More Information! Conditional Probability and Inverse Methods

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Harvard Atmospheres Journal Club
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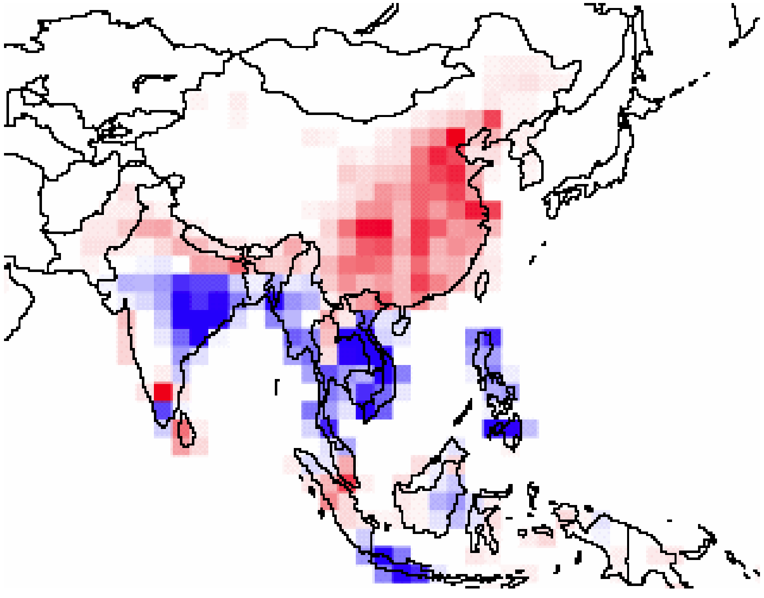


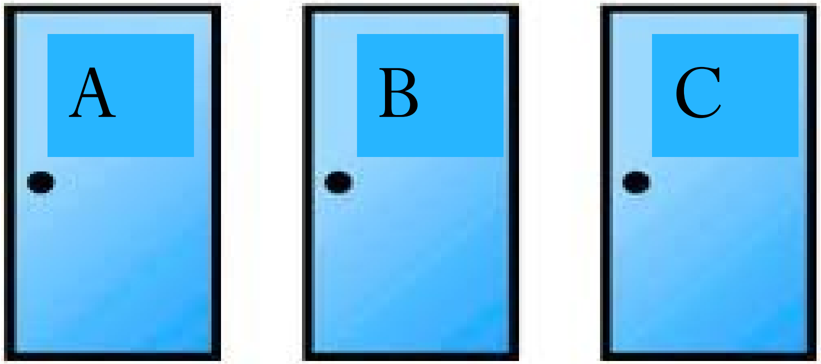

Bayesian Analysis – not very intuitive!

- Humans are bad at combining a priori information with new observations
- Conditional Probability examples
- 2-box inversion to constrain emissions
- Arellano et al paper on application of these methods

The Monty Hall Problem

You're given the choice of three doors:

Behind one door is a car; behind the others, goats. You pick a door, say door A [but the door is not opened], and the host, who knows what's behind the doors, opens another door, say door B, which has a goat. He then says to you, "Do you want to pick door C?" Is it to your advantage to switch your choice?



