The Emergence, Motion, and Disappearance of Magnetic Null Points

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Introduction

- Magnetic reconnection frequently occurs at and around null points: locations where the magnetic field strength equals zero
- Models of reconnection often assume symmetry such that the magnetic null point coincides with a flow stagnation point
- However, reconnection in nature and the laboratory is typically asymmetric (e.g., Cassak & Shay 2007; Murphy et al. 2010)
- Simulations of reconnection with asymmetry typically show a gap between the null and stagnation points
  - Consequently, there are often non-ideal flows across null points (e.g., Oka et al. 2008; Murphy 2010; Wyper & Jain 2013)
- In this poster, we:
  - Derive an exact expression for the 3D motion of null points
  - Discuss how non-ideal effects lead to flows across null points
  - Discuss the appearance and disappearance of null points
  - Show that an expression for the motion of a separator cannot be derived using solely local quantities

Definitions

- The time-dependent position of an isolated null point is

$$\mathbf{x}_n(t) \tag{1}$$

- The null point's velocity is:

$$\mathbf{U} \equiv \frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t} \tag{2}$$

- The Jacobian matrix of the magnetic field at the null point is

$$\mathbf{M} \equiv \left(\begin{array}{ccc} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{array} \right)_{\mathbf{x}_n} \tag{3}$$

  The local magnetic field structure near the null is given by $\mathbf{B} = \mathbf{M}\mathbf{r}$ where $\mathbf{r}$ is the position vector.

- We only consider null points because null lines and null planes are structurally unstable

We derive an expression for the motion of a null point in an arbitrary time-varying vector field with smooth derivatives

- First we take the derivative of the magnetic field following the motion of the magnetic field null,

$$\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} + \left.(\mathbf{U}\cdot\nabla)\,\mathbf{B}\right|_{\mathbf{x}_n} = 0 \tag{4}$$

  The RHS equals zero because the magnetic field will not change from zero as we follow the null point.
- Solving for $\mathbf{U}$ provides an expression for a null point's velocity

$$\mathbf{U} = -\mathbf{M}^{-1}\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} \tag{5}$$

  - Independent of Maxwell's equations
  - Assumes C^1 continuity of $\mathbf{B}$ about $\mathbf{x}_n$
  - Unique null point velocity when $\mathbf{M}$ is non-singular
- Geometric information about the magnetic field is contained within $\mathbf{M}$. It is easier to change the position of a root of a function by a vertical shift if the slope is locally shallow.

We use Faraday's law to get an expression for the motion of a null point that remains independent of Ohm's law

- Faraday's law is given by

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{6}$$

- By applying Faraday's law to Eq. 5, we arrive at

$$\mathbf{U} = \mathbf{M}^{-1} \left. \nabla \times \mathbf{E} \right|_{\mathbf{x}_n} \tag{7}$$

In resistive MHD, null point motion results from a combination of advection by the bulk plasma flow and resistive diffusion of the magnetic field

- Next, we apply the resistive MHD Ohm's law,

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \tag{8}$$

  where we assume the resistivity to be uniform.

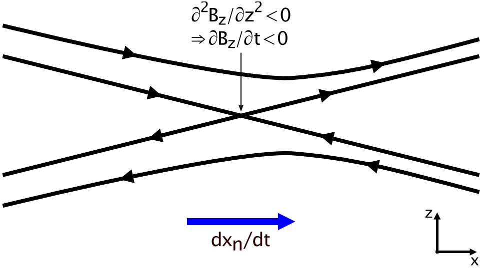
- The expression for the rate of motion of a null point becomes

$$\mathbf{U} = \mathbf{V} - \eta \, \mathbf{M}^{-1} \, \nabla^2 \mathbf{B} \tag{9}$$

  where all quantities are evaluated at the magnetic null point. The terms on the RHS represent null point motion by

  - Bulk plasma flow
  - Resistive diffusion of the magnetic field

Murphy (2010): 1D X-line retreat via resistive diffusion



- B_z is negative above and below the X-line
- Diffusion of B_z leads to the current X-line position having negative B_z at a slightly later time
- The X-line moves to the right as a result of diffusion of the normal component of the magnetic field
- The 1D expression for X-line motion is:

