Resistive MHD Simulations of X-Line Retreat and Competing Reconnection Sites

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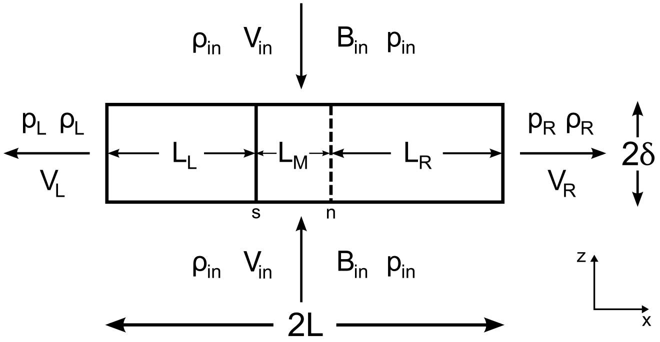
Introduction

- X-line motion is frequently observed during magnetic reconnection in nature and the laboratory. Examples include:
  - X-line retreat in Earth's magnetotail (e.g., Runov *et al.* 2003; Eastwood *et al.* 2010)
  - Current sheets that form in the wakes of coronal mass ejections (CMEs)
  - Radial motion of the diffusion region during spheromak merging (e.g., Inomoto *et al.* 2006; Gray *et al.* 2010; Ono *et al.* 1997; see also Murphy & Sovinec 2008)
- Despite the prevalence of X-line retreat, it is standard practice to compare *in situ* measurements to particle-in-cell (PIC) simulations of roughly stationary reconnection layers
- In this poster, we present resistive MHD simulations of X-line retreat and competing X-lines and derive an expression for the rate of X-line retreat

Background: Murphy, Sovinec, & Cassak (2010) developed a scaling model for asymmetric outflow reconnection



- The above figure represents a long and thin reconnection layer with asymmetric downstream pressure. Reconnection is assumed to be steady in an inertial reference frame.
- 'n' denotes the magnetic field null and 's' denotes the flow stagnation point

Scaling relations for asymmetric outflow reconnection

- Following Cassak & Shay (2007), we integrate over this control volume and find relations approximating conservation of mass, momentum, and energy

$$
\begin{aligned}
2\rho_{in} V_{in} L &\sim \rho_L V_L \delta + \rho_R V_R \delta \\
\rho_L V_L^2 + p_L &\sim \rho_R V_R^2 + p_R \\
2 V_{in} L \left(\alpha p_{in} + \frac{B_{in}^2}{\mu_0} \right) &\sim V_L \delta \left(\alpha p_L + \frac{\rho_L V_L^2}{2} \right) \\
&\quad + V_R \delta \left(\alpha p_R + \frac{\rho_R V_R^2}{2} \right)
\end{aligned}
$$

where $\alpha \equiv \gamma/(\gamma - 1)$ and we ignore upstream kinetic energy/downstream kinetic energy and assume the contribution from tension along the boundary is small or even.

Deriving the outflow velocity and reconnection rate

- In the incompressible limit

$$V_{L,R}^2 \sim \sqrt{4\left(c_{in}^2 - \frac{\bar{p}}{\rho}\right)^2 + \left(\frac{\Delta p}{2\rho}\right)^2} \pm \frac{\Delta p}{2\rho}$$

using $\bar{p} \equiv \frac{p_L + p_R}{2}$, $\Delta p \equiv p_R - p_L$, and $c_{in}^2 = \frac{B_{in}^2}{\mu_0 \rho_{in}} + \frac{\alpha p_{in}}{\rho_{in}}$

- By assuming resistive dissipation, the electric field is then given by

$$E_y \sim B_{in}\sqrt{\frac{\eta\left(V_L + V_R\right)}{2\mu_0 L}}$$

Implications of scaling model

- The scaling relations show that the reconnection rate is weakly sensitive to asymmetric downstream pressure
  - If one outflow jet is blocked, reconnection will be almost as quick
  - Reconnection will slow down greatly only if both outflow jets are blocked
  - The current sheet responds to asymmetric downstream pressure by changing its thickness or length
- However, this analysis makes three major assumptions:
  - The current sheet is stationary
  - The current sheet thickness is uniform
  - Magnetic tension contributes symmetrically along the boundaries
- To make further progress, we must do numerical simulations

