Global Two-Fluid Simulations of Magnetic Reconnection

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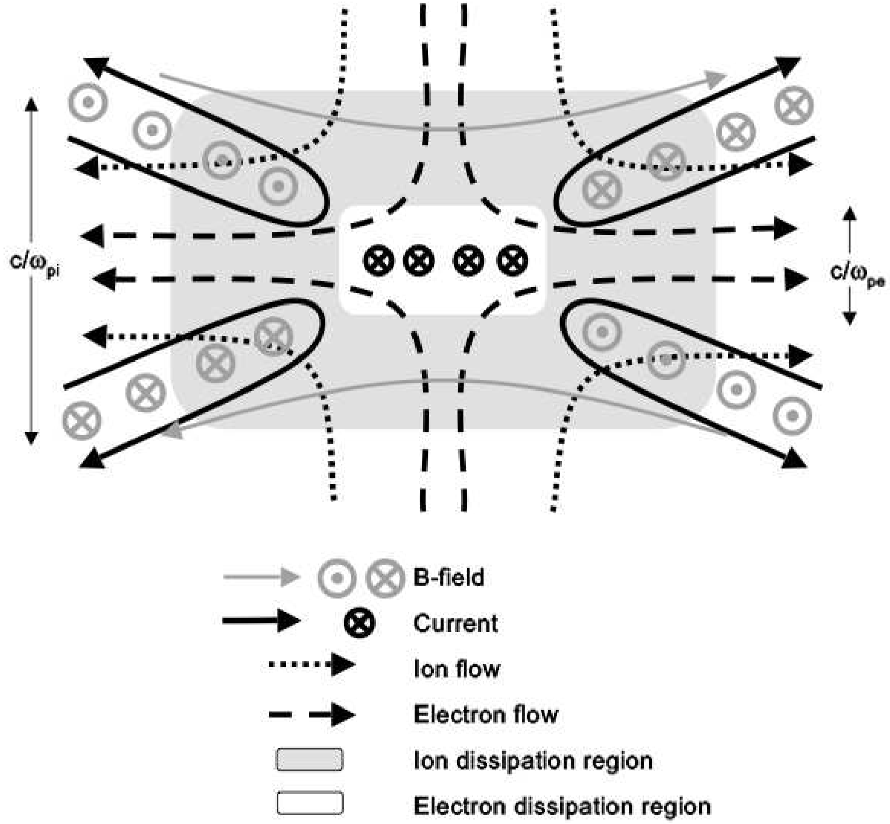
Outline

- Local vs. global issues in reconnection
- Introduction to the NIMROD code
- Simulations of the Magnetic Reconnection Experiment (MRX)
  - Radial motion of current sheet due to toroidicity
  - Effect of toroidicity on quadrupole shape
  - Reconnection rate dependence on mode of operation and physical model
  - Symmetry breaking due to the Hall effect in counter-helicity merging

Motivation

- Most two-fluid studies of reconnection focus on local reconnection physics (e.g. Birn *et al.*, 2001) and most global studies use resistive MHD (e.g. Lukin *et al.*, 2001)
- However, the reconnection layer can communicate with the global magnetic field through both MHD and non-MHD effects
- Changes in the global magnetic field topology can in turn affect the reconnection rate
- NIMROD has the capability to perform two-fluid global simulations in a realistic geometry to examine the impact of large-scale effects
- The choice of MRX as a global case allows benchmarks to aid the simulation effort and provides computational support to the experiment

Two-fluid Reconnection



- On scales below the ion inertial length, the ions and electrons become decoupled
- The double-Y point prevalent in the Sweet-Parker model becomes an X-point as the outflow opens up due to whistler physics
- The signature of two-fluid antiparallel reconnection is a quadrupole out-of-plane magnetic field

NIMROD's Non-Ideal Hall MHD Model

NIMROD solves the equations of extended MHD cast in a single fluid form. The model below is used in simulations of MRX and SSX to study the interplay between local and global effects in the reconnection process.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne}\mathbf{J} \times \mathbf{B} - \frac{1}{ne}\nabla p_e \right) + \kappa_{divb} \nabla\nabla \cdot \mathbf{B}$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$

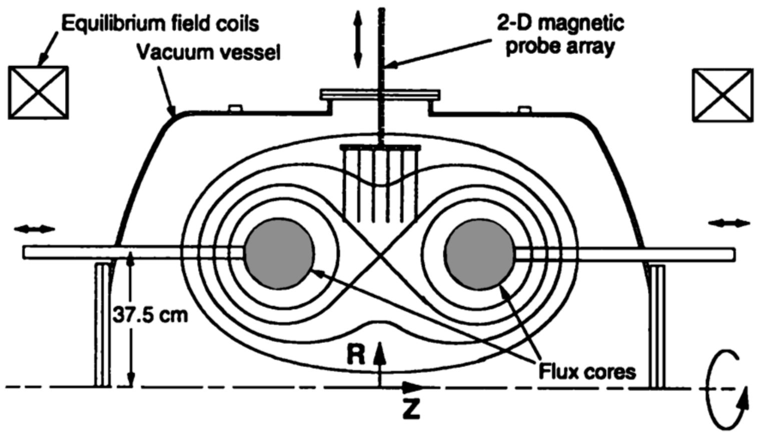
$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \rho \nu \nabla \mathbf{V}$$

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n$$

$$\frac{n}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q$$

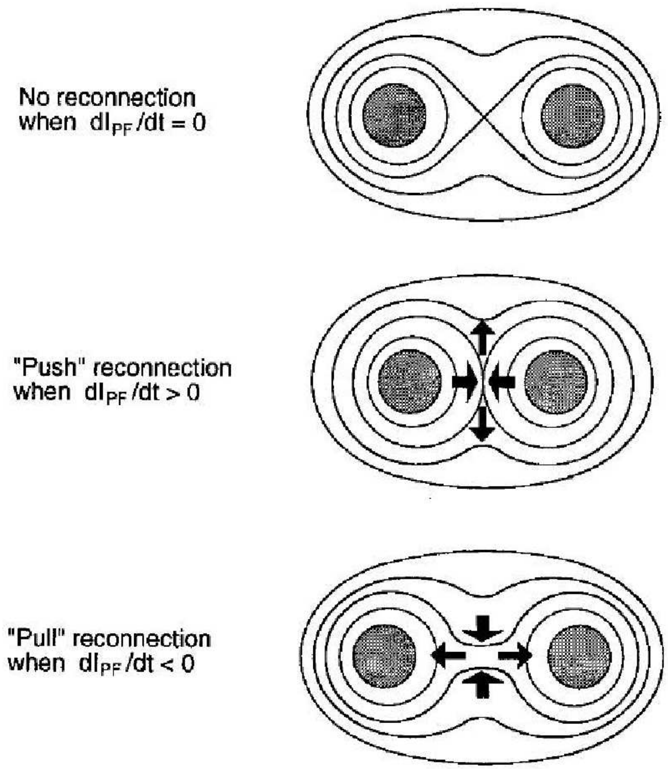
- Two-fluid effects are included via the Hall term and the electron pressure gradient in the generalized Ohm's law (blue). The terms in red are included for numerical purposes.