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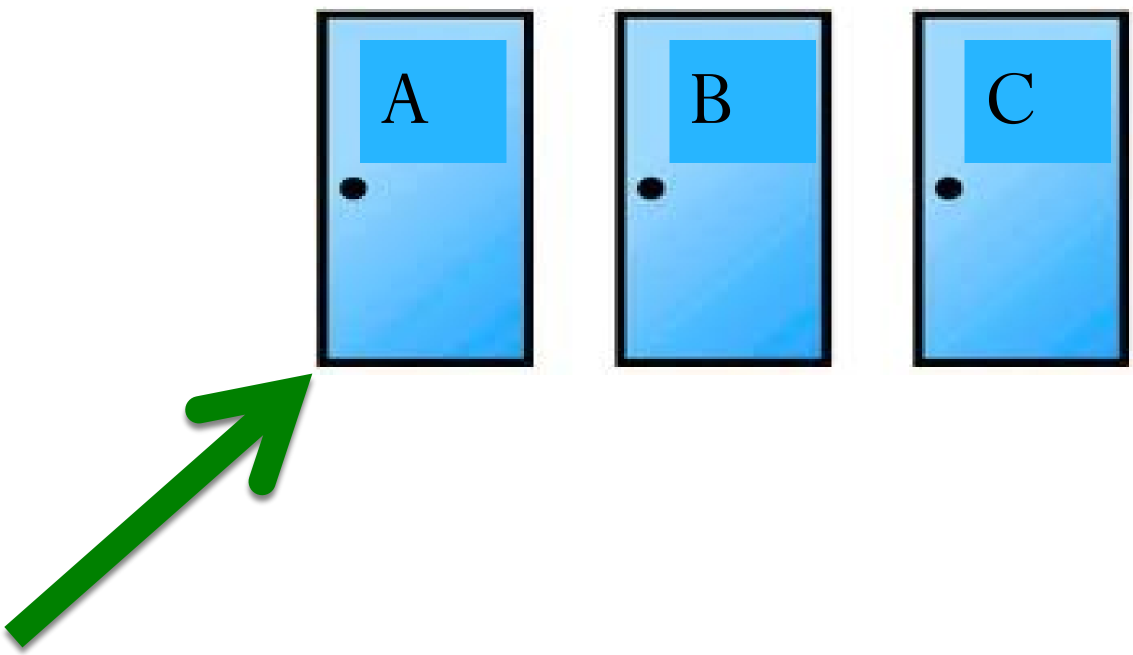

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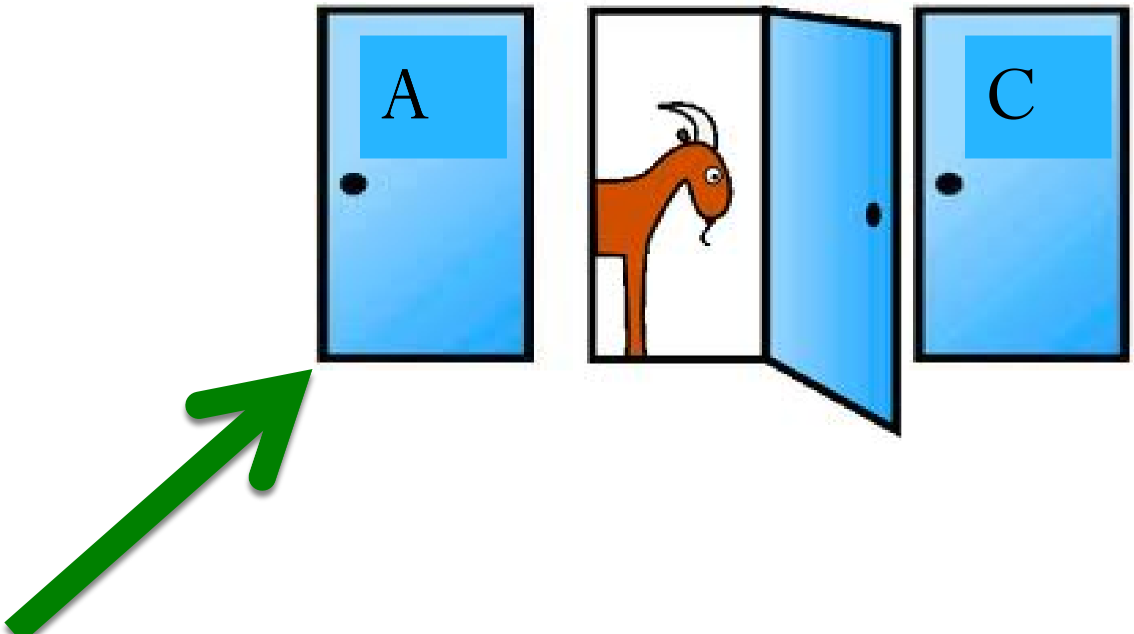

Bayes' Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$P(\text{car is behind A}|\text{door B opened}) = \frac{P(\text{door B opened}|\text{car is behind A})P(\text{car is behind A})}{P(\text{door B opened})}$$

$$P(\text{door B opened}|\text{car is behind A}) = 0.5$$

$$P(\text{car is behind A}|\text{door B opened}) = \frac{0.5 * 0.33}{0.5} = 0.33$$

Why is door C different?