We perform resistive MHD simulations of two initial X-lines which retreat from each other as reconnection develops (see Murphy 2010)

- The 2-D simulations start from a periodic Harris sheet which is perturbed at two nearby locations ($x = \pm 1$)
- Domain: $-30 \leq x \leq 30$, $-12 \leq z \leq 12$
- Simulation parameters: $\eta = 10^{-3}$, $\beta_\infty = 1$, $\mathrm{S} = 10^3$–10^4, $\mathrm{Pm} = 1$, $\gamma = 5/3$, $\delta_0 = 0.1$
- Define:
  - x_n is the position of the X-line
  - x_s is the position of the flow stagnation point
  - $V_x(x_n)$ is the velocity *at* the X-line
  - $\frac{\mathrm{d}x_n}{\mathrm{d}t}$ is the velocity *of* the X-line
- $\hat{\mathbf{x}}$ is the outflow direction, $\hat{\mathbf{y}}$ is the out-of-plane direction, and $\hat{\mathbf{z}}$ is the inflow direction
- We show only $x \geq 0$ since the simulation is symmetric

NIMROD solves the equations of extended MHD using a finite element formulation (Sovinec *et al.* 2004)

- In dimensionless form, the equations used for these simulations are

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \mathbf{J} - \mathbf{V} \times \mathbf{B}) + \kappa_{divb} \nabla \nabla \cdot \mathbf{B} \tag{1}$$

$$\mathbf{J} = \nabla \times \mathbf{B} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

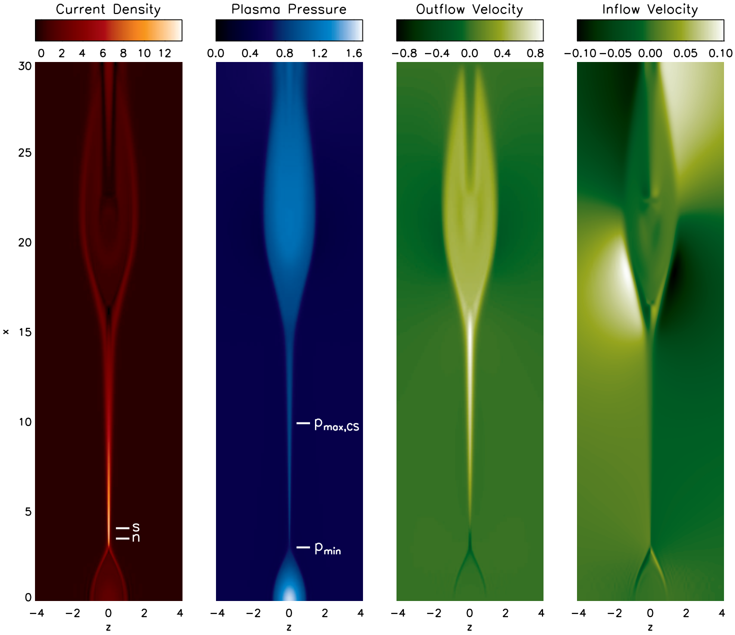
$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \rho \nu \nabla \mathbf{V} \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = \nabla \cdot D \nabla \rho \tag{5}$$

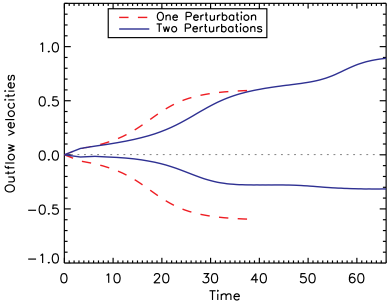
$$\frac{\rho}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q \tag{6}$$

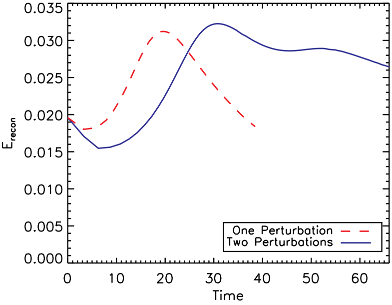
- Divergence cleaning is used to prevent the accumulation of divergence error