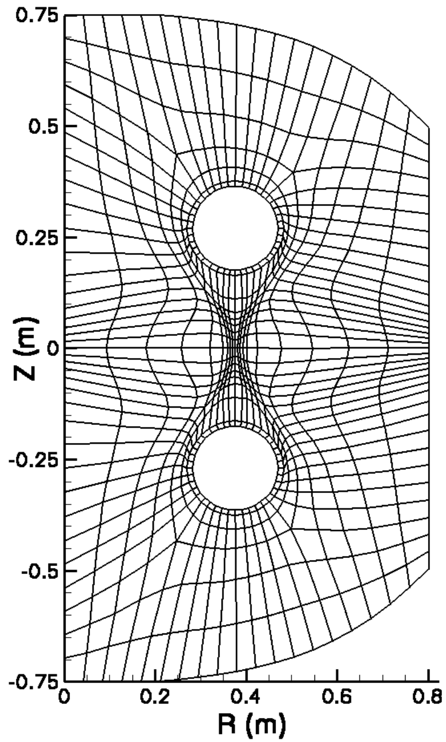
The Magnetic Reconnection Experiment



- The Magnetic Reconnection Experiment (MRX) is located at the Princeton Plasma Physics Laboratory and is designed to study controlled axisymmetric magnetic reconnection.
- Plasma parameters: $T \sim 5 - 15$ eV, $B \sim 200 - 500$ G, $S \sim 200 - 2500$, and $n \sim 0.1 - 1 \times 10^{14}$ cm^{-3}.
- MRX is a good candidate for computational study because the spatial scale separation is small and plasma parameters are not too extreme

MRX Experimental Setup



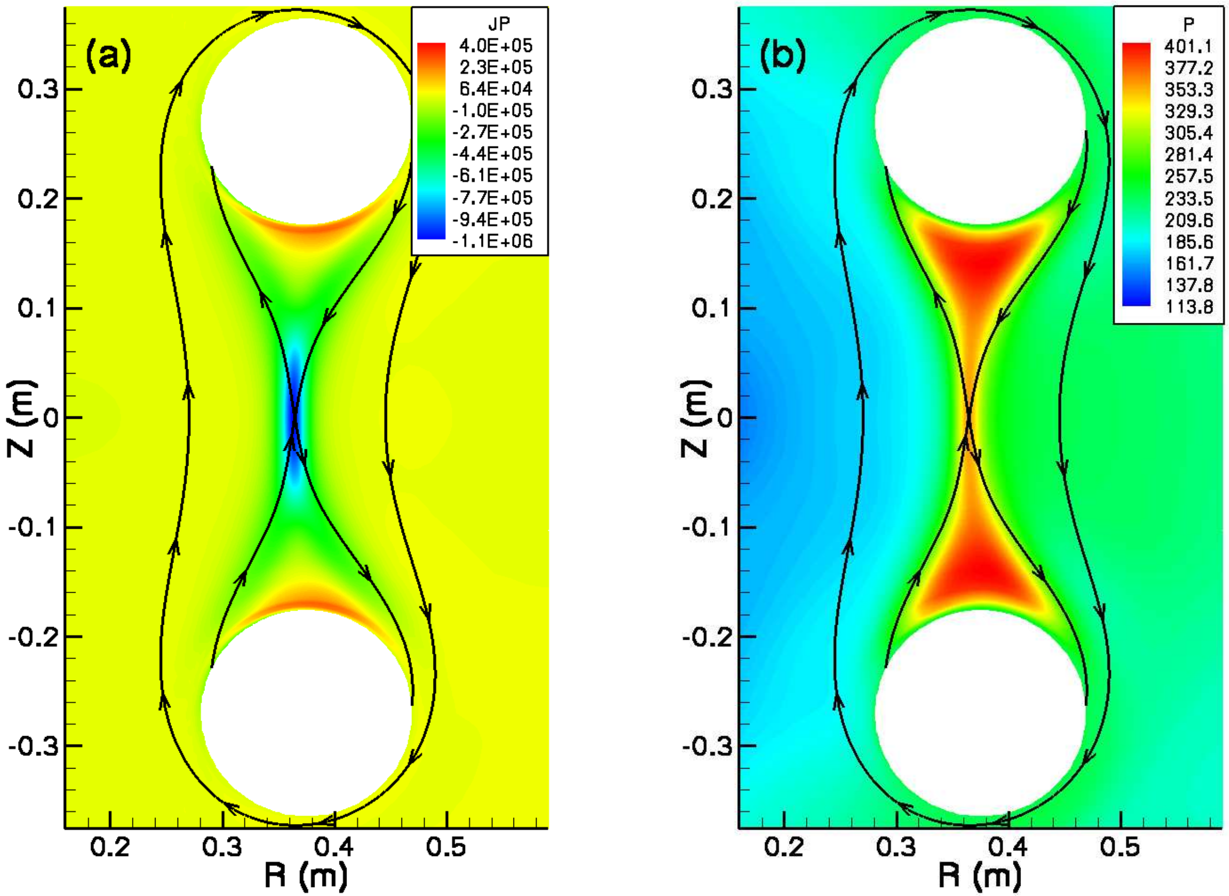


- *Left:* By changing the currents in the flux cores, two distinct modes of reconnection can be induced in MRX (e.g. Yamada *et al*., 1997). Our NIMROD simulations of MRX investigate both of these modes of operation.
- *Right:* A sample finite element grid used for simulations of two-fluid pull reconnection in MRX. The grid is produced with the assistance of a grid smoothing technique. Simulations using a linear geometry contain the flux cores but have a rectangular outer boundary.

Simulating MRX provides a chance to study global effects on reconnection

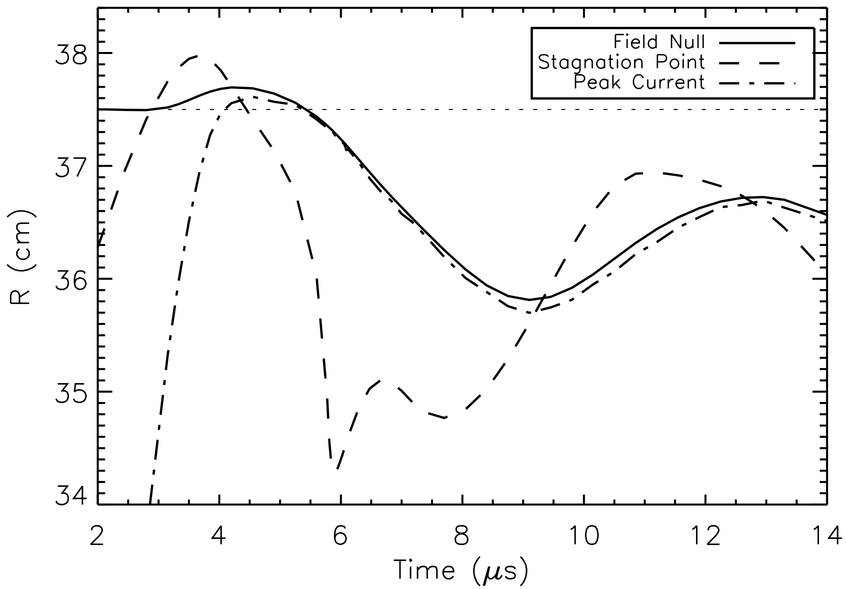
- Simulations of MRX are axisymmetric and have similar physical parameters to the experiment ($S \sim 250 - 500$, $n \sim 5 \times 10^{19}$ m^{-3}, and $T_i = T_e = 15$ eV with isotropic heat conduction and $Pm = 1$)
- The initial magnetic field is set up through coils at the centers of the flux cores, and outside the domain to set up a vertical field
- An applied electric field on the flux core surfaces drives reconnection, and if desired can also induce toroidal magnetic field
- Density and temperature are kept constant on the flux core surface, and the velocity is set by the $E \times B$ drift
- The exterior of the domain is assumed to be a perfect conductor with no-slip boundary conditions
- Simulations are performed for both toroidal and linear geometries, allowing a detailed investigation of the role of toroidicity

