Simulation and Analysis of Magnetic Reconnection in a Laboratory Plasma Astrophysics Experiment

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Thesis Defense

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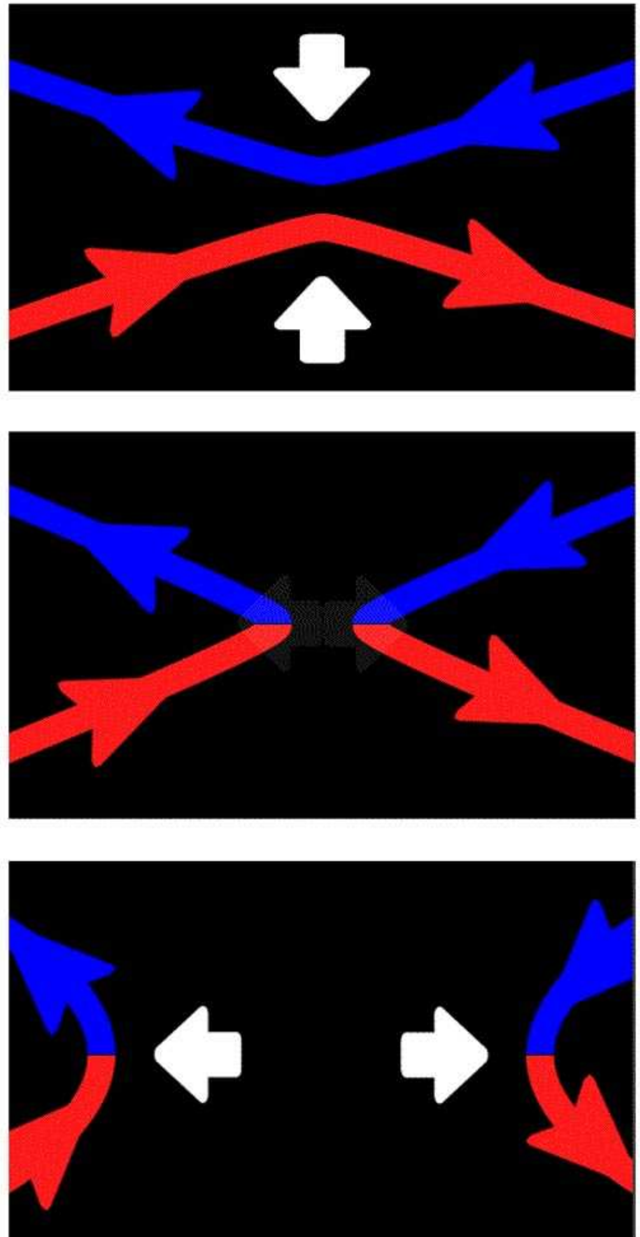
Outline

- Introduction to magnetic reconnection
- Resistive MHD and two-fluid simulations of a laboratory plasma astrophysics experiment
- Magnetic reconnection with asymmetry in the outflow direction
- Conclusions, astrophysical significance, and future work

Introduction

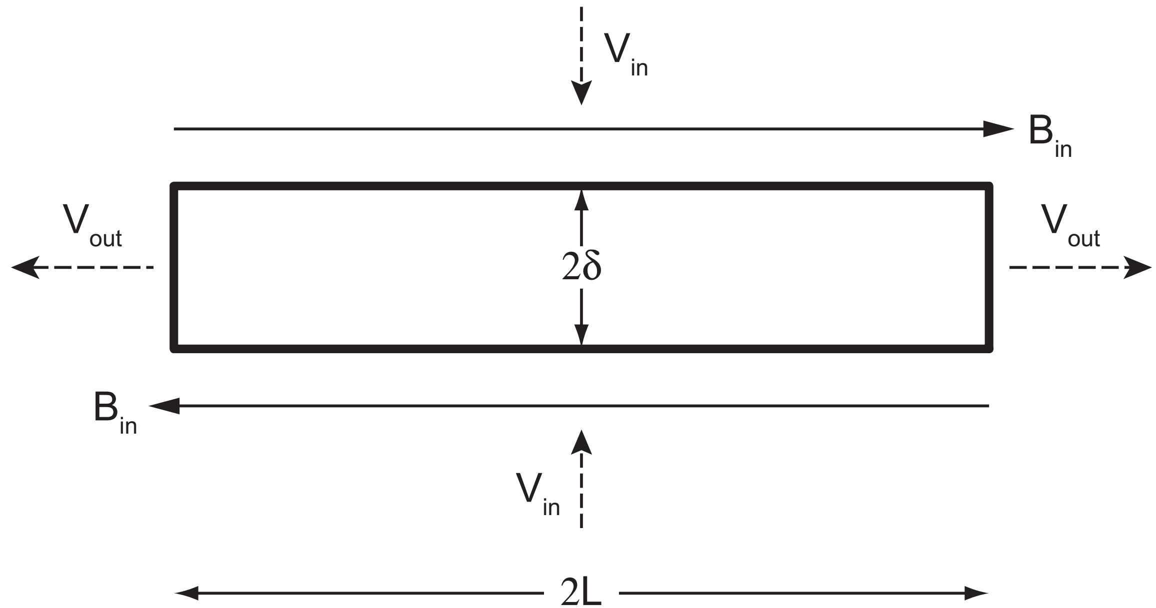
- In ideal magnetohydrodynamics (MHD), the magnetic field is said to be “frozen-in” to the plasma
  - The magnetic field attached to a parcel of plasma moves along with that parcel
- Real plasmas, however, allow some magnetic field slippage
- *Magnetic reconnection* is the process where magnetic field lines are broken and rejoined in a highly conducting plasma
- This process occurs in the solar atmosphere, the solar wind, the Earth’s magnetosphere, cometary magnetotails, the ISM, astrophysical disks, turbulence, and laboratory plasmas
- Direct or *in situ* measurements of reconnection are not possible for most astrophysical situations, so it is essential to find connections with space and laboratory plasmas

Magnetic reconnection involves a change in magnetic topology



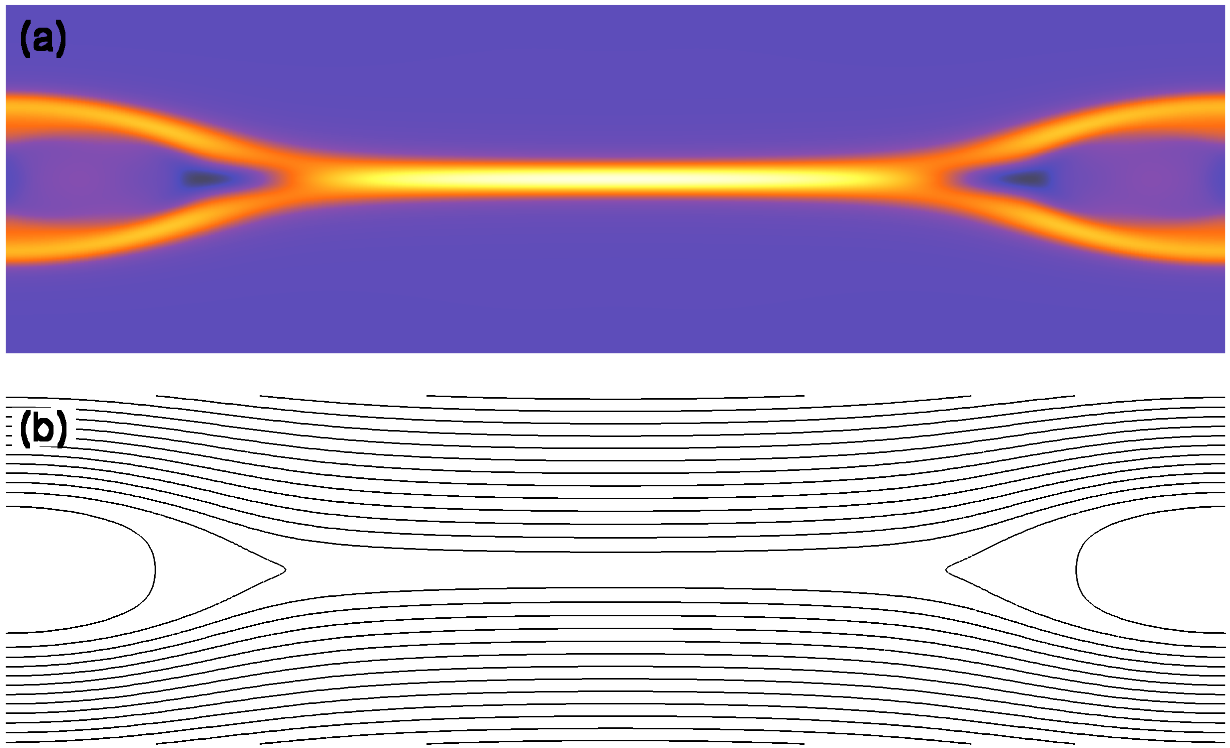
Credits: Center for Visual computing, Univ. of California Riverside

