71$, as well as the change averaged over latitude and their power spectra (all, middle and high latitudes are shown in black, red, and, green respectfully). The data sets used (left to right) are the MDI standard pipe-line, the MDI improved pipe-line asymmetric fit, my fit to MDI using 182-day long series, my fit to improved MDI 576-day (8x72) long series, and the GONG pipe-line with their polynomial fit, 9 coefficients.