Sweet-Parker-like current sheets show up when $\mathbf{E}+\mathbf{V}\times\mathbf{B}=\eta\mathbf{J}$



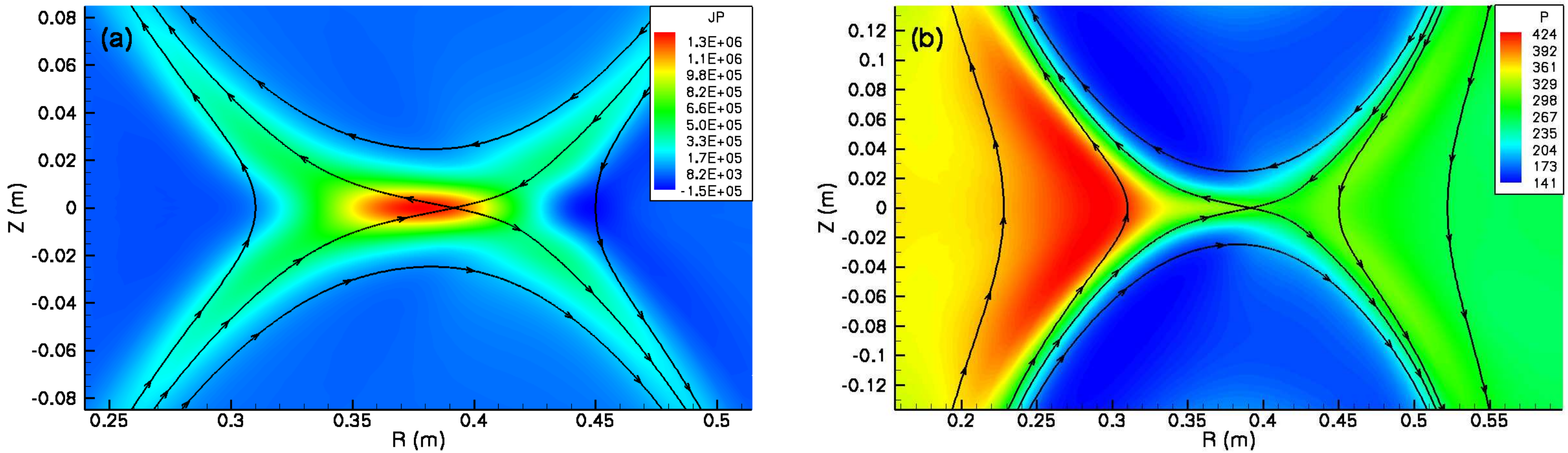
- Toroidal current density contours (left) and plasma pressure contours (right) along with streamtraces of the magnetic field during resistive MHD pull reconnection
- The arms of toroidal current density are diamagnetic currents $\mathbf{J}_* = \frac{\mathbf{B}\times\nabla p}{B^2}$ located along the separatrix where there is a strong pressure gradient between the low pressure inflow region and the high pressure outflow region

The effects of toroidicity include a radial motion of the current sheet



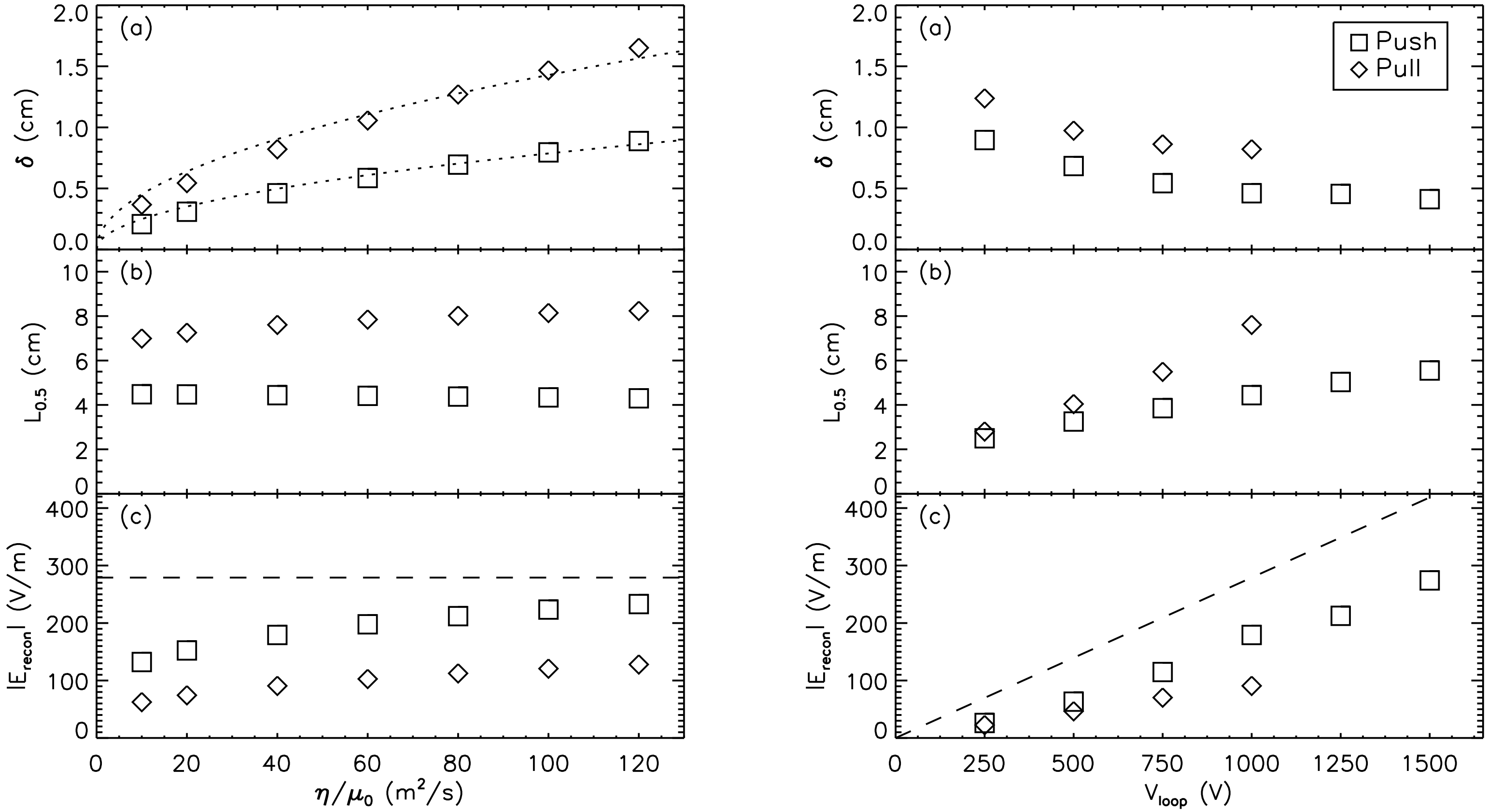
- This radial motion occurs for both resistive MHD (above) and two-fluid models during pull reconnection
- The stagnation point and the field null do not coincide (except when the motion of the field null ceases)
- The drift does not occur if $\beta \equiv 0$ or if a linear geometry is used, tying the drift to toroidicity and pressure effects
- The density on the inboard side becomes depleted quickly due to the lesser volume than on the outboard side
- Inflow coming from the inboard side has to undergo a larger jump in pressure than inflow from the outboard side, resulting in a net inward force