The Sweet-Parker model provides a basis for understanding resistive reconnection



- From conservation of mass: $V_{in} L \sim V_{out} \delta$
- From conservation of energy: $V_{in} L \left(\frac{B_{in}^2}{2\mu_0} \right) \sim V_{out} \delta \left(\frac{\rho V_{out}^2}{2} \right)$
- The outflow velocity scales as the Alfvén speed: $V_{out} \sim V_A \equiv \frac{B_{in}}{\sqrt{\mu_0 \rho}}$
- The ideal electric field outside the layer matches the resistive electric field within the layer: $V_{in} B_{in} \sim \eta J \implies V_{in} \sim \frac{\eta}{\mu_0 \delta}$ (using $J \sim \frac{B_{in}}{\mu_0 \delta}$)
- The reconnection rate scales as $\frac{V_{in}}{V_A} \sim \frac{1}{S^{1/2}}$, where the Lundquist number is given by $S \equiv \frac{\mu_0 L V_A}{\eta} = \frac{\tau_{\mathrm{res}}}{\tau_{\mathrm{Alf}}}$

Resistive MHD simulations show the development of extended diffusion regions



- Shown are (a) out-of-plane current density and (b) magnetic flux contours for a characteristic NIMROD simulation of resistive MHD reconnection starting from a Harris sheet equilibrium

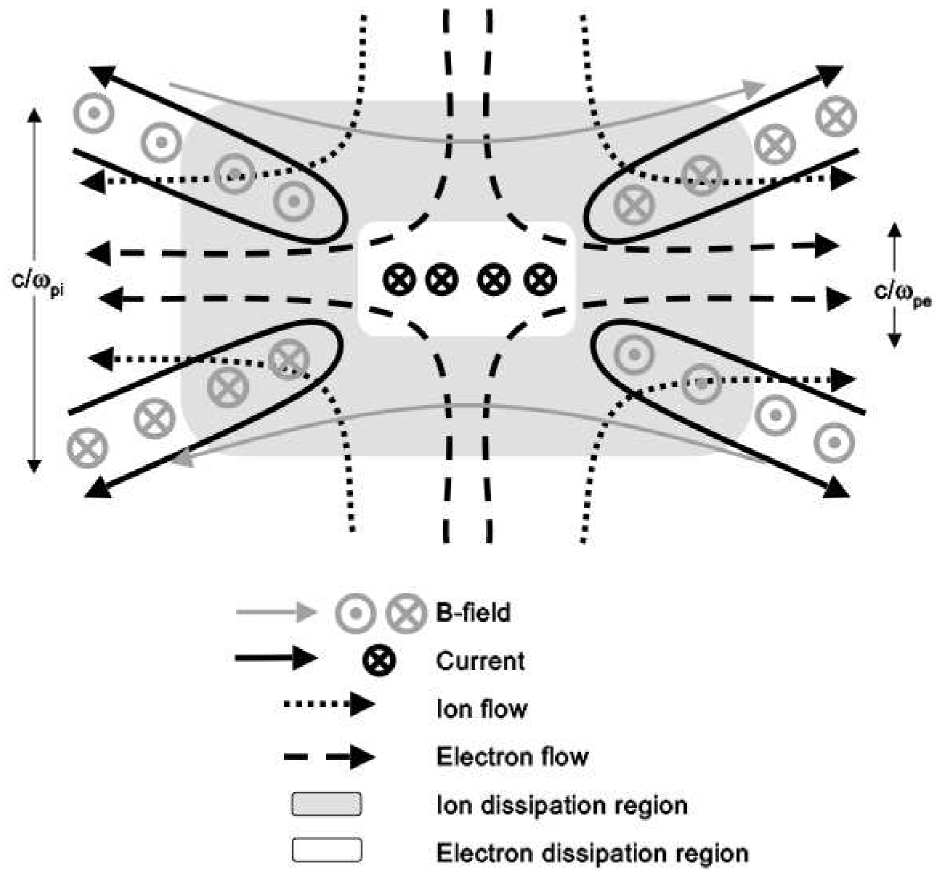
The generalized Ohm's law includes non-MHD terms that facilitate fast reconnection

- The generalized Ohm's law is given by

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} + \frac{\mathbf{J} \times \mathbf{B}}{ne} - \frac{\nabla \cdot \mathsf{\mathbf{P}}_e}{ne} + \frac{m_e}{ne^2} \left[\frac{\partial \mathbf{J}}{\partial t} + \nabla \cdot \left(\mathbf{JV} + \mathbf{VJ} - \frac{\mathbf{JJ}}{ne} \right) \right] \quad (1)$$

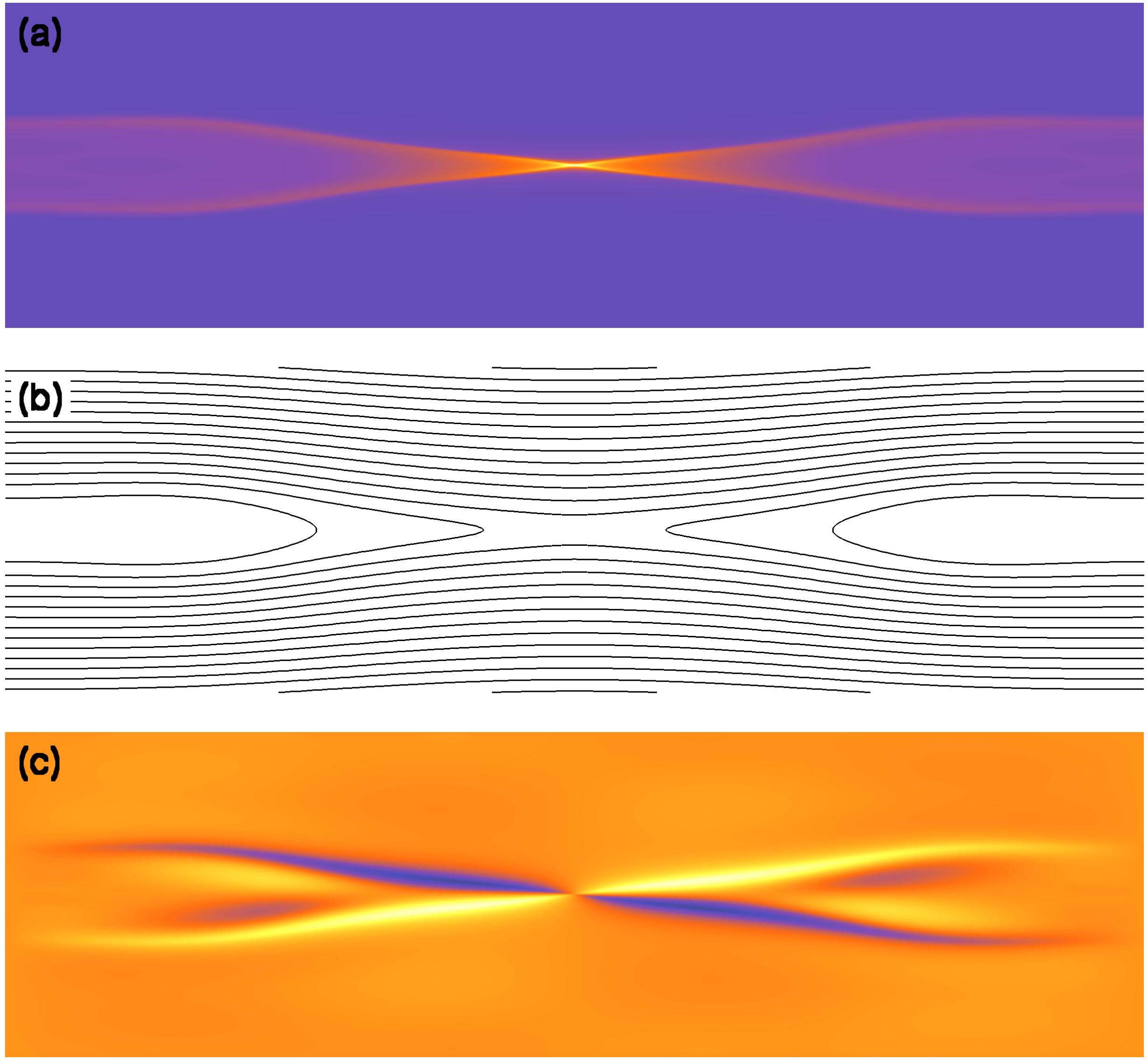
  - $\mathbf{V} \times \mathbf{B}$ represents the ideal electric field
  - $\eta \mathbf{J}$ represents resistive diffusion (with $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$)
  - $\frac{\mathbf{J} \times \mathbf{B}}{ne}$ is the Hall term which acts to freeze the magnetic field into the electron fluid
  - $\frac{\nabla \cdot \mathsf{\mathbf{P}}_e}{ne}$ is the electron pressure gradient term
  - The term including the brackets represents electron inertia
- Magnetic topology can be changed by resistivity, electron inertia, and off-diagonal (nongyrotropic) components of the electron pressure tensor