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Why is door C different?

$$P(\text{door B opened}|\text{car is behind C}) = 1$$

Updating an Initial Guess

- A cab was involved in a hit-and-run accident at night
  - 85% of the cabs in the city are Green and 15% are Blue
  - A witness identified the cab as Blue. Tests show that this witness correctly identifies the color of the cab 80% of the time

What is the probability the cab was involved in the accident was Blue?

Set-up from Thinking, Fast and Slow (Kahneman)

Combine observation with a priori information

$$P(\text{car is Blue} \mid \text{witnessed Blue}) = \frac{P(\text{witnessed Blue} \mid \text{car is Blue})P(\text{car is blue})}{P(\text{witnessed Blue})}$$

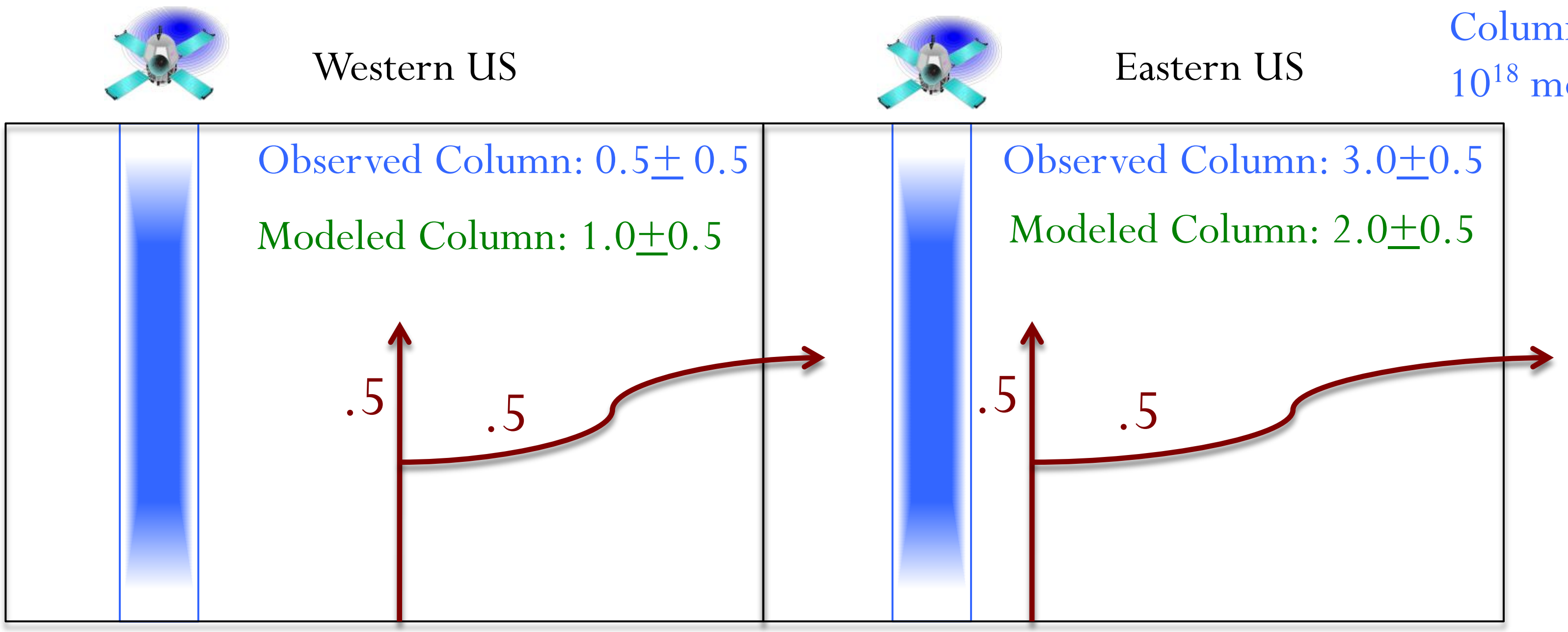
$$P(\text{car is blue}) = 0.15$$

$$P(\text{witnessed Blue} \mid \text{car is Blue}) = 0.8$$

$$P(\text{witnessed Blue}) = 0.8 * 0.15 + 0.2 * 0.85 = 0.29$$

$$P(\text{car is Blue} \mid \text{witnessed Blue}) = \frac{0.8 * 0.15}{0.29} = 0.41$$

