Accurate Characterization of High-Degree Modes using MDI data

S.G. Korzennik[(1)], M.C. Rabello-Soares[(2,3)], J. Schou[(3)], and, T. Larson[(3)]
1: Harvard-Smithsonian Center for Astrophysics, Cambridge, MA
2: Physics Department, Universidade Federal de Minas Gerais, Minas Gerais, Brazil
3: Hansen Experimental Physics Laboratory, Stanford University, CA

Introduction

- Inclusion of high degree modes ($\ell \leq 1000$) has the potential to constrain the outer 4% of the solar interior
  - ▹ Stratification: Rabello-Soares et al., 2000,
  - ▹ Rotation: Korzennik & Eff-Darwich, 1999.
- Modes merge into ridges at high degrees:
  - ▹ $\ell > 300$ for f-modes, $\ell > 200$ for low order p-modes.
- Ridge can be fitted, but:
  - ▹ the resulting characteristics ($\tilde{\nu}$, $\tilde{\Gamma}$, $\tilde{A}$, $\tilde{\alpha}$) are **not** the underlying mode characteristics.
- Fit ridges and correct the ridge characteristics, using a precise and sophisticated ridge modeling procedure.

Data Analysis and Ridge Fitting

- Analyzed MDI's *Dynamics 2001*: 90 days of full disk images, with a 97% fill factor, after gap filling (95.8% prior).
  - ▹ Performed *best* possible spatial decomposition, including both components of the image distortion (Korzennik et al., 2004).

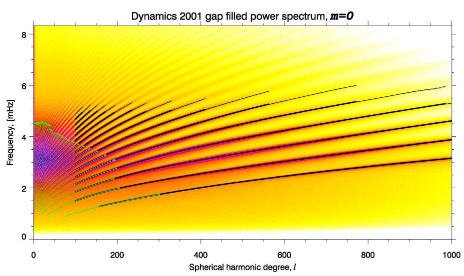

Fig. 1: *Gap-filled Dynamics 2001 power spectrum, displayed on a logarithmic scale, for the zonal modes ($m = 0$).*

- ▹ Used a 61 term sine multi-taper spectral estimator:
  - (a) ridges are wide ⇒ no need for high spectral resolution,
  - (b) mode widening ⇒ modes blend into ridges at lower degrees.
- Fitted power spectra for $100 \leq \ell \leq 1000$
  - ▹ Fit a sum of modified (asymmetric) Lorentzian,
  - ▹ use a multi-step least-squares minimization,
  - ▹ fitted all ℓ and all m,
  - ▹ contrast limit on n (1:500 ratio & reject highest n).

> - Resulting in 5.2×10^6 singlets (n, ℓ, m), or 5,780 multiplets (n, ℓ).
> - Tables are available at
>   `https://www.cfa.harvard.edu/~sylvain/research/tables/HiL/`

Ridge Modeling

- Developed a precise and sophisticated model of the ridge (Korzennik et al., 2004, Rabello-Soares et al., 2008, Korzennik et al, 2012 *submitted*).
  - ▹ Refined to adjust iteratively most input parameters to produce a model that matches the observations.
  - ▹ Perturbed all input parameters to derive model sensitivity and precision of the correction scheme.

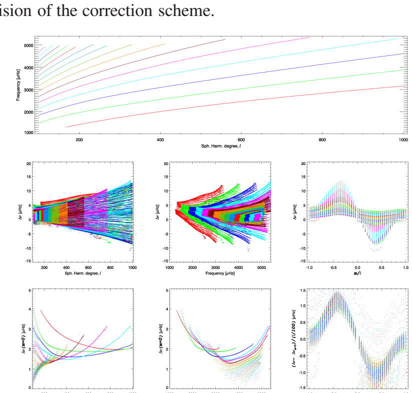

Fig. 2: *Top panel: coverage in the $\ell - \nu$ of the fitting and modeling. Middle panel: frequency offsets, Δ^ν, between mode and ridge frequency as a function of degree, ℓ (left), frequency, ν (middle), and relative azimuthal order, m/ℓ (right). Bottom panels: frequency offsets, Δ^ν, for $m = 0$ only, plotted as a function of degree (left) or frequency (middle), and frequency offsets with respect to the corresponding zonal value and divided by $\ell/100$, also plotted versus relative azimuthal order, m/ℓ (right).*

Results

Frequencies & Frequency Splittings

- Mode frequency is estimated as follows:

$$\nu_{n,\ell,m} = \tilde{\nu}_{n,\ell,m} - \Delta^\nu_{n,\ell,m} \qquad (1)$$

where $\tilde{\nu}$ is the ridge frequency and Δ^ν the ridge to mode offset:

$$\Delta^\nu_{n,\ell,m} = \tilde{\nu}^M_{n,\ell,m} - \nu^M_{n,\ell,m} \qquad (2)$$

ν^M being the mode frequency and $\tilde{\nu}^M$ the resulting ridge frequency in our model.

- Overlap between mode and ridge fitting allows validation.