Two-fluid effects alter the structure of the diffusion region



- On scales shorter than the ion inertial length c/ω_{pi} the magnetic field is carried by the electrons rather the bulk plasma (figure from Drake & Shay 2006)
- The electrons pull the magnetic field into a much smaller diffusion region, enabling fast reconnection
- The out-of-plane electron flow pulls in-plane magnetic field lines in the out-of-plane direction, resulting in an out-of-plane quadrupole magnetic field

Two-fluid simulations show the development of X-point geometry

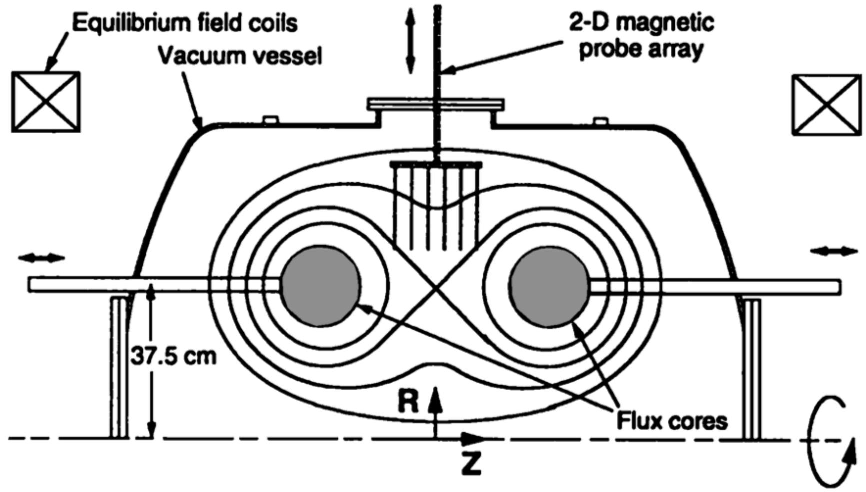


- Shown are (a) out-of-plane current density, (b) magnetic flux contours, and (c) out-of-plane magnetic field for a characteristic NIMROD simulation of two-fluid reconnection starting from a Harris sheet equilibrium

Magnetic reconnection is an inherently multiscale process

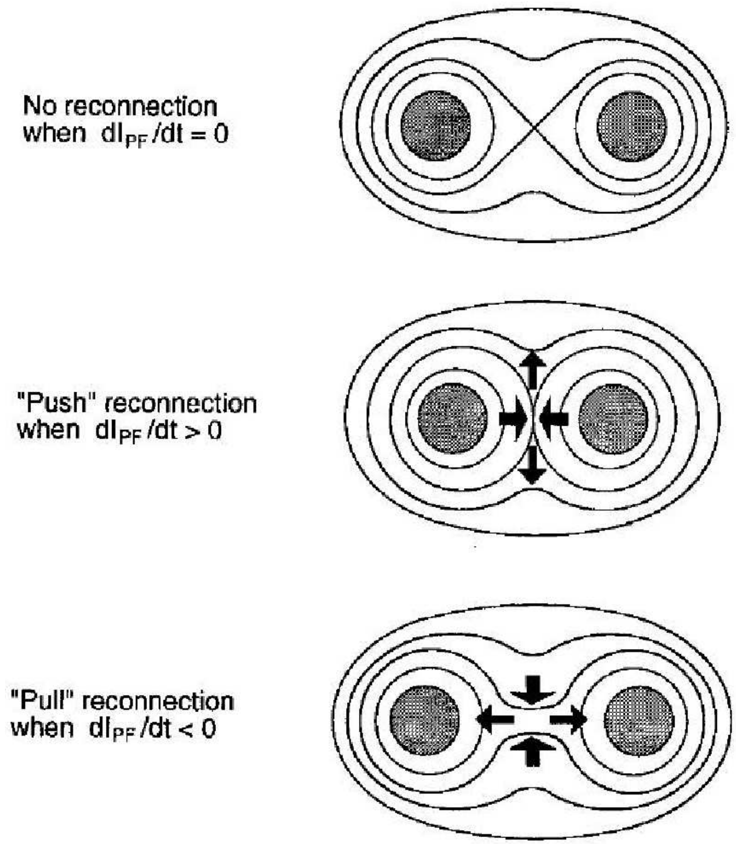
- Reconnection at small scales feeds back on dynamics on global scales
- Global scales then help regulate what happens at small scales
- Two-fluid effects can facilitate fast reconnection, but are not commonly included in large-scale simulations
- Large-scale effects such as asymmetry can also affect how the reconnection process occurs and where the energy goes
- Objectives of this computational and analytical study include
  - Extended MHD simulations of reconnection in an experimental geometry
  - A model describing reconnection with asymmetry in the outflow direction

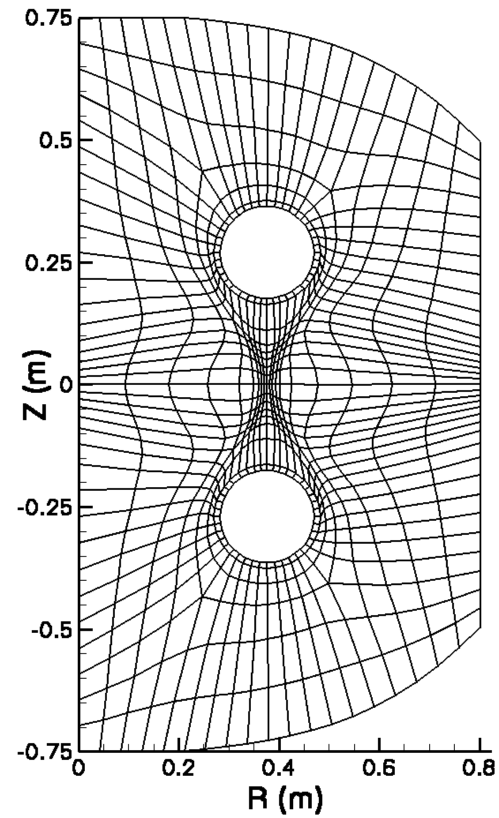
The Magnetic Reconnection Experiment



- The Magnetic Reconnection Experiment (MRX) at the Princeton Plasma Physics Laboratory is designed to study controlled axisymmetric magnetic reconnection
- Plasma parameters: $T \sim 5 - 20$ eV, $B \sim 200 - 500$ G, $S \equiv \mu_0 L V_A / \eta \sim 250 - 2500$, and $n \sim 0.1 - 1 \times 10^{14}$ cm^{-3}
- MRX is a good candidate for computational study because, unlike astrophysical plasmas, spatial scale separation is moderate and plasma parameters are numerically tractable

MRX Experimental Setup





- *Left:* By changing the currents in the flux cores, two distinct modes of reconnection can be induced in MRX (Yamada et al. 1997). These simulations of MRX investigate both of these modes of operation (Murphy & Sovinec 2008).
- *Right:* A sample finite element grid used for simulations of two-fluid pull reconnection in MRX.

NIMROD's Non-Ideal Hall MHD Model

The NIMROD code (Sovinec et al. 2004) solves the equations of extended MHD cast in a single fluid form. This model is used in simulations of MRX to study the interplay between local and global effects in reconnection.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{ne} - \frac{\nabla p_e}{ne} \right) + \kappa_{divb} \nabla \nabla \cdot \mathbf{B} \qquad (2)$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B} \qquad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (4)$$

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \rho \nu \nabla \mathbf{V} \qquad (5)$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n \qquad (6)$$

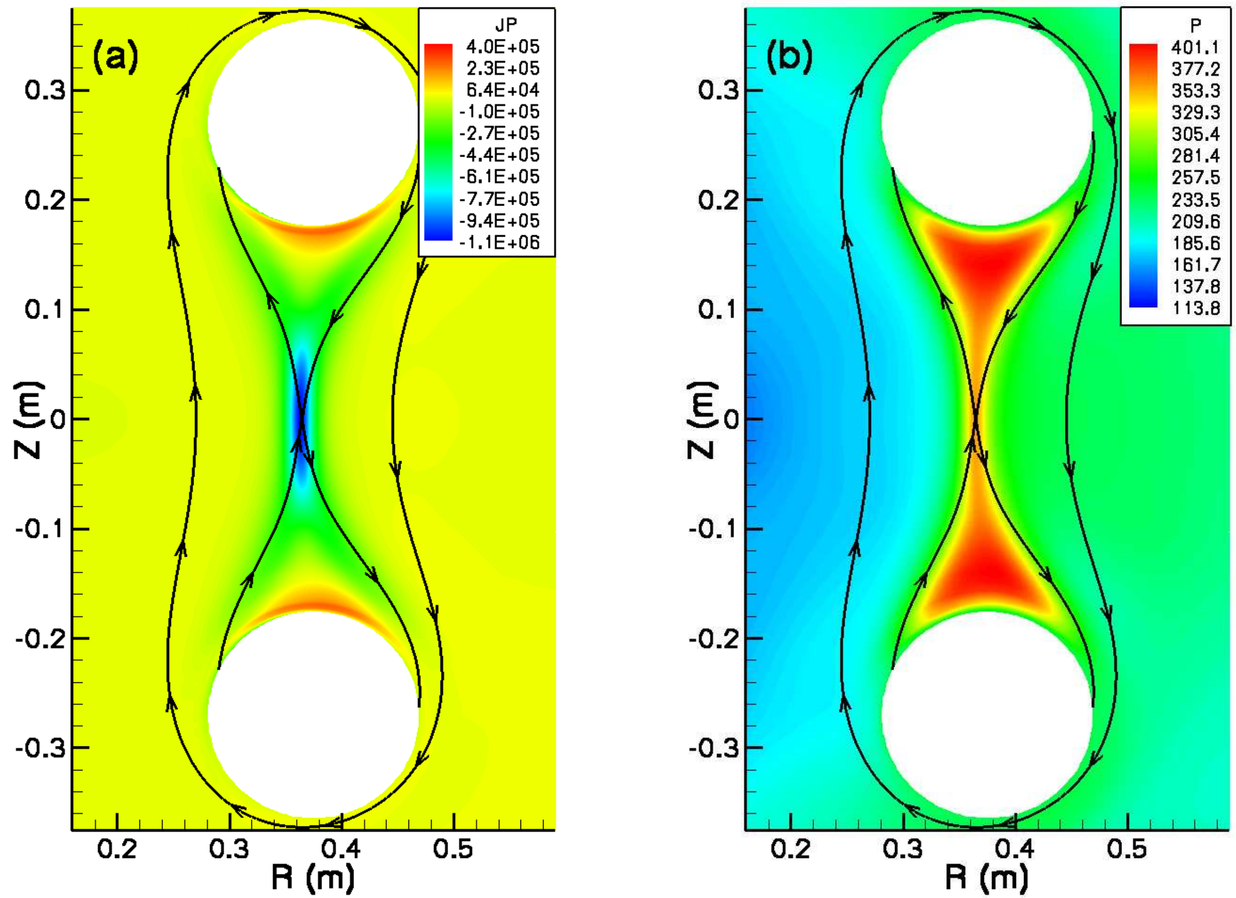
$$\frac{n}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q \qquad (7)$$

- Two-fluid effects are included via the Hall and electron pressure gradient terms in the generalized Ohm's law (blue). The terms in red are included for numerical purposes.

Simulating MRX provides a chance to study global effects on reconnection

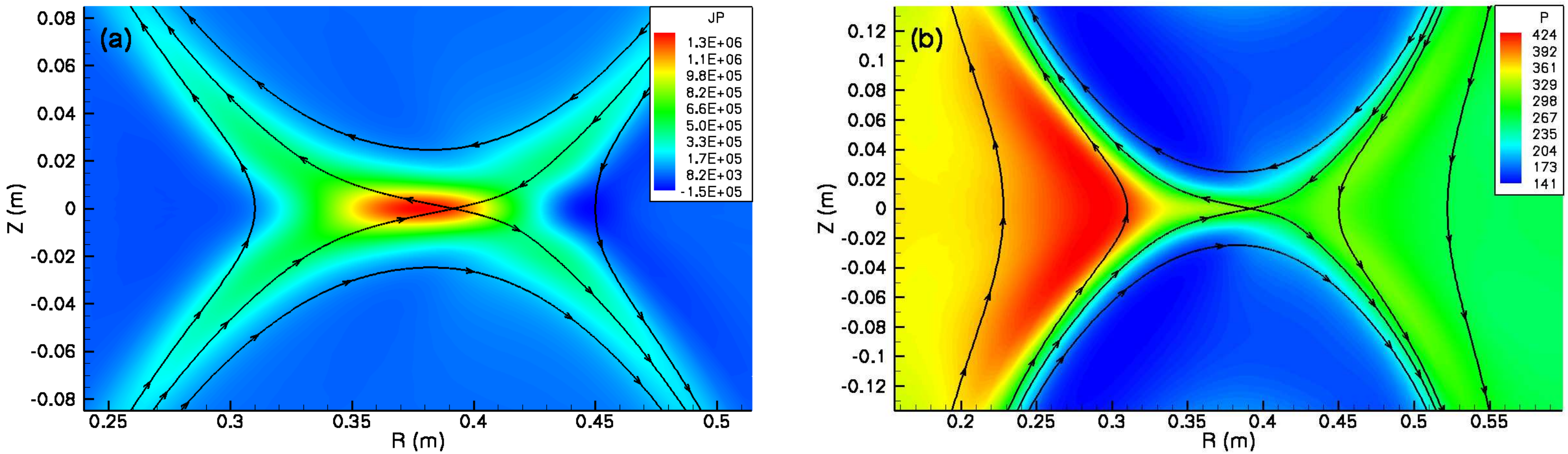
- Simulations of MRX are axisymmetric and have similar physical parameters to the experiment ($S \sim 250 - 500$, $n \sim 5 \times 10^{19}$ m^{-3}, and $T_i = T_e = 15$ eV with isotropic heat conduction and $Pm = 1$)
- The initial magnetic field is set up through coils at the centers of the flux cores, and outside the domain to set up a vertical field
- An applied electric field on the flux core surfaces drives reconnection, and if desired can also induce azimuthal magnetic field
- Density and temperature are kept constant on the flux core surface, and the velocity is set by the $E \times B$ drift
- Perfectly conducting no-slip boundary conditions
- Simulations are performed for both cylindrical and linear geometry

Simulations of pull reconnection show global effects are important



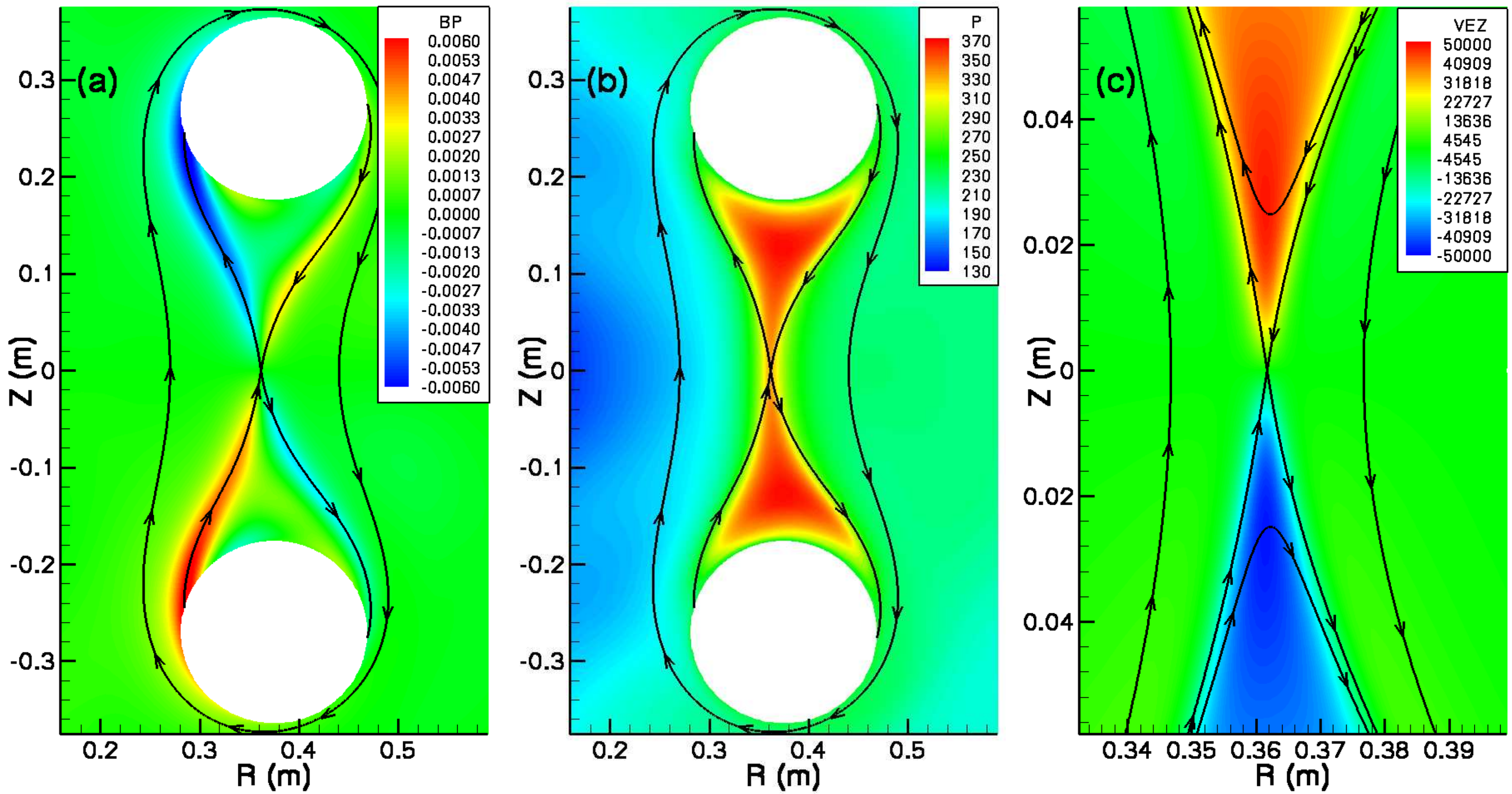
- The reconnection layer length is limited by the flux core separation and the outflow is greatly slowed due to high downstream pressure (resistive MHD simulation)
- Outflow is confined between the flux cores and separatrix
- Density is quickly depleted on the inboard side due to the low available volume
- Higher pressure on the outboard side of the reconnection layer results in a radially inward motion of the X-point

Global pressure differences are also important in push reconnection



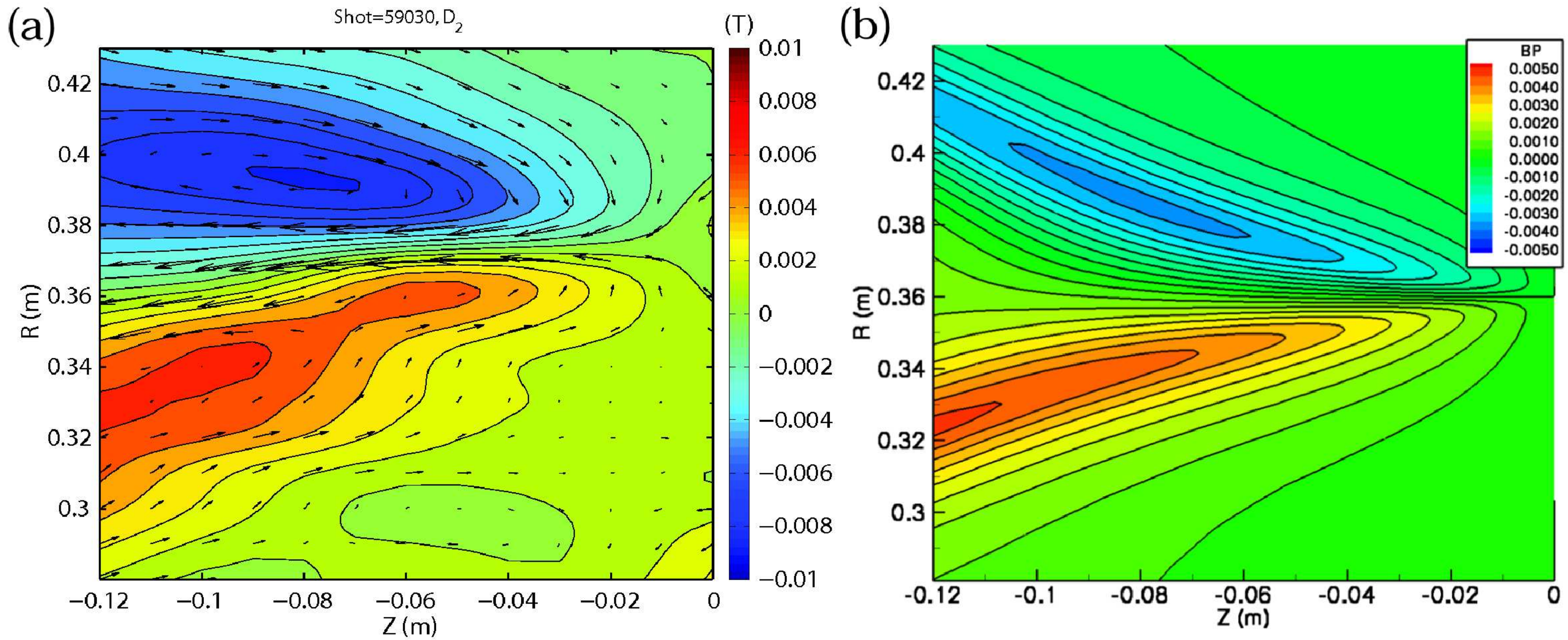
- Due to the same volume effects as in pull reconnection, the inboard downstream region quickly develops high pressure, pushing the X-point to larger radii (resistive MHD simulation)
- The position of the X-point near the outboard side of the reconnection layer allows a stronger inward-directed tension force to overcome the steeper pressure gradient
- This is an example of reconnection with asymmetry in the outflow direction (see also discussion of asymmetry due to the Hall effect during counter-helicity merging in Section 2.6)

The quadrupole B-field is induced in two-fluid simulations of MRX



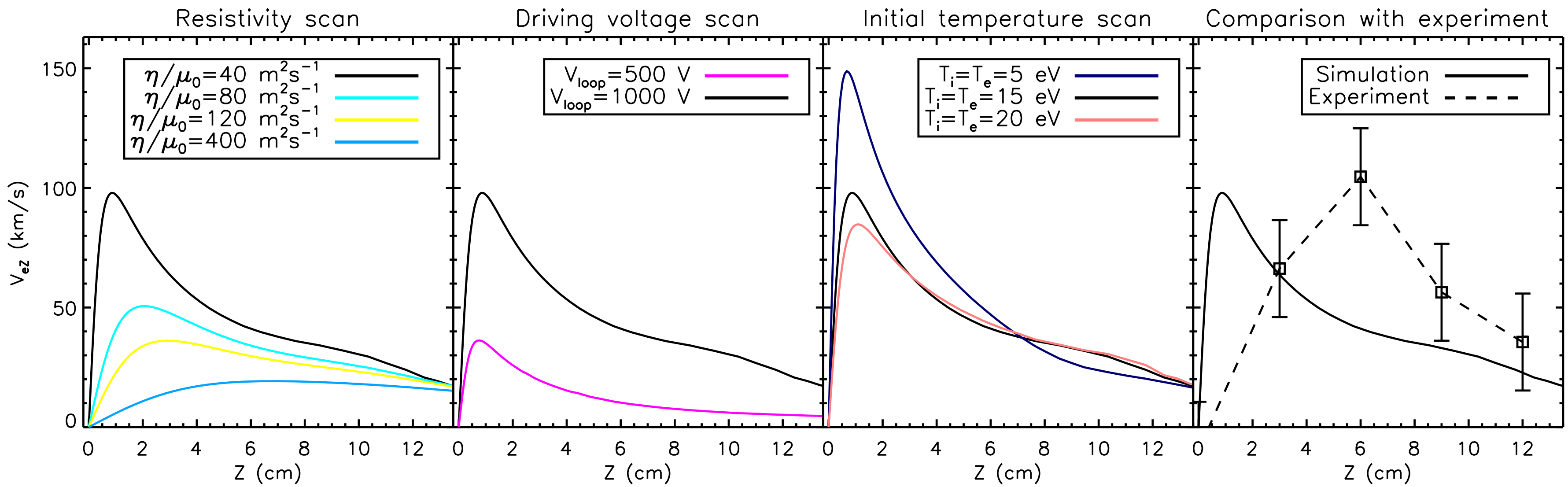
- From left to right are contours of azimuthal magnetic field, plasma pressure, and the vertical (outflow) component of electron velocity during the pull mode of reconnection
- The separatrix marks the location of the quadrupole, the strongest pressure gradients, and the boundary between the inflow and outflow regions

The quadrupole shape compares favorably with experiment



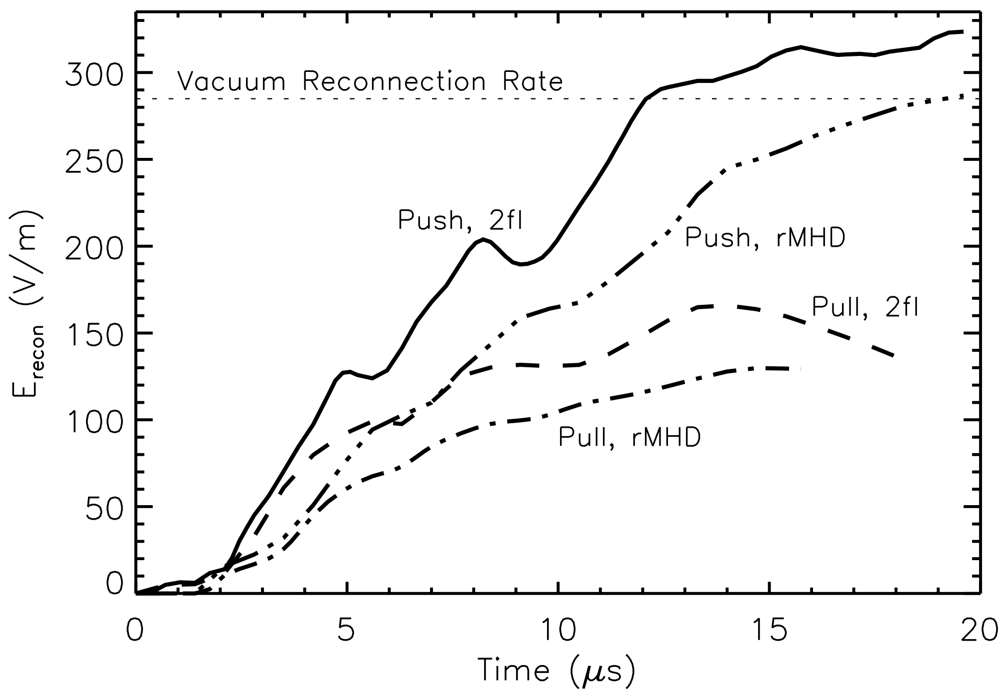
- In both experiment (left) and in our two-fluid simulations (right), the outboard quadrupole lobe peaks closer to the X-point than the inboard quadrupole lobe
- The higher density on the outboard side corresponds to weaker two-fluid effects
- Here, the asymmetry in density results from cylindrical geometry effects
  - See also Pritchett (2008) for fully kinetic simulations of asymmetric reconnection in linear geometry

The outflow component of electron velocity depends on η, V_{loop}, and T_{init}



- The electron outflow peaks closer to the X-point in simulation than in experiment
- Changing simulation parameters such as resistivity, driving voltage, and initial temperature do not enable a match between simulation and experiment
- This result suggests that physics beyond our two-fluid axisymmetric model are needed to explain the electron outflow profile in experiment
- Recent kinetic simulations show the development of an elongated electron diffusion region not present in fluid simulations (e.g., Daughton et al. 2006, Fujimoto 2006, Shay et al. 2007, Karimabadi et al. 2007, Dorfman et al. 2008)

In this case, geometry influences the reconnection rate more than two-fluid effects

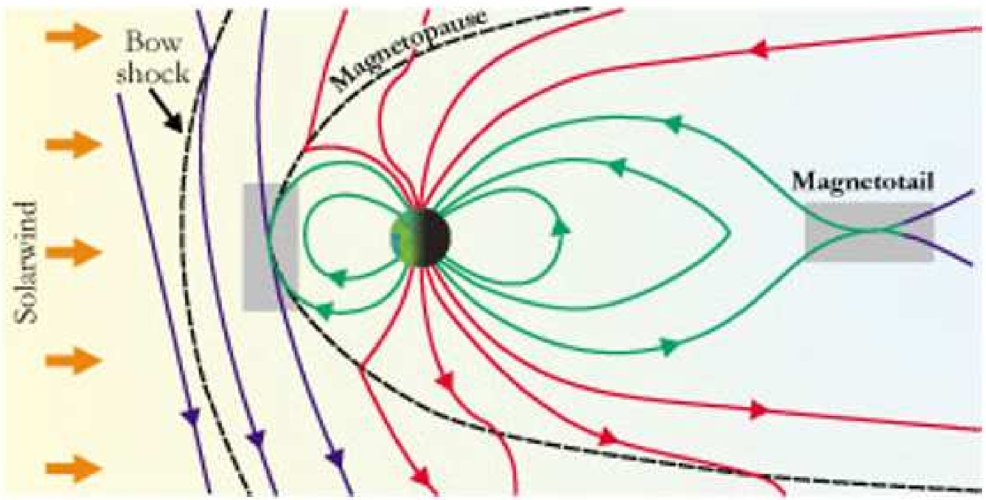


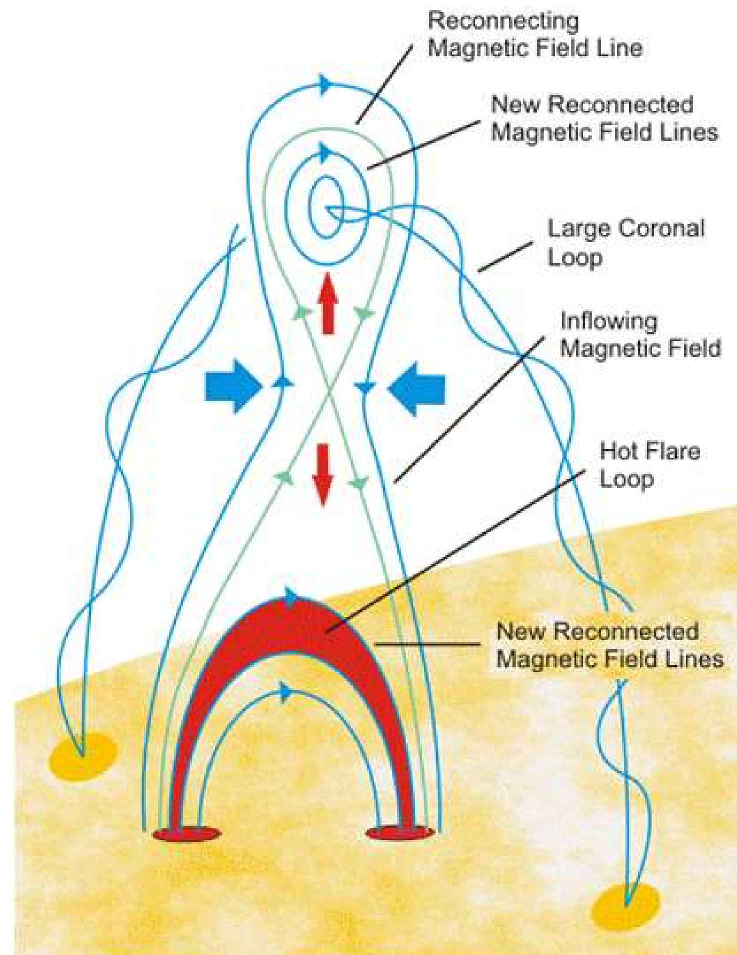
- For a given physical model, push reconnection is always quicker than pull reconnection due to the effects of downstream pressure and geometry
  - Only outflow from one side of the reconnection layer is blocked during push reconnection, whereas both sides are blocked during pull reconnection
- For a given mode of operation, two-fluid reconnection is always quicker than resistive MHD reconnection
- The reconnection rate in this setup depends more on geometric mode of operation than it does on the inclusion of two-fluid effects in our model

Reconnection with asymmetry in the outflow direction

- Reconnection in physically realistic scenarios often have asymmetry in the outflow direction
  - Reconnection in planetary magnetotails
  - Coronal mass ejections, solar flares, and flux cancellation events
  - Magnetically channeled disks in the winds of massive stars
  - Reconnection with multiple competing reconnection sites
  - Laboratory merging of toroidal plasma configurations (e.g., MRX)
- Cassak & Shay (2007) extend the Sweet-Parker model to describe reconnection between plasmas with different upstream magnetic field strengths and/or different densities
- A similar analysis is performed here for reconnection with asymmetric downstream pressure and outflow

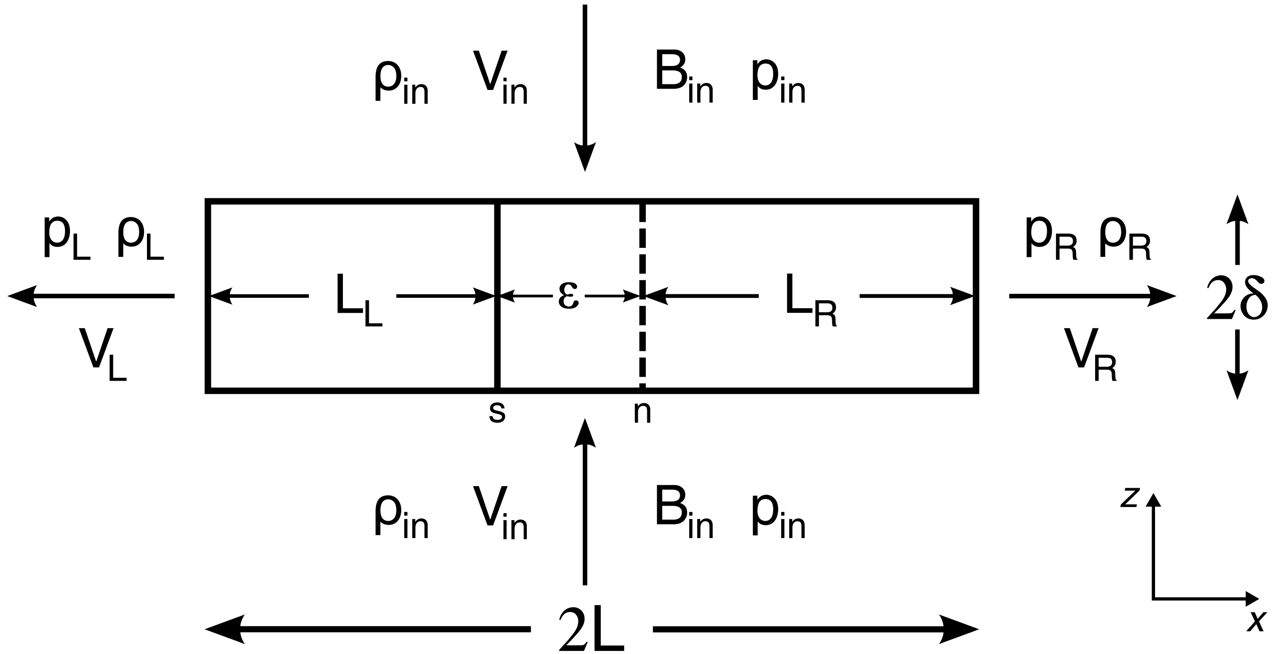
Reconnection with asymmetry in the outflow direction





- The asymmetry in the outflow direction in the Earth's magnetotail (left) and in coronal mass ejections (right) involves one outflow jet propagating into a higher density medium

Developing a model for asymmetric reconnection



- The above figure represents a long and thin reconnection layer with asymmetric downstream pressure
- The current sheet length is given by $2L \equiv L_L + \epsilon + L_R$
- The solid vertical bar represents the flow stagnation point, and the dashed bar represents the magnetic field null
- The reconnection process is assumed to be steady within an inertial reference frame

Equations of steady-state MHD in integral form

- The equations of steady-state resistive MHD can be written as

$$\oint_S \mathrm{d}\mathbf{S} \cdot (\rho \mathbf{V}) = 0 \tag{8}$$

$$\oint_S \mathrm{d}\mathbf{S} \cdot \left[\rho \mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{2\mu_0} \right) \hat{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} \right] = 0 \tag{9}$$

$$\oint_S \mathrm{d}\mathbf{S} \cdot \left[\left(\frac{\rho V^2}{2} + \frac{\gamma p}{\gamma - 1} \right) \mathbf{V} + \left(\frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right) \right] = 0 \tag{10}$$

$$\oint_S \mathrm{d}\mathbf{S} \times \mathbf{E} = 0 \tag{11}$$

- In order, these represent conservation of mass, momentum, energy, and flux
- These relations are valid for any closed volume in steady-state MHD

Finding scaling relations for asymmetric outflow

- The surface integrals from the last slide are integrated over the entire volume of the reconnection layer
- Conservation of mass gives

$$2\rho_{in}V_{in}L \sim \rho_L V_L \delta + \rho_R V_R \delta \tag{12}$$

- Conservation of momentum in the outflow direction gives

$$\rho_L V_L^2 + p_L \sim \rho_R V_R^2 + p_R \tag{13}$$

- Ignoring upstream kinetic energy/downstream magnetic energy and defining $\alpha \equiv \gamma/(\gamma-1)$, conservation of energy gives

$$2V_{in}L\left(\alpha p_{in} + \frac{B_{in}^2}{\mu_0}\right) \sim V_L\delta\left(\alpha p_L + \frac{\rho_L V_L^2}{2}\right) + V_R\delta\left(\alpha p_R + \frac{\rho_R V_R^2}{2}\right) \tag{14}$$

These relations are used to find the outflow velocity

- If equations (12), (13), and (14) are met exactly, then the relation

$$C_{6L}V_L^6 + C_{4L}V_L^4 + C_{2L}V_L^2 + C_{0L} = 0 \quad (15)$$

  can be used with equation (13) to find V_L and V_R. The coefficients are included in equations (3.24)–(3.27) in my thesis.

- In the incompressible limit ($\rho_L = \rho_R = \rho_{in} \equiv \rho$ with $\gamma \rightarrow \infty$), the outflow velocities are given by

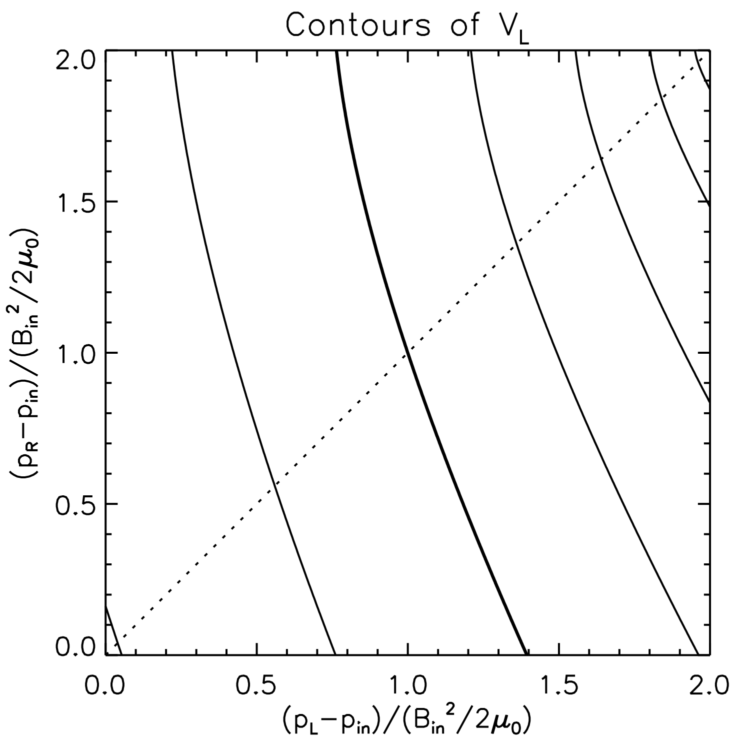
$$V_{L,R}^2 \sim \sqrt{4\left(c_{in}^2 - \frac{\bar{p}}{\rho}\right)^2 + \left(\frac{\Delta p}{2\rho}\right)^2} \pm \frac{\Delta p}{2\rho} \quad (16)$$

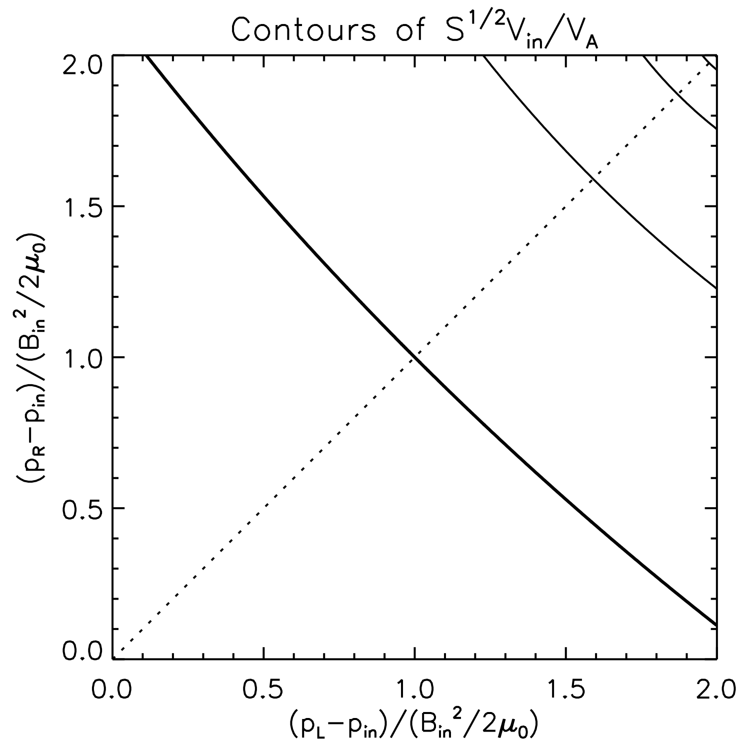
  using $\bar{p} \equiv \frac{p_L + p_R}{2}$, $\Delta p \equiv p_R - p_L$, and $c_{in}^2 = \frac{B_{in}^2}{\mu_0 \rho_{in}} + \frac{\alpha p_{in}}{\rho_{in}}$

- By assuming resistive dissipation, the electric field is then given by

$$E_y \sim B_{in}\sqrt{\frac{\eta\left(V_L + V_R\right)}{2\mu_0 L}} \quad (17)$$

Solution plots for V_L and $S^{1/2}V_{in}/V_A$ (incompressible case)





- *Left:* Solution contours for the magnitude of the leftward-directed outflow velocity V_L as a function of p_L and p_R. Contours are separated by $0.2V_A$.
- *Right:* Contours of the normalized reconnection rate $S^{1/2}\frac{V_{in}}{V_A} = \sqrt{\frac{V_L+V_R}{2V_A}}$ separated by 0.2. These plots assume that the scaling factors in relations (11)–(13) are unity.
- The outflow from one end depends only weakly on the downstream pressure on the other end. The reconnection rate is greatly affected only when outflow from both sides of the current sheet is blocked. The current sheet thickness δ increases for greater downstream pressure.

The flow stagnation point is found through conservation of mass

- Conservation of mass inside the current sheet gives the relations

$$\rho_{in} V_{in} L_L \sim \rho_L V_L \delta, \tag{18}$$

$$\rho_{in} V_{in} \left(\epsilon + L_R \right) \sim \rho_R V_R \delta \tag{19}$$

- The position of the flow stagnation point is then given by

$$L_L \sim 2L \left(\frac{\rho_L V_L}{\rho_L V_L + \rho_R V_R} \right), \tag{20}$$

$$\epsilon + L_R \sim 2L \left(\frac{\rho_R V_R}{\rho_L V_L + \rho_R V_R} \right). \tag{21}$$

The magnetic field null is approximated with the momentum equation

- The outflow component of the momentum equation (ignoring magnetic pressure) is given by

$$\rho V_x \frac{\partial V_x}{\partial x} = \frac{B_z}{\mu_0}\frac{\partial B_x}{\partial z} - \frac{\partial p}{\partial x} \tag{22}$$

- Evaluating this at the flow stagnation point, x_s, and using $\partial B_x/\partial z \sim B_{in}/\delta$, we find

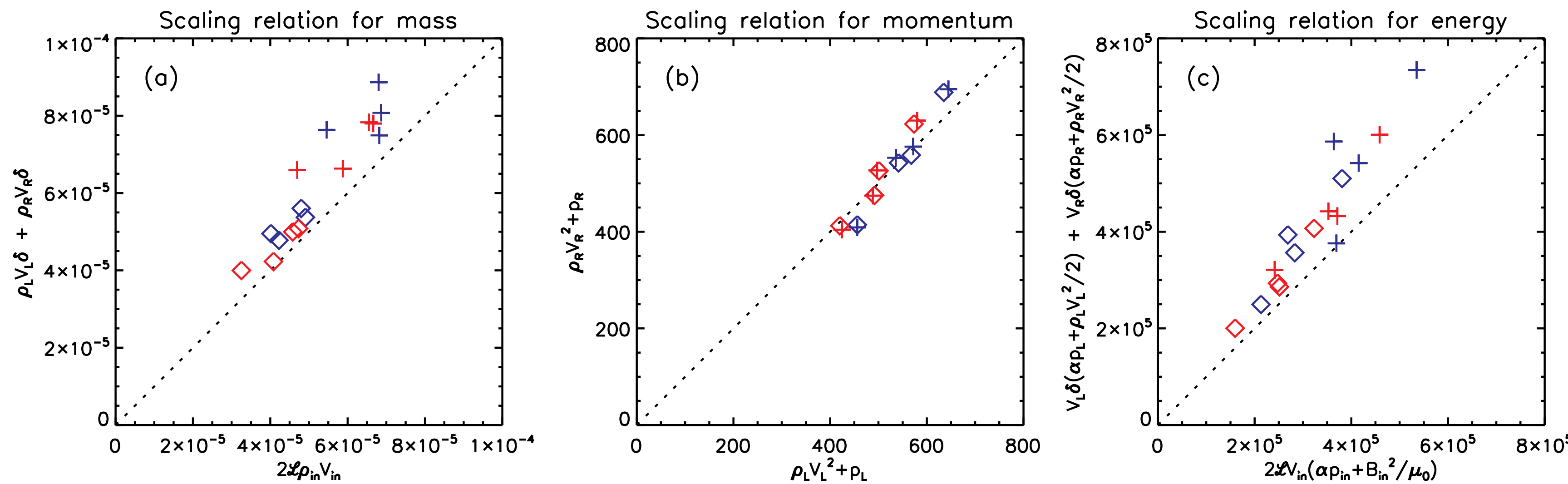
$$\frac{B_z(x_s)}{\mu_0}\frac{B_{in}}{\delta} \sim \left.\frac{\partial p}{\partial x}\right|_{x=x_s} \tag{23}$$

- Given that $B_z(x_n) = 0$ at the magnetic field null position, x_n, a Taylor expansion around $x = x_n$ gives $B_z(x_s) = (x_n - x_s)\left.\frac{\partial B_z}{\partial x}\right|_{x_n} + \mathcal{O}[(x_n - x_s)^2]$
- We then approximate the position of the magnetic field null by

$$\epsilon \equiv x_n - x_s \sim -\frac{\mu_0 \delta}{B_{in}}\left(\frac{\partial p/\partial x|_{x_s}}{\partial B_z/\partial x|_{x_n}}\right) \tag{24}$$

- The field null and stagnation point will not coincide in a steady state unless the pressure gradient is zero at the flow stagnation point
- If the flow stagnation point and magnetic field null are separated, there will be a Poynting flux across the flow stagnation point

MRX simulation results are used to check the assumptions of the model



- Linear geometry simulations of MRX are performed with one downstream wall closer to the reconnection layer than the other
  - The red case is more asymmetric than the blue case
- The data are extracted when the out-of-plane current goes down by a factor of e (diamonds) or e^2 (plus signs) of its peak value
- The current sheet position is constrained by the position of the flux cores (no X-line retreat)
- Straight lines indicate that profiles do not vary significantly as conditions change

Most predicted values are accurate to within ∼15–30%

Parameter	Blue Case	Model	Red Case	Model
V_L (km s^{-1})	39.4	46.8	27.0	19.6
V_R (km s^{-1})	43.4	50.3	33.2	23.0
x_s (cm)	0.73	0.78	1.41	1.22
ϵ (cm)	0.20	0.38	0.41	0.11
E (V m^{-1})	115	101	103	83

- The presence of a local pressure maximum near x_s and x_n complicates predicting ϵ

This model is applicable to flux cancellation events and the magnetotail

- Using equilibrium models for the solar atmosphere by Gary (2001), it is possible to make a direct application to photospheric reconnection

Z	500–600 km	V_{lower}	7.7 km s^{-1}
B	200 G	V_{upper}	8.3 km s^{-1}
n	5×10^{15} cm^{-3}	V_{in}	57 m s^{-1}
γ	1.2	δ	35 m

- Similarly, equilibrium models of the magnetotail show that the downstream pressure difference can be large if the length scale of a diffusion region is $\sim 100 c/\omega_{pi}$ rather than $\sim 10 c/\omega_{pi}$
  - X-point motion and two-fluid effects are likely to be important
- However, as seen from simulations of MRX, confinement of exhaust can significantly impact downstream pressure on one or both ends

Conclusions – I

- Extended MHD simulations of the Magnetic Reconnection Experiment (MRX) allow an investigation of how small and large scales interact in realistic geometry during reconnection
- Much of the interplay and feedback between small and large scales is due to pressure gradients that develop as a result of the reconnection process
- Depending on the mode of operation, reconnection in MRX is asymmetric in either the inflow or outflow direction because of geometric effects
- In these simulations, the experimental mode of operation determines the reconnection rate more than the inclusion of two-fluid effects

Conclusions – II

- Magnetic reconnection with asymmetry in the outflow direction occurs in many situations in nature and the laboratory
- Scaling relations are derived for the outflow velocity, electric field, and interior structure of a long and thin reconnection layer with asymmetric downstream pressure
- Reconnection will be greatly slowed only when outflow from both ends of the reconnection layer is blocked
- The flow stagnation point and magnetic field null will be separated in a steady state in the presence of a pressure gradient
- A similar model is possible for cylindrical geometry with outflow aligned with the radial direction (Section 3.4)

Astrophysical Significance

- Finding connections with space and laboratory plasmas is essential to understanding astrophysical reconnection
- Simulations of MRX show that geometric effects associated with downstream pressure and pressure differences feed back on the reconnection process
  - The ability of the downstream magnetic field configuration to confine outflow affects the reconnection rate and structure of the reconnection region
- The model for reconnection with asymmetric outflow is applicable to situations in the solar atmosphere, astrophysical disks, and planetary magnetotails

Open Problems and Future Work

- Advances in computing power and numerical methods will improve our ability to simulate greater separation of scales
- What sets the aspect ratio of the electron diffusion region during collisionless reconnection?
- Investigate time-dependent effects such as current sheet motion during reconnection with asymmetry in the outflow direction (e.g., through simulations)
- Apply asymmetric reconnection models to quantitative observations in nature and the laboratory
- I will be starting a postdoctoral position with John Raymond at the Harvard-Smithsonian Center for Astrophysics in August to investigate heating of coronal mass ejections