Constraining Emissions



Column Units: 10^{18} molec/cm^2

Emissions = 1 ± 1

Emissions = 1 ± 1

Emissions units 100 Tg/yr

Modeled Column ~ Western Emissions

Modeled Column ~ Western Emissions + Eastern Emissions

Bayesian Inversion

$$P(X(\text{emissions})\,|\,Y(\text{observations})) = \frac{P(Y\,|\,X)P(X)}{P(Y)}$$

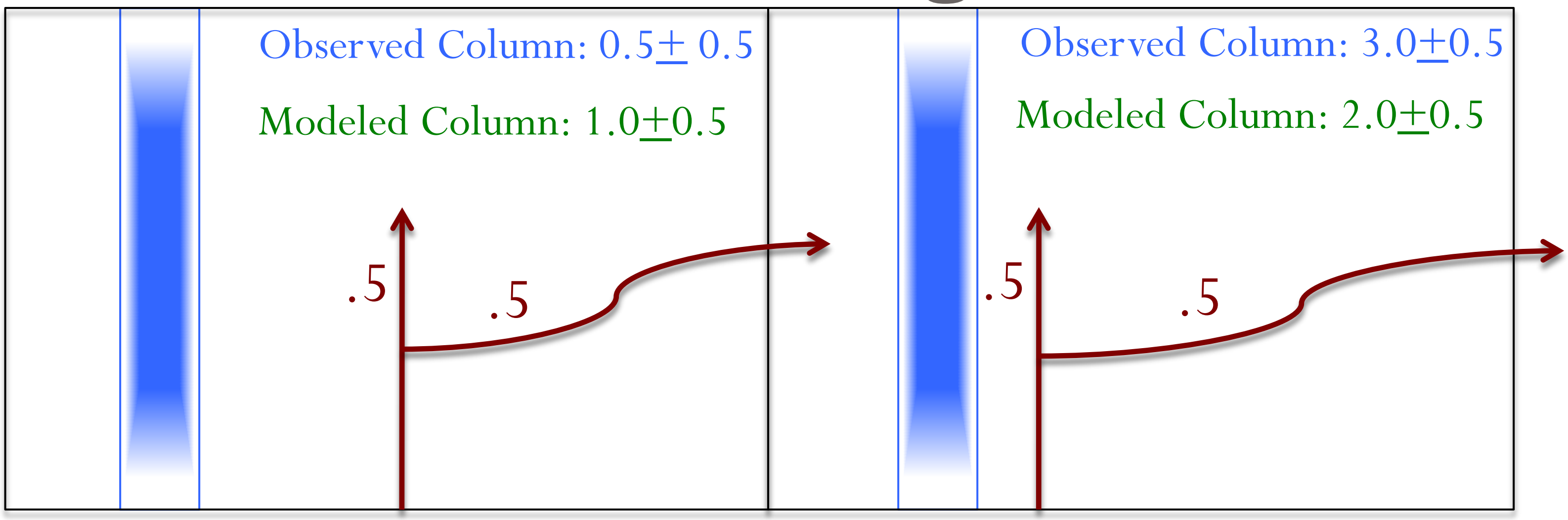
$$\mathbf{K}^{\mathrm{T}}\mathbf{S}_{\varepsilon}^{-1}(\mathbf{K}\hat{\mathbf{x}}-\mathbf{y})+\mathbf{S}_{a}^{-1}(\hat{\mathbf{x}}-\mathbf{x}_{\mathbf{a}})=0$$

$$\mathbf{y}=\mathbf{K}\mathbf{x}=\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix}=\begin{pmatrix}x_1\\ x_1+x_2\end{pmatrix}$$

$$\hat{\mathbf{x}}=\mathbf{x}_{\mathbf{a}}+\mathbf{G}(\mathbf{y}-\mathbf{K}\hat{\mathbf{x}}_{a})$$

$$\mathbf{G}=\mathbf{S}_{a}\mathbf{K}^{\mathrm{T}}\left(\mathbf{K}\;\mathbf{S}_{a}\mathbf{K}^{\mathrm{T}}+\mathbf{S}_{\varepsilon}\right)^{-1}$$