Resistive MHD push reconnection in MRX



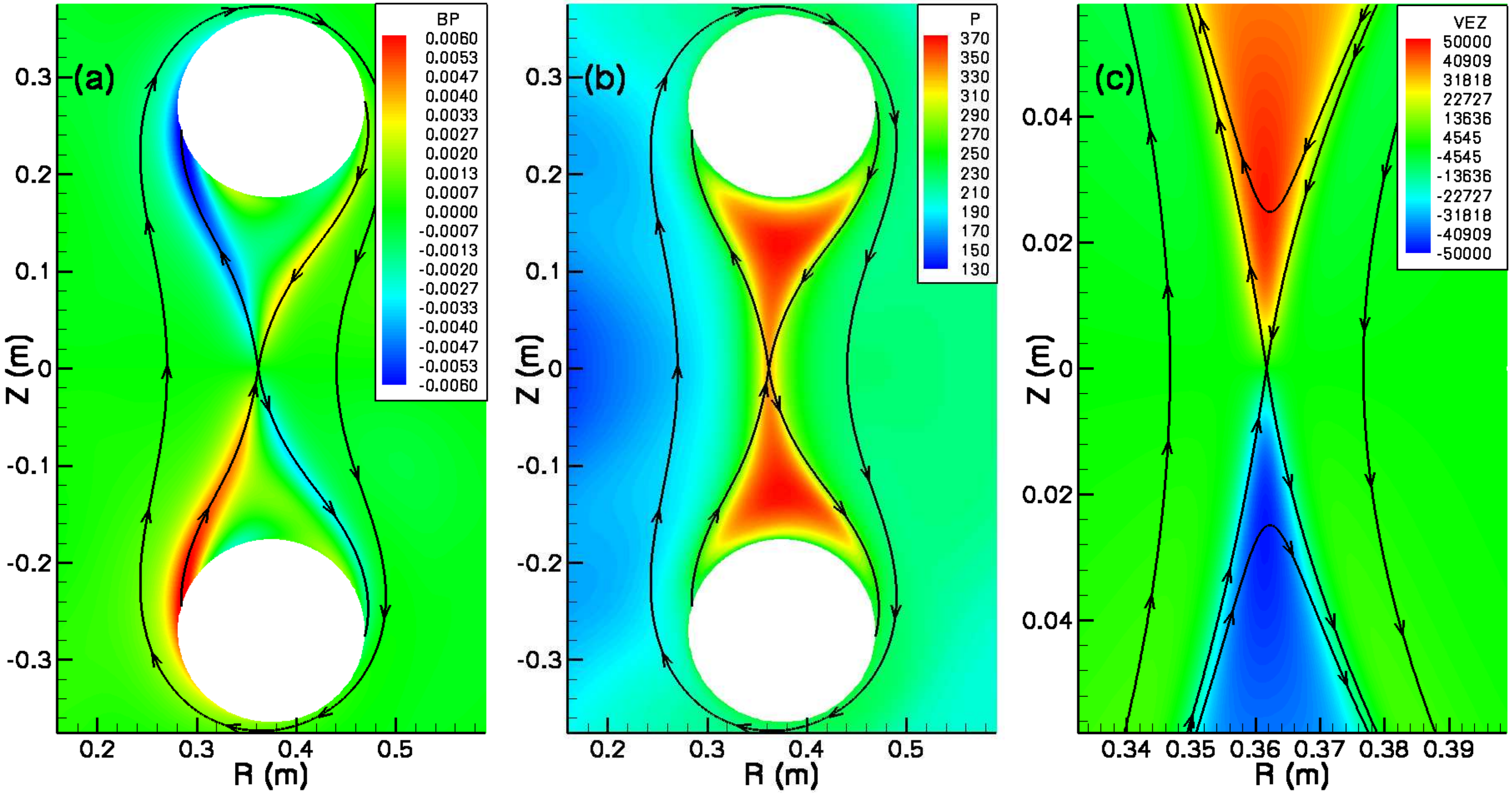
- Due to the same volume effects as in pull reconnection, the inboard side develops high pressure quickly, pushing the X-point to higher radii
- The X-point position is shifted towards the outboard side of the current sheet, allowing a stronger inward-directed tension force to overcome the steeper pressure gradient
- For the same driving voltage strength, push reconnection is 1.5-2 times faster than pull reconnection
    - The difference is largely attributed to differences in downstream pressure
    - While outflow in pull reconnection is trapped between the separatrix and the flux cores, the outflow here can move along field lines outside the separatrix

Resistive MHD scaling studies



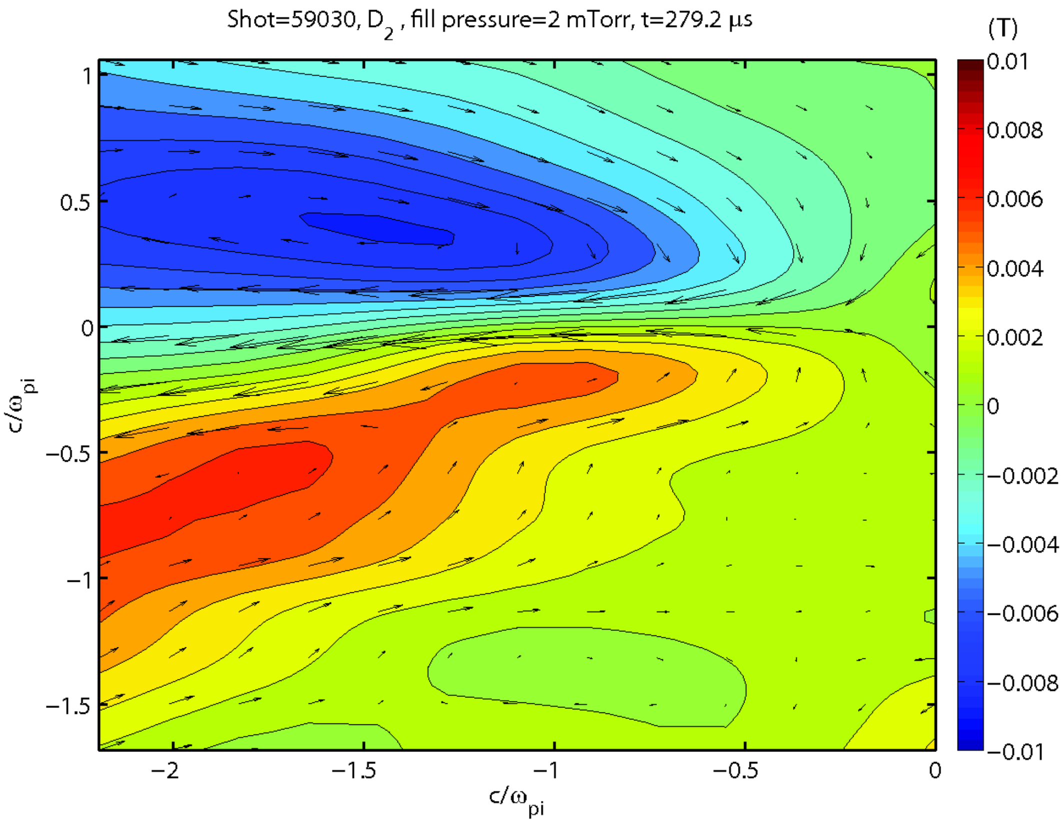
- The current sheet width δ is proportional to $\eta^{1/2}$, but also depends on V_{loop}
- The current sheet length $L_{0.5}$ is largely independent of η but depends highly on V_{loop}
- The reconnection rate is roughly proportional to V_{loop}, and gets closer to the vacuum reconnection rate with higher η
- The current sheet length is determined by global parameters, while the width is determined by both local and global effects

The well-known quadrupole is present in two-fluid simulations of MRX



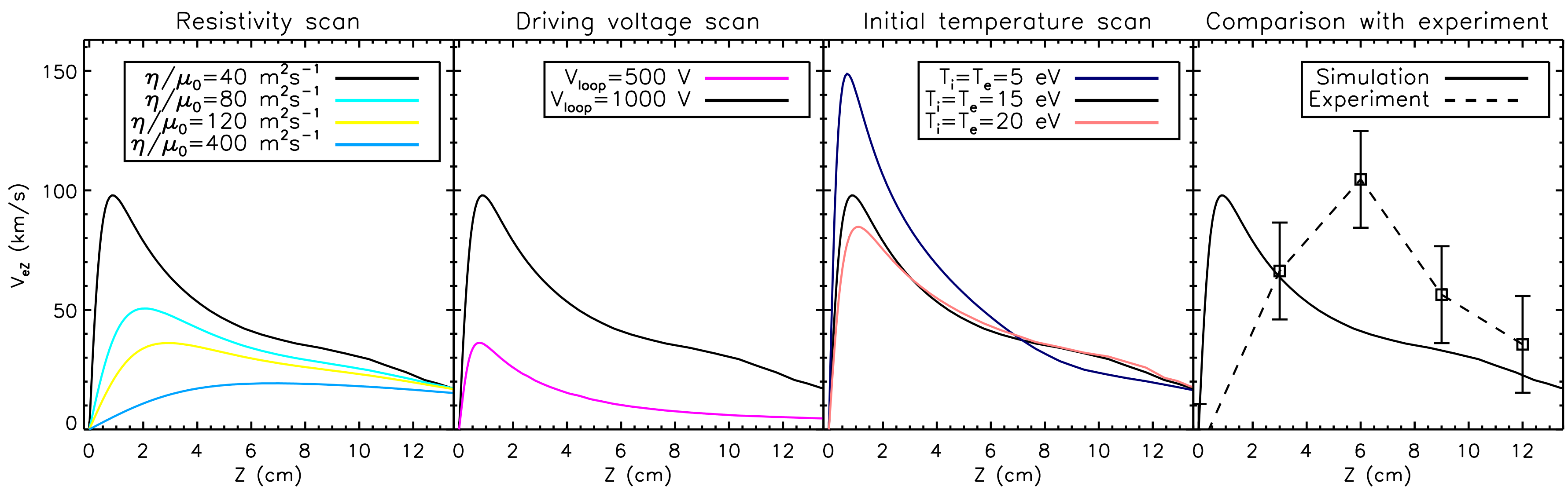
- From left to right are contours of toroidal magnetic field, plasma pressure, and the vertical (outflow) component of electron velocity during the pull mode of reconnection
- The separatrix marks the location of the quadrupole, the strongest pressure gradients, and the boundary between the inflow and outflow regions

The quadrupole shape compares favorably with experiment



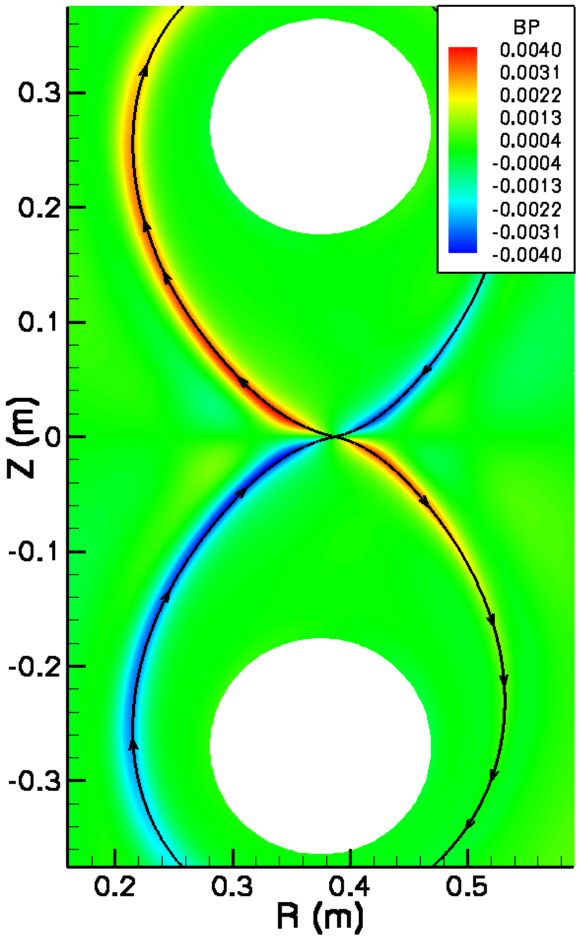
- The outboard quadrupole lobe peaks closer to the X-point than the inboard quadrupole lobe, consistent with simulation; however, it is possible that toroidal field remains from the process that drives reconnection in experiment
- Note that the orientation of the experimental results are rotated with respect to simulation
- Y. Ren, Ph.D. Thesis, 2007

The outflow component of electron velocity depends on η, V_{loop}, and T_{init}



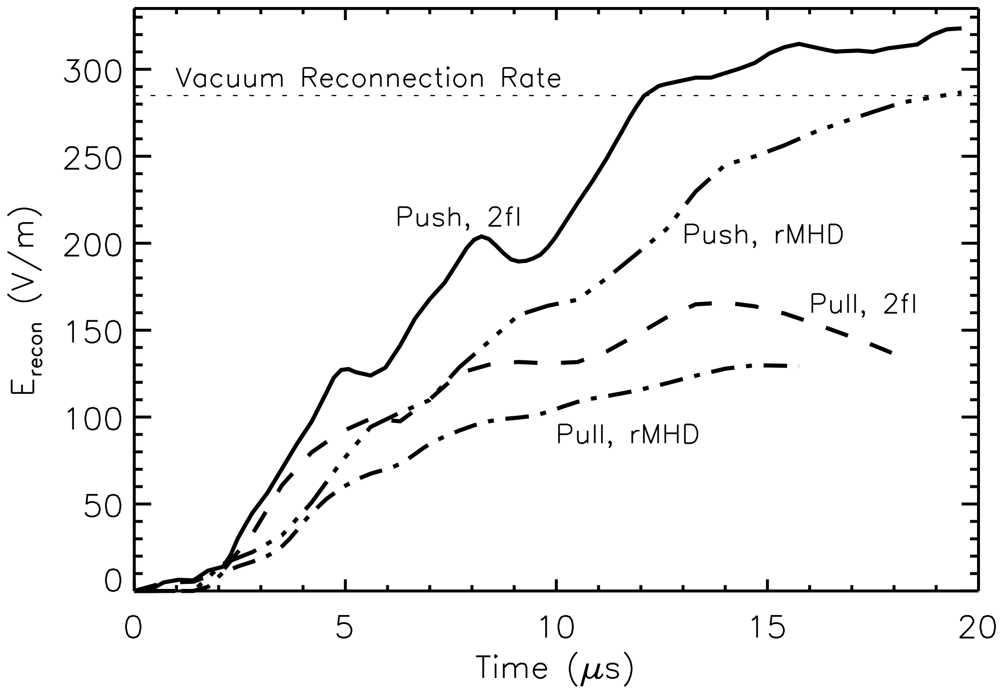
- Increasing resistivity weakens the outflow strength while moving the peak further away
- Increasing the driving voltage greatly increases the electron outflow speed
- Reducing the initial temperature increases the peak outflow speed by lessening the effects of downstream pressure
- The electron outflow in experiment is stronger further away from the X-point than in most simulations

The quadrupole during push reconnection is narrower



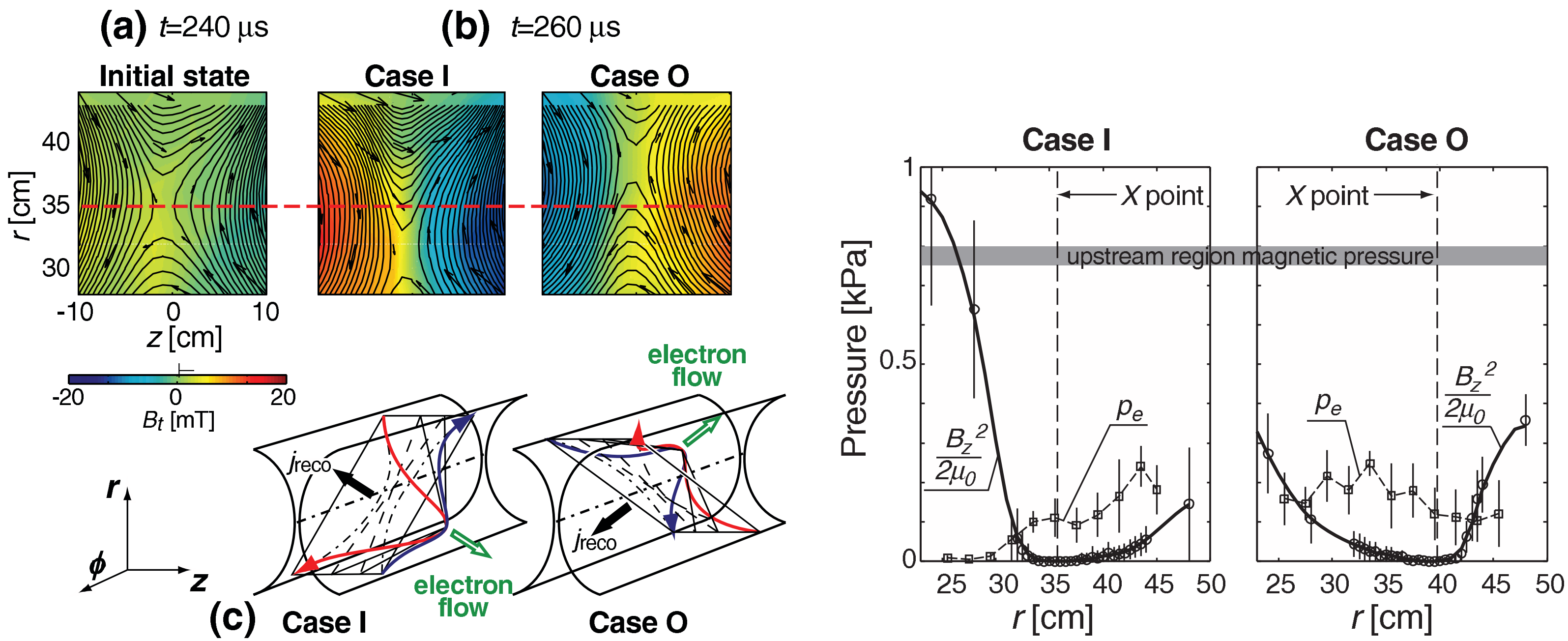
- As in pull reconnection, the quadrupole field is located along the separatrix
- Push reconnection is generally not studied in MRX, so there is no comparison of quadrupole field shape to experiment

Comparison of reconnection rate



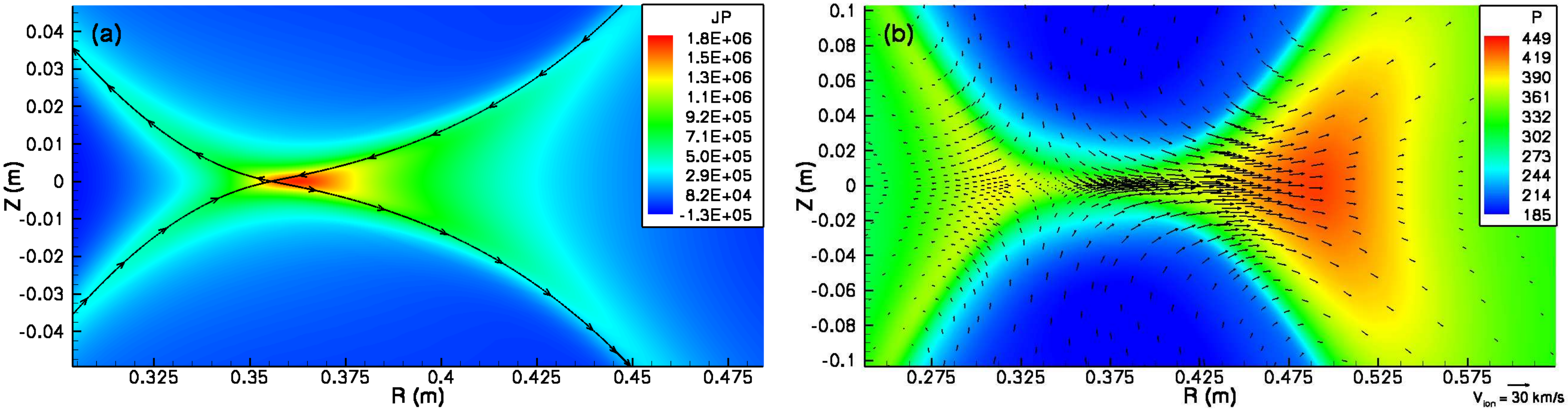
- For a given physical model, push reconnection is always faster than pull reconnection due to the effects of downstream pressure and geometry
- For a given mode of operation, two-fluid reconnection is always faster than resistive MHD reconnection
- The reconnection rate depends more on geometric mode of operation than it does on physical model

X-point motion is observed during spheromak merging in MRX



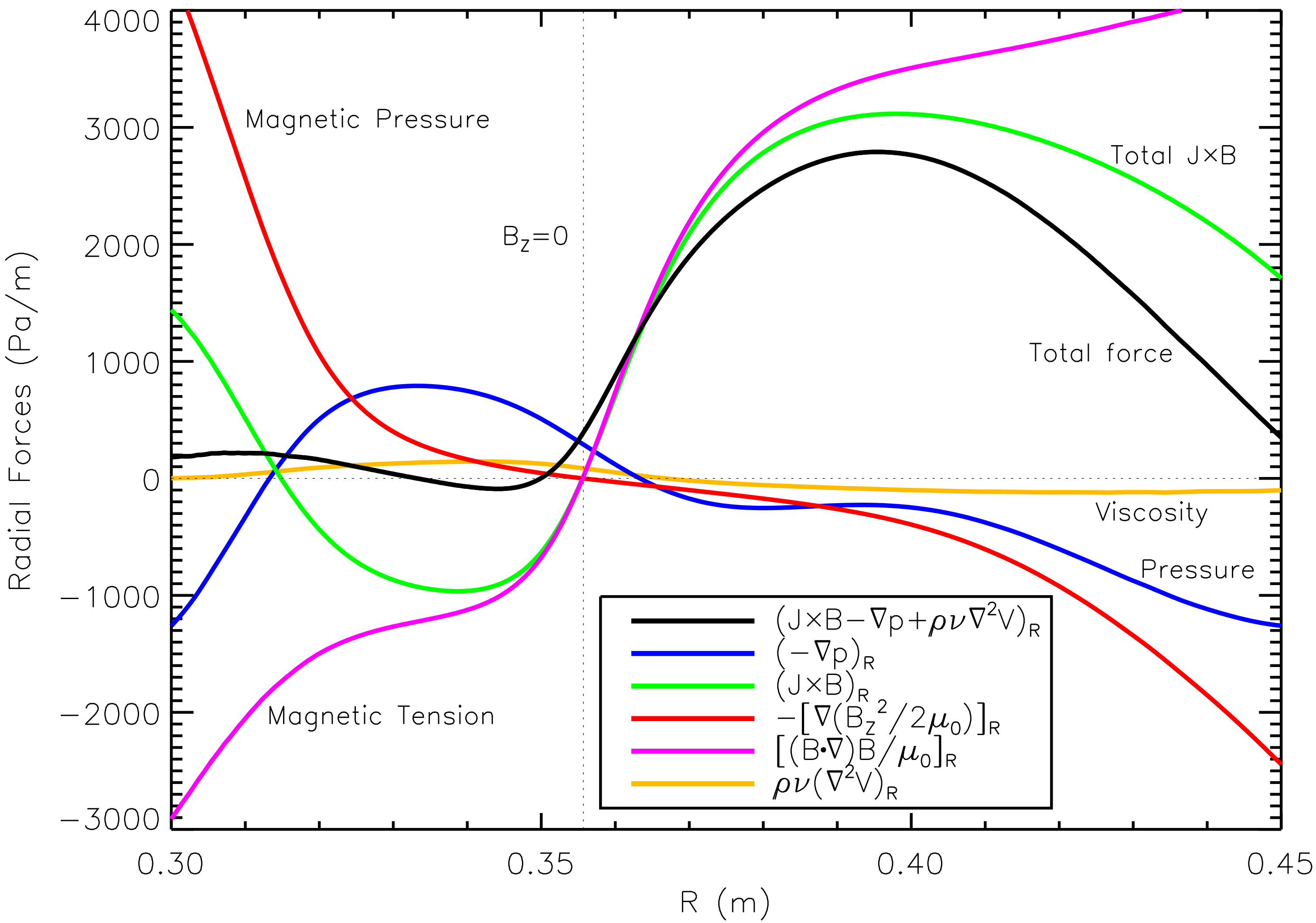
- Hall symmetry breaking is apparent when the orientation of the toroidal field is changed during two-fluid reconnection in MRX (Inomoto *et al.* 2006)
- The electron motion in the reconnection current pulls the magnetic field in a direction perpendicular to the plane of reconnection, which is projected back in the poloidal plane as a radial shift in position
- The Hall symmetry breaking causes asymmetric outflow patterns, which can be investigated in simulation

Counter-helicity push reconnection



- The left plot shows out-of-plane current density contours with a magnetic field stream trace, and the right plot shows pressure contours with V_i vectors for a counter-helicity simulation of push reconnection in a linear geometry
- The current sheet position becomes shifted with respect to 37.5 cm and there is asymmetric outflow, in accordance with the results from Inomoto *et al.* (2006).
- Several effects contribute to asymmetric outflow
  - On the left, increased magnetic pressure impedes the outflow (e.g. Inomoto *et al.* (2006))
  - On the right, increased magnetic tension facilitates outflow
  - Above and below, diamagnetic effects result in strong ion flow to the right

Terms from the momentum equation along $Z = 0$ show the importance of tension



- On the right side, the total force is dominated by tension
- On the left side, tension is almost balanced the pressure gradient
- Magnetic pressure acts indirectly, by forcing a buildup of pressure which inhibits outflow

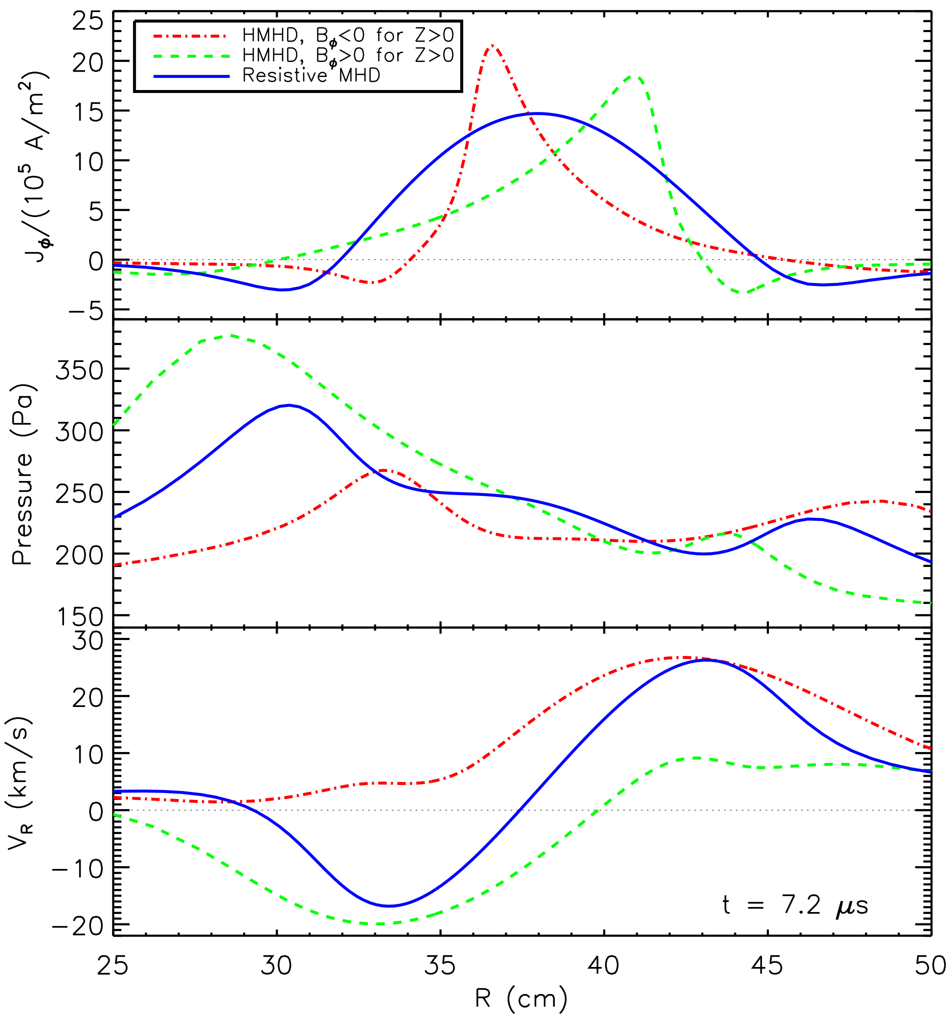
Diamagnetic flows are present above and below the current sheet

- The diamagnetic drift is given by

$$\mathbf{V}_{*j} = \frac{\mathbf{B} \times \nabla p_j}{q_j n_j B^2}$$

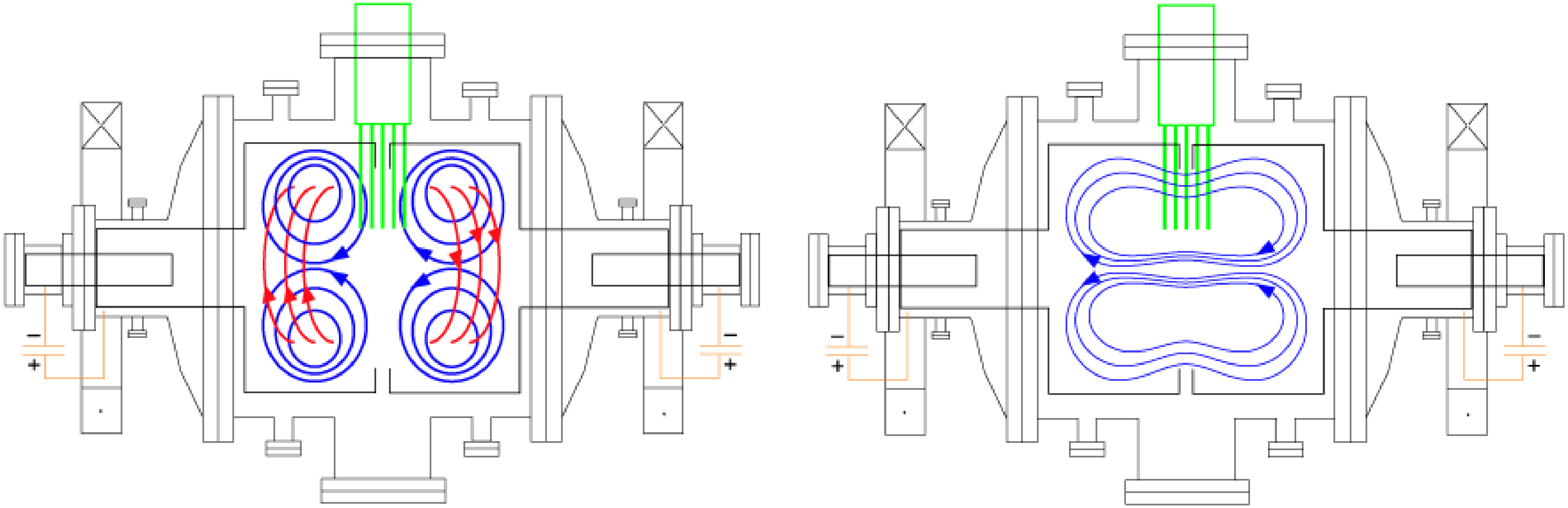
- Above and below the current sheet there is a large pressure gradient in the inflow direction
- The out-of-plane $\mathbf{B}$ is out-of-page above and into the page below the current sheet
- Ion diamagnetic flow is to the right and electron flow is to the left with speeds comparable to the outflow velocity
- The pressure buildup is associated with the ion outflow
- Note that the diamagnetic drift velocity acts only above and below the current sheet and not at $Z = 0$

Toroidal symmetry breaking can act with or against Hall symmetry breaking



- The toroidal current density, pressure profile, and outflow velocity (seen at $Z = 0$) all depend on both toroidal and Hall symmetry breaking
- Symmetry breaking due to toroidicity and the Hall effect can work together or oppositely

Swarthmore Spheromak Experiment



- NIMROD Simulations are also performed of the Swarthmore Spheromak Experiment (SSX)
- Two spheromaks are formed by plasma guns, which then merge and reconnect at the midplane of the flux conserver (Cothran *et al.*, 2003)
- Plasma parameters are similar to MRX ($n \lesssim 10^{15}$ cm^{-3}, $T \sim 20$ eV, and $S \sim 1000$)
- The prolate flux conserver is currently being replaced with an oblate flux conserver

Conclusions

- MRX conclusions include:
  - A radially inward drift of the current sheet results from fast density depletion on the inboard side
  - The outboard quadrupole lobes peak closer to the X-point than the inboard quadrupole lobes
  - The reconnection rate is affected more by geometry than by the inclusion of two-fluid effects
  - Counter-helicity push reconnection results in asymmetric outflow due to multiple effects (magnetic pressure, tension, diamagnetic flows) and can work with or against toroidal symmetry breaking