The current sheets have characteristic single wedge shapes



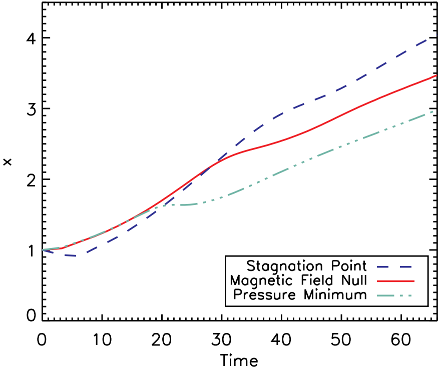
The outflow away from the obstruction is much faster





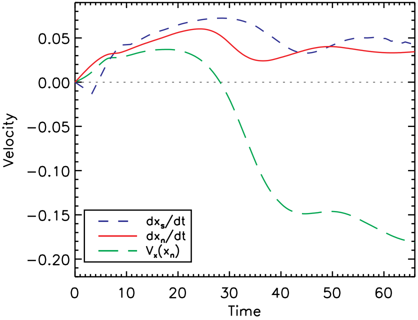
- In agreement with Seaton (2008) and Reeves *et al.* (2010), most of the energy goes away from the obstructed exit
- Eventually, reconnection proceeds more quickly in retreat simulations than in otherwise equivalent symmetric, non-retreating simulations
- The comparison with the single perturbation case is halted around $t \approx 40$ because of the formation of an island at $x = 0$

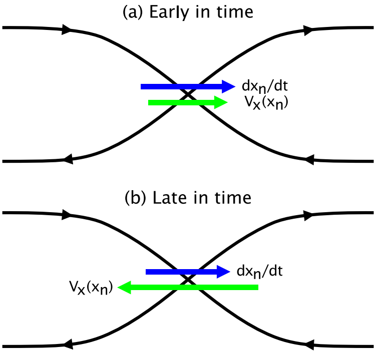
The flow stagnation point and X-line are not colocated



- Surprisingly, the relative positions of the X-line and flow stagnation point switch!
- This occurs so that the stagnation point will be located near where the tension and pressure forces cancel
- Reconnection develops slowly because the X-line is located near a pressure minimum early in time

Late in time, the X-line diffuses against strong plasma flow





- The stagnation point retreats more quickly than the X-line
- Any difference between $\frac{dx_n}{dt}$ and $V_x(x_n)$ must be due to diffusion (e.g., Seaton 2008)
- The velocity *at* the X-line is not the velocity *of* the X-line

What sets the rate of X-line retreat?

- The inflow (z) component of Faraday's law for the 2D symmetric inflow case is

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \tag{7}$$

- The convective derivative of B_z at the X-line taken at the velocity of X-line retreat, $\mathrm{d}x_n/\mathrm{d}t$, is

$$\left.\frac{\partial B_z}{\partial t}\right|_{x_n} + \frac{\mathrm{d}x_n}{\mathrm{d}t} \left.\frac{\partial B_z}{\partial x}\right|_{x_n} = 0 \tag{8}$$

The RHS of Eq. (8) is zero because $B_z(x_n, z = 0) = 0$ by definition for this geometry.

Deriving an expression for X-line retreat

- From Eqs. 7 and 8:

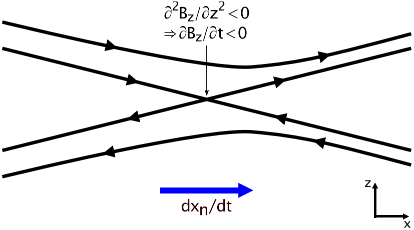
$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = \left.\frac{\partial E_y/\partial x}{\partial B_z/\partial x}\right|_{x_n} \tag{9}$$

- Using $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}$, we arrive at

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = V_x(x_n) - \eta \left[\frac{\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2}}{\frac{\partial B_z}{\partial x}} \right]_{x_n} \tag{10}$$

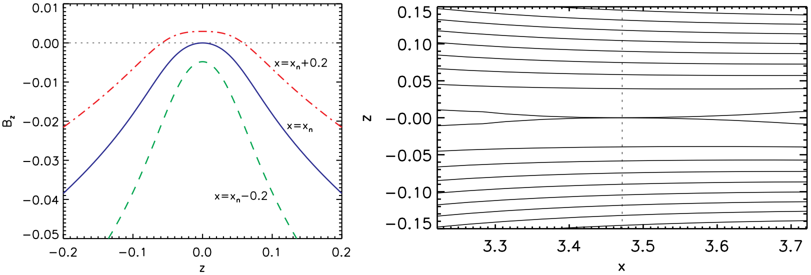
- In the simulations $\frac{\partial^2 B_z}{\partial z^2} \gg \frac{\partial^2 B_z}{\partial x^2}$, so X-line retreat is caused by diffusion of the normal component of the magnetic field along the inflow direction
- Equation (9) can also be evaluated using additional terms in the generalized Ohm's law

