The Appearance, Motion, and Disappearance of 3D Magnetic Null Points

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Introduction

- **Null points** are locations where the magnetic field equals zero
- **Magnetic skeletons** are the boundaries that divide a plasma into distinct domains
  - Include null points, separatrix surfaces, spines, and separators
  - Quasi-separatrix layers are analogs if no nulls are present
  - Preferred locations for reconnection
  - Evolve **geometrically** and **topologically**
- During **asymmetric reconnection**, there is usually a gap between the null point (when present) and flow reversal
  - **Non-ideal flows across topological boundaries!**
- Key question: **How do null points move?**
- We present exact expressions for the **velocity of a null point** and describe important properties of **null point bifurcations**.

Definitions

- The time-dependent **position** of a null point is

$$\mathbf{x}_n(t) \tag{1}$$

- The null point's **velocity** is

$$\mathbf{U} \equiv \frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t} \tag{2}$$

- The **linear magnetic field structure** near the null is

$$\mathbf{B} = \mathbf{M} \cdot \delta\mathbf{x} \tag{3}$$

where $\delta\mathbf{x} \equiv \mathbf{x} - \mathbf{x}_n(t)$. The **Jacobian matrix** of **B** at x_n is

$$\mathbf{M} \equiv \begin{pmatrix} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{pmatrix}_{\mathbf{x}_n} \tag{4}$$

Null point motion in a smooth time-varying vector field

- Take the derivative of $\mathbf{B}$ following the motion of the null:

$$\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} + \left.(\mathbf{U}\cdot\nabla)\,\mathbf{B}\right|_{\mathbf{x}_n} = 0 \qquad (5)$$

  The RHS equals zero because **the magnetic field does not change from zero as we follow the null point**.

- Solving for $\mathbf{U}$ gives the **null point's velocity**:

$$\mathbf{U} = -\mathbf{M}^{-1}\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} \qquad (6)$$

  This expression is **independent of Maxwell's equations** and provides a **unique null point velocity** when $\mathbf{M}$ is non-singular

Use Faraday's law to get an expression for null point motion that remains independent of Ohm's law

- Faraday's law is given by

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{7}$$

- By applying Faraday's law to Eq. 6, we arrive at

$$\mathbf{U} = \mathbf{M}^{-1} \left. \nabla \times \mathbf{E} \right|_{\mathbf{x}_n} \tag{8}$$

which may be evaluated for an **arbitrary Ohm's law**

Null point motion in resistive MHD

- The resistive MHD Ohm's law with uniform resistivity is

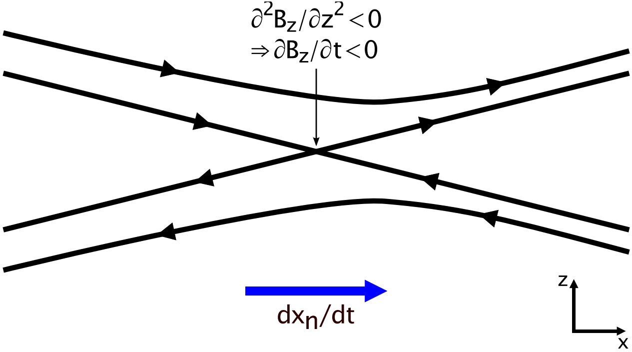
$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \tag{9}$$

- The null point velocity is

$$\mathbf{U} = \mathbf{V}(\mathbf{x}_n) - \eta\, \mathbf{M}^{-1}\, \nabla^2 \mathbf{B}\Big|_{\mathbf{x}_n} \tag{10}$$

- Null point motion results from a combination of
  - **Advection by the bulk plasma flow**
  - **Resistive diffusion of the magnetic field**

1D null point motion via resistive diffusion



- B_z is negative above and below the X-point
- Diffusion of B_z leads to the current X-point position having negative B_z at a slightly later time
- **The X-point moves to the right as a result of diffusion of the normal component of the magnetic field**
- Murphy (2010) simulations: X-point moving in opposite direction of plasma flow

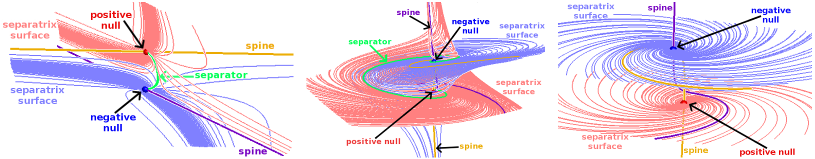
What does it mean for a magnetic null point to move?

- **Null points are not objects and cannot be 'pushed'** (e.g., by pressure gradient forces).
  - However, there may be indirect coupling between the momentum and induction equations.
- **Plasma is not permanently affixed to nulls in non-ideal cases**
  - Non-ideal plasma flows across topological boundaries
- **Null point motion depends intrinsically on local parameters evaluated at the null.**
  - However, global dynamics help set the local conditions.
- **Practical applications**
  - Exact constraint for models of asymmetric reconnection
  - Convergence test for simulations
  - Application to in situ observations and experiment?

Null points appear and disappear through bifurcations

- Linear null points are **structurally stable** (Greene 1988)
- Linear null points appear and disappear through **bifurcations of degenerate null points**
  - The Jacobian matrix $\mathbf{M}$ is singular (usually with $\operatorname{rank}\mathbf{M} = 2$)
  - Need quadratic terms to describe local magnetic structure
- Need $\mathbf{B}$ and $\frac{\partial \mathbf{B}}{\partial t}$ to be **oppositely directed** at location of impending null
- Usually an **infinite instantaneous velocity of separation or convergence** of a bifurcating null-null pair
  - Finite or zero instantaneous velocities are possible but unlikely

Properties of separators during bifurcations



- Separators are field lines that connect two null points
  - Given by the intersection of two separatrix surfaces
- Separators are generally **not straight field lines**
  - Curved field line between bifurcating radial nulls (left)
  - Wound up field line between bifurcating spiral nulls (center)
- Separators between bifurcating nulls **do not always exist**
  - Bifurcating spiral nulls with separatrix surfaces on parallel planes (right)
- **An expression for separator motion must account for connectivity changes along the entire field line**

Conclusions

- Just as we must be careful when using field line velocities, we must be careful when using null point velocities
- Exact expressions for null point motion provide insight into non-ideal flows across topological boundaries
- In resistive MHD, the position of a null point can change via bulk plasma flow or resistive diffusion of the magnetic field.
- The instantaneous velocity of convergence or separation of a bifurcating null-null pair is typically infinite
- Separators between bifurcating null points do not necessarily exist, and are generally not straight field lines
- **More work is needed to understand the geometrical and topological evolution of magnetic skeletons during reconnection**

Important Questions

- What is the interplay between small and large scales during null point motion?
- How do null points diffuse in and out of existence in realistic systems?
- How do we perform a general analysis of the bifurcation of second order null points?
- How do we find the initial orientation of separator(s) just after a null point bifurcation?
- How do we describe the motion of separators in 3D?
  - What is the nature of non-ideal flows across separators?
  - What is the nature of separator bifurcations?
- How do we apply these results to observation and experiment?