Constraining Emissions



Emissions = 1 ± 1 Emissions = 1 ± 1

$$\mathbf{S}_a = \begin{pmatrix} 1^2 & 0 \\ 0 & 1^2 \end{pmatrix}$$

$$\mathbf{S}_\varepsilon = \begin{pmatrix} 2*0.5^2 & 0 \\ 0 & 2*0.5^2 \end{pmatrix}$$

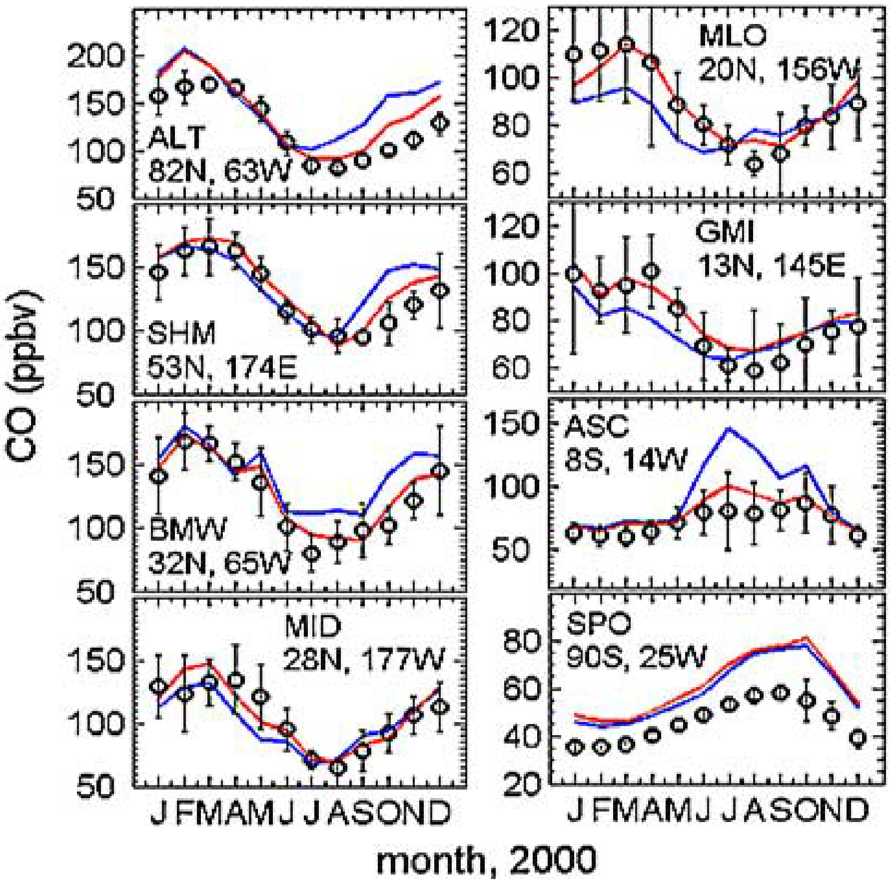
$$\mathbf{G} = \mathbf{S}_a\mathbf{K}^T\left(\mathbf{K}\ \mathbf{S}_a\mathbf{K}^T + \mathbf{S}_\varepsilon\right)^{-1} = \begin{pmatrix} .54 & .18 \\ -.36 & .54 \end{pmatrix}$$

$$(\mathbf{y} - \mathbf{K}\hat{\mathbf{x}}_a) = \begin{pmatrix} .5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -.5 \\ 1 \end{pmatrix}$$

$$\hat{\mathbf{x}} = \mathbf{x}_\mathbf{a} + \mathbf{G}(\mathbf{y} - \mathbf{K}\hat{\mathbf{x}}_a) = \begin{pmatrix} .91 \\ 1.72 \end{pmatrix}$$

Estimating CO Sources

- Why CO?
  - Strong link between free troposphere and regional emissions
  - Relatively easy to observe from space
- Important to compare results using optimized sources to independent data sets
- CO can be used to optimize emissions/concentrations of other species
  - CO_2, ozone



[Arellano et al 2004]