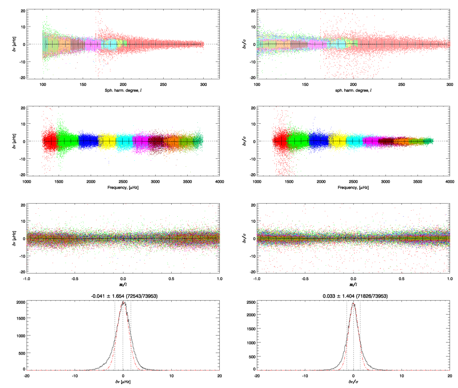

Fig. 3: *Comparison of estimate of mode singlet frequencies derived from ridge fitting to actual mode frequency computed by fitting resolved modes at intermediate degrees.*

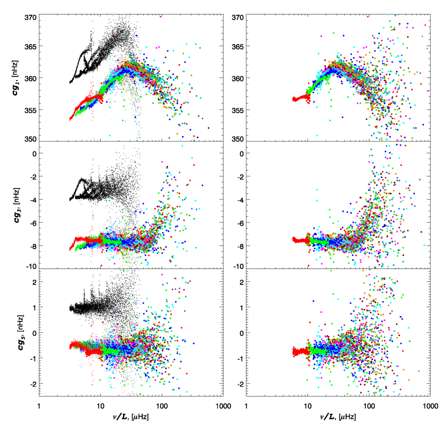

Fig. 4: *Left panels: comparison of estimates of mode frequencies splittings, parametrized as Clebsch-Gordan coefficients, plotted as a function of the ratio ν/L, a proxy for the mode inner turning point, derived from ridge fitting after correcting for the ridge to mode offset (small dots) to frequency splitting coefficients computed by fitting resolved modes (large dots). Right panels: same Clebsch-Gordan coefficients computed by fitting resolved modes, only (same large dots).*

FWHM

- Ridge width is a function of m.
- Zonal ridge width is not a good estimate of the mode width.
- Correction scheme use a Gaussian widening model:

$$\tilde{\Gamma}^2_{n,\ell} = (\Gamma^E_{n,\ell})^2 + \mathcal{W}^2_{n,\ell}$$

where $(\Gamma^E_{n,\ell})^2 = \Gamma^2_{n,\ell} + W^2_{N,T}$ and $W_{N,T}$ is the spectral resolution.

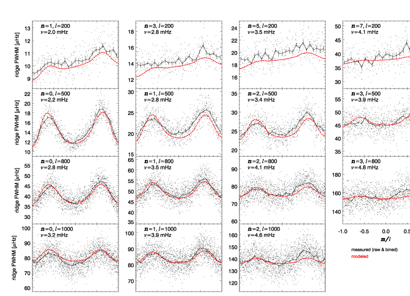

Fig. 5: *Comparison of ridge FWHM for a selection of multiplets, as a function of the ratio m/ℓ. The red curves correspond to our best model of the ridge width.*

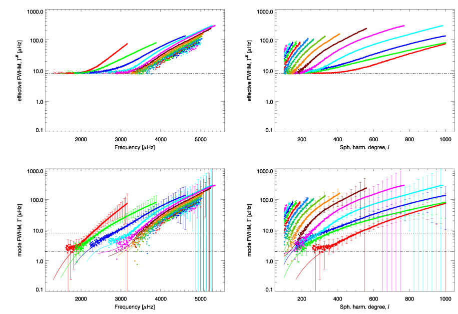

Fig. 6: *Top panels: effective FWHM, estimated from the ridge width, using the widening model, as a function of frequency or degree (left and right panels, respectively). Bottom panels: mode FWHM, estimated from the effective width, and their associated uncertainty, as a function of frequency or degree (left and right panels, respectively).*

Amplitude

- Ridge amplitude is a function of m.
- Model ridge amplitude does not match observations
  - ▹ indicative of poor knowledge of PSF

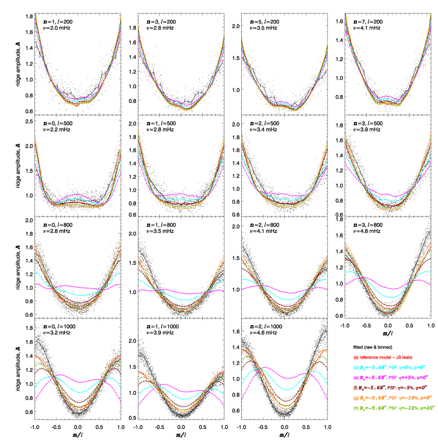

Fig. 7: *Comparison of ridge amplitude, as a function of the ratio m/ℓ. The dots correspond to individual ridge power amplitude measurements, and the black curves the corresponding binned values. The colored curves correspond to different models of the ridge amplitude, resulting from different models computed for different leakage matrices.*

Conclusions

> - Derived most accurate high-ℓ mode characteristics using MDI
> - Limiting factor is poor knowledge of MDI's PSF
> - Co-eval HMI & MDI data
>   - ▹ should allow us to better characterize MDI's PSF
> - Method should be
>   - ▹ extended to more MDI *Dynamics* segments,
>   - ▹ ported to HMI.

References

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Acknowledgments

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