Asymmetric Magnetic Reconnection and the Motion of Magnetic Null Points

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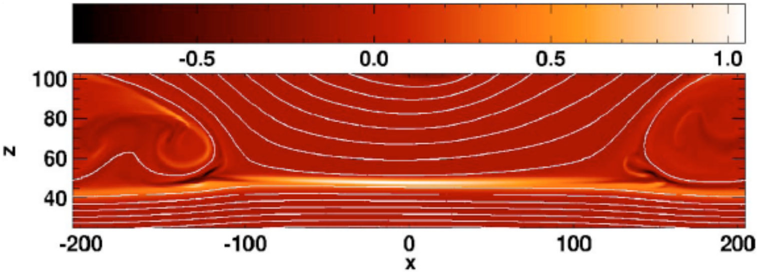
Outline

- Background information
  - Asymmetric magnetic reconnection
  - Standard model of solar flares
- Recent results
  - Observational signatures of asymmetric reconnection in solar flares and coronal mass ejections (CMEs)
  - The plasmoid instability during asymmetric inflow reconnection
  - What does it mean for a magnetic null point to move?

Introduction

- Most models of reconnection assume symmetry
- However, asymmetric magnetic reconnection occurs in the solar atmosphere, solar wind, space/astrophysical plasmas, and laboratory experiments
- *Asymmetric inflow reconnection* occurs when the upstream magnetic fields and/or plasma parameters differ
  - Earth's dayside magnetopause
  - Tearing in tokamaks, RFPs, and other confined plasmas
  - Solar jets
  - 'Pull' reconnection in MRX
- *Asymmetric outflow reconnection* occurs when conditions in the outflow regions are different
  - Solar flare and CME current sheets
  - Earth's magnetotail
  - Spheromak merging
  - 'Push' reconnection in MRX

Cassak & Shay (2007) consider the scaling of asymmetric inflow reconnection

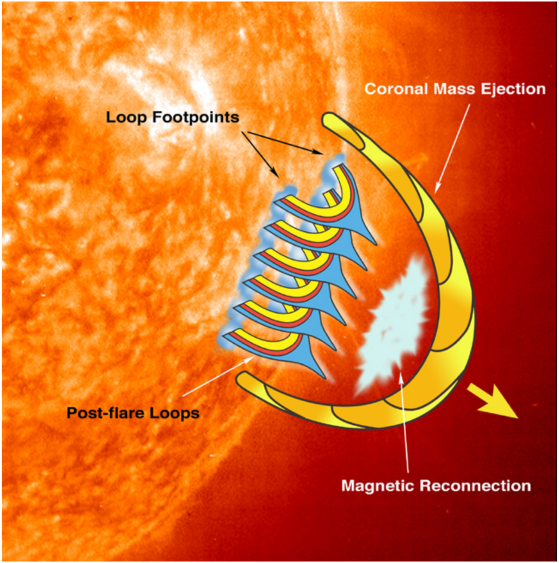


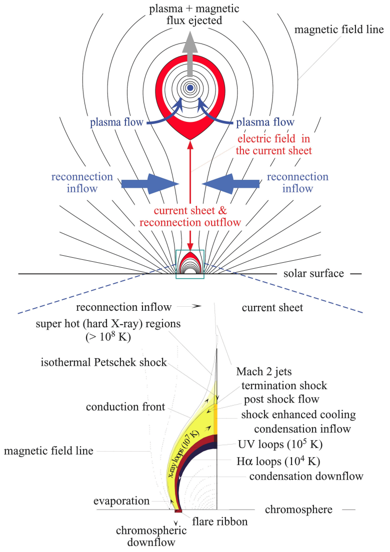
- Assume Sweet-Parker-like reconnection with different upstream magnetic fields (B_L, B_R) and densities (ρ_L, ρ_R)
- The outflow velocity scales as a hybrid Alfvén velocity:

$$V_{out} \sim V_{Ah} \equiv \sqrt{\frac{B_L B_R \left(B_L + B_R\right)}{\rho_L B_R + \rho_R B_L}} \tag{1}$$

- The X-point and flow stagnation point are not colocated

Flux rope models of CMEs predict a current sheet behind the rising flux rope





Observational signatures of solar flare reconnection

- Flare loop arcade structure of newly reconnected loops
- Ray-like structures in white light, EUV, and X-rays
  - Connecting flare loops to rising flux rope
  - Propagating blobs
  - Line widths constrain level of turbulence
- Supra-arcade downflows (and hot supra-arcade plasma)
- Inflow/outflow patterns
- Bidirectional jets
- Above-the-loop-top hard X-ray sources
- Footpoint emission from hot particles impacting chromosphere
- Termination shock (rarely observed)

- *The coronal magnetic field is very difficult to observe*
- It is not yet possible to reliably distinguish between different models of the small-scale physics of solar reconnection

Part I: Observational Signatures of Asymmetric Reconnection in Solar Eruptions