Mechanism and implications



- The X-line moves in the direction of increasing total reconnection electric field strength
- X-line retreat occurs through a combination of advection by bulk plasma flow and diffusion of the normal component of the magnetic field
- X-line motion depends intrinsically on local parameters evaluated at the X-line

The magnetic field structure near the X-line is characteristic of this mechanism for X-line retreat



- Negative B_z diffuses from above and below into the region of the X-line
- *Left:* $B_z(z)$ along three locations near the X-line.
- *Right:* Magnetic flux contours near the X-line.

Simulations of multiple X-line reconnection

- NIMROD resistive MHD simulations with multiple initial X-line perturbations were analyzed by A. K. Young
  - Isolated or strong perturbations are more likely to survive
  - Initial X-lines surrounded by other initial X-lines are less likely to survive, but X-lines which have room to develop in one outflow direction do have a better chance of developing (see also two-fluid simulations by Nakamura *et al.* 2010)
  - Early on, winning X-lines have plasma pressure facilitating outflow rather than impeding it
  - When an X-line is located near one exit of the current sheet, the flow stagnation point is located between the X-line and a central plasma pressure maximum
  - Flow across the X-line in the opposite direction of X-line motion does occur, but infrequently
- SHASTA simulations by C. C. Shen and J. Lin show that the direction of plasmoid ejection is related to the relative locations of the flow stagnation point and X-line
  - If $x_s > x_n$, then usually $V_{\text{plasmoid}} > 0$

In resistive MHD when inflow is symmetric, the number of X-lines can only change by resistive diffusion of B_z

- The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{V} - \mathbf{B} \left(\nabla \cdot \mathbf{V} \right) + \eta \nabla^2 \mathbf{B}, \qquad (11)$$

where in our 2-D geometry,

$$\left[\eta \nabla^2 \mathbf{B} \right]_z = \eta \left[\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2} \right]. \qquad (12)$$

- The term $\eta \frac{\partial^2 B_z}{\partial x^2}$ acts to smooth out the $B_z(x)$ profile. This term can move or reduce the number of X-lines (where $B_z = 0$), but not create new X-lines.
- The term $\eta \frac{\partial^2 B_z}{\partial z^2}$ brings in B_z along the inflow direction. This term can move or increase the number of X-lines, but by symmetry cannot cause pre-existing X-lines to disappear.

Conclusions

- X-line motion occurs frequently during reconnection in space, astrophysics, and the laboratory
- Resistive MHD simulations of X-line retreat/asymmetric outflow reconnection show that most of the energy is directed away from the obstructed exit
- Late in time there is significant flow across the X-line in the opposite direction of X-line retreat
- An expression for the rate of X-line retreat shows that X-line motion is due to either advection by the bulk plasma flow or by diffusion of the normal component of the magnetic field
- Simulations of multiple X-line reconnection show that:
  - Initial X-lines with room to develop on at least one side are more likely to survive
  - The flow stagnation point is typically located between the X-line and a central plasma pressure maximum

Future work on X-line retreat

- Simulation:
  - Two-fluid simulations of X-line retreat
  - 3-D simulations of X-line retreat with initial perturbations offset from each other in the out-of-plane direction
  - Examine X-line behavior during the plasmoid instability
  - Investigate X-line dynamics in two and three dimensions for realistic geometries such as coronal jets, CME current sheets, planetary magnetotails, and spheromak merging experiments
  - Investigate X-line motion in asymmetric inflow reconnection
- Theory:
  - Derive expressions for neutral line and X-line motion in 3-D
  - Determine the rate of retreat of the flow stagnation point
  - Examine the consequences of a separation between 3-D magnetic field nulls and flow stagnation points
- Experiment and observation (suggestions for others):
  - Characterize X-line motion during laboratory reconnection
  - Find signatures of X-line retreat in magnetotail (M. Oka)
  - Observe effects of asymmetric outflow in CME current sheets