The Emergence, Motion, and Disappearance of Magnetic Null Points

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http://www.cfa.harvard.edu/∼namurphy/Presentations/Murphy_SPD_2013.pdf

Introduction

- Magnetic reconnection frequently occurs at and around null points: locations where the magnetic field strength equals zero
- Models of reconnection often assume symmetry such that the magnetic null point coincides with a flow stagnation point
- However, reconnection in nature and the laboratory is typically asymmetric (e.g., Cassak & Shay 2007; Murphy et al. 2010)
- Simulations of reconnection with asymmetry typically show a gap between the null and stagnation points
  - Consequently, there are often non-ideal flows across null points (e.g., Oka et al. 2008; Murphy 2010; Wyper & Jain 2013)
- In this poster, we:
  - Derive an exact expression for the 3D motion of null points
  - Discuss how non-ideal effects lead to flows across null points
  - Discuss the appearance and disappearance of null points
  - Show that an expression for the motion of a separator cannot be derived using solely local quantities

Definitions

- We define $\mathbf{x}_n(t)$ as the time-dependent position of an isolated null point. By definition,

$$\mathbf{B}\left(\mathbf{x}_n\right) \equiv 0 \tag{1}$$

- We define $\mathbf{U}$ as the velocity of this null point

$$\mathbf{U} \equiv \frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t} \tag{2}$$

- The Jacobian matrix of the magnetic field evaluated at the null point is given by

$$\mathbf{M} \equiv \left(\begin{array}{ccc} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{array} \right)_{\mathbf{x}_n} \tag{3}$$

  The local magnetic field structure near the null is given by $\mathbf{B} = \mathbf{M} \cdot \mathbf{r}$ where $\mathbf{r}$ is the position vector relative to the null.

We derive an expression for the motion of a null point in an abitrary time-varying vector field with smooth derivatives

- First we take the derivative of the magnetic field following the motion of the magnetic field null,

$$\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} + \left.(\mathbf{U}\cdot\nabla)\,\mathbf{B}\right|_{\mathbf{x}_n} = 0 \tag{4}$$

  The RHS equals zero because the magnetic field will not change from zero as we follow the null point.
- By solving for **U** in Eq. 4, we arrive at the exact relation

$$\mathbf{U} = -\mathbf{M}^{-1}\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} \tag{5}$$

- A null is structurally stable and will continue to exist when **M** is non-singular and structurally unstable when **M** is singular.
- Geometric information about the magnetic field is contained within **M**. It is easier to change the position of a root of a function by a vertical shift if the slope is shallow.

We derive an exact expression for the rate of motion of an isolated magnetic null point independent of Ohm's law

- Faraday's law is given exactly by

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{6}$$

- By applying Faraday's law to Eq. 5, we arrive at

$$\mathbf{U} = \mathbf{M}^{-1} \left. \nabla \times \mathbf{E} \right|_{\mathbf{x}_n} \tag{7}$$

  This expression is independent of choice of Ohm's law, and does not even require the existence of an Ohm's law.

In resistive MHD, null point motion results from a combination of advection by the bulk plasma flow and resistive diffusion of the magnetic field

- Next, we apply the resistive MHD Ohm's law,

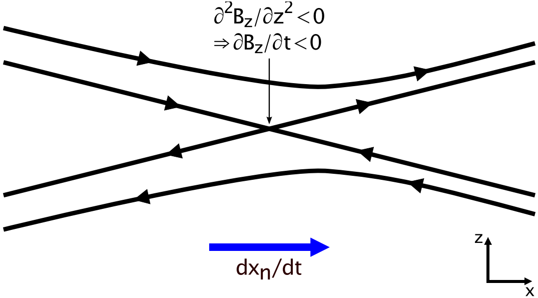
$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \tag{8}$$

  where we assume the resistivity to be uniform.
- The expression for the rate of motion of a null point becomes

$$\mathbf{U} = \mathbf{V} - \eta \, \mathbf{M}^{-1} \, \nabla^2 \mathbf{B} \tag{9}$$

  where all quantities are evaluated at the magnetic null point.
  - The first term on the RHS represents the bulk plasma flow velocity at the null point.
  - The second term on the RHS represents motion of the null point via resistive diffusion.