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = V_x(x_n) - \eta \left[\frac{\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2}}{\frac{\partial B_z}{\partial x}} \right]_{x_n} \qquad (10)$$

Additional effects in the generalized Ohm's law

- Additional terms in the generalized Ohm's law can be incorporated by re-evaluating Eq. 7.
- For example, if we choose our Ohm's law to be

$$\mathbf{E} + \mathbf{V}_i \times \mathbf{B} = \eta \mathbf{J} + \frac{\mathbf{J} \times \mathbf{B}}{n_e e} - \frac{\nabla P_e}{n_e e} \tag{11}$$

with $\mathbf{J} = n_e e \left(\mathbf{V}_i - \mathbf{V}_e \right)$, then Eq. 7 becomes

$$\mathbf{U} = \mathbf{V}_e - \eta \, \mathbf{M}^{-1} \, \nabla^2 \mathbf{B} + \mathbf{M}^{-1} \left(\frac{\nabla n_e \times \nabla P_e}{n_e^2 e} \right) \tag{12}$$

Again, all terms are evaluated at the null point.

  - The relevant plasma velocity becomes the electron velocity rather than the bulk plasma velocity.
  - The last term corresponds to the Biermann battery.

What does it mean for a magnetic null point to move?

- The velocity of a null point depends intrinsically on *local* plasma parameters evaluated at the null
- Global dynamics help set the local conditions
- A unique null point velocity exists if $\mathbf{M}$ is non-singular
- Nulls are not objects and cannot be pushed by, e.g., pressure gradient forces
  - Indirect coupling between the momentum equation and the combined Faraday/Ohm's law
  - Plasma not permanently affixed to nulls in non-ideal cases
- Our expression provides a further constraint on the structures of asymmetric diffusion regions (Cassak & Shay 2007)

Appearance and disappearance of null points

- In resistive MHD, nulls must diffuse in and out of existence
  - Not accounted for in bifurcation theory/topological analysis
- At instant of formation or disappearance, a null is degenerate so $\mathbf{M}$ is singular
- Nulls bifurcate in directions along which $\mathbf{B}$ and $\frac{\partial \mathbf{B}}{\partial t}\Big|_{\mathbf{x}_n}$ are oppositely directed
- If no such directions exist, then the degenerate null disappears
- Instantaneous velocity of separation/convergence is infinite

Can we perform a similar local analysis to describe the motion of separators?

- A separator is a magnetic field line connecting two null points
  - These are often important locations for reconnection.
- Suppose that there is non-ideal behavior only along one segment of a separator.
- At a slightly later time, the field line in the ideally evolving region will in general no longer be the separator, even though the evolution was locally ideal.
- Therefore, it is not possible to find an exact expression describing separator motion based solely on local parameters.
- However, a global approach could lead to an exact expression by taking into account connectivity changes along the separator as well as motion of its endpoints.

Conclusions

- We derive an exact expression for the motion of a magnetic null point that depends solely on parameters evaluated at the null. This expression can be applied for arbitrary Ohm's law.
- In resistive MHD, the position of a null point can change via bulk plasma flow or resistive diffusion of the magnetic field.
- Magnetic null points must diffuse in and out of existence through non-ideal effects. Upon formation, a new null is degenerate before bifurcating into a null-null pair. These new nulls have an instantaneously infinite velocity of separation.
- An expression for the motion of separators must include information on plasma motion and connectivity changes along its entire length.

Open Questions: Null Point Motion

- How do we connect this local expression to global models?
  - X-line retreat in magnetotail
  - Solar flares
  - Laboratory reconnection experiments
- How do we connect this into models of the structure of diffusion regions? (e.g., Cassak & Shay 2007)
- Can we derive an expression for how the Jacobian evolves in time in non-ideal cases? (cf. Hornig & Schindler 1996)
- What mathematical tools can we use to derive an expression for the motion of separators? What insights will this provide into separator bifurcations?