Murphy (2010): X-line retreat in resistive MHD



- X-line retreat in 1-D occurs due to a combination of bulk plasma flow along the outflow direction and diffusion of the normal component of the magnetic field:

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = V_x(x_n) - \eta \left[\frac{\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2}}{\frac{\partial B_z}{\partial x}} \right]_{x_n} \tag{10}$$

- $B_z < 0$ both above and below the X-line. Diffusion of B_z causes the current X-line position to have negative B_z at a slightly later time. The X-line therefore moves to the right.

Additional effects in the generalized Ohm's law

- Additional terms in the generalized Ohm's law can be incorporated by re-evaluating Eq. 7.
- For example, if we choose our Ohm's law to be

$$\mathbf{E} + \mathbf{V}_i \times \mathbf{B} = \eta \mathbf{J} + \frac{\mathbf{J} \times \mathbf{B}}{n_e e} - \frac{\nabla P_e}{n_e e} \tag{11}$$

with $\mathbf{J} = n_e e \left(\mathbf{V}_i - \mathbf{V}_e \right)$, then Eq. 7 becomes

$$\mathbf{U} = \mathbf{V}_e - \eta \, \mathbf{M}^{-1} \nabla^2 \mathbf{B} + \mathbf{M}^{-1} \left(\frac{\nabla n_e \times \nabla P_e}{n_e^2 e} \right) \tag{12}$$

Again, all terms are evaluated at the null point.

  - The relevant plasma velocity becomes the electron velocity rather than the bulk plasma velocity.
  - The last term corresponds to the Biermann battery.

Appearance and disappearance of nulls

- Much of the work on the appearance and disappearance of nulls uses bifurcation theory and topological analysis (e.g., Priest, Lonie, & Titov 1996).
- Null points must come in and out of existence through diffusion or other non-ideal effects: therefore, non-ideal effects must be included for a self-consistent description of the appearance and disappearance of nulls.
- Nulls are degenerate at the instant when they appear or disappear before bifurcating into a null-null pair.
- The Jacobian matrix of the null is singular, so the velocity of separation (convergence) is instantaneously infinite at the beginning (end) of a null's existence.

Can we perform a similar local analysis to describe the motion of separators?

- A separator is a magnetic field line connecting two null points
  - These are often important locations for reconnection.
- The locations of separators change as the plasma evolves.
  - If the evolution is ideal, then the motion of the separator will be given by its field line velocity.
  - If the evolution is non-ideal, then topology changes must be considered.
- Suppose that there is non-ideal behavior along one segment of the separator but not along the rest of it. At a slightly later time, the field line in the ideally evolving region will in general no longer be the separator even though the evolution was locally ideal.
  - Therefore, it is not possible to find an exact expression describing separator motion based solely on local parameters.
  - However, a global approach could lead to an exact expression by taking into account connectivity changes along the separator as well as motion of its endpoints.

Conclusions

- We derive an exact expression for the motion of a magnetic null point that depends solely on parameters evaluated at the null. This expression can be applied for arbitrary Ohm's law
- In resistive MHD, the position of a null point can change via bulk plasma flow or resistive diffusion of the magnetic field.
- Magnetic null points must diffuse in and out of existence through non-ideal effects. Upon formation, a new null is degenerate before bifurcating into a null-null pair. These new nulls have an instantaneously infinite velocity of separation.
- An expression for the motion of separators must include information on plasma motion and connectivity changes along its entire length.

Puns about computational magnetohydrodynamics

- This topic is pretty narrow, so we may have to allow for some divergence error.
- Fortunately, the standards for these puns are pretty Lax.
- Momentum transfer between ions and neutrals, however, is kind of a drag.
- I could have put in more puns, but I'm being conservative.
- Thank you for converging at my poster. I am glad we did not get a null result.
- And remember, if you ever need someone to numerically solve a system of vector partial differential equations that you're embarrassed about not being able to solve analytically, I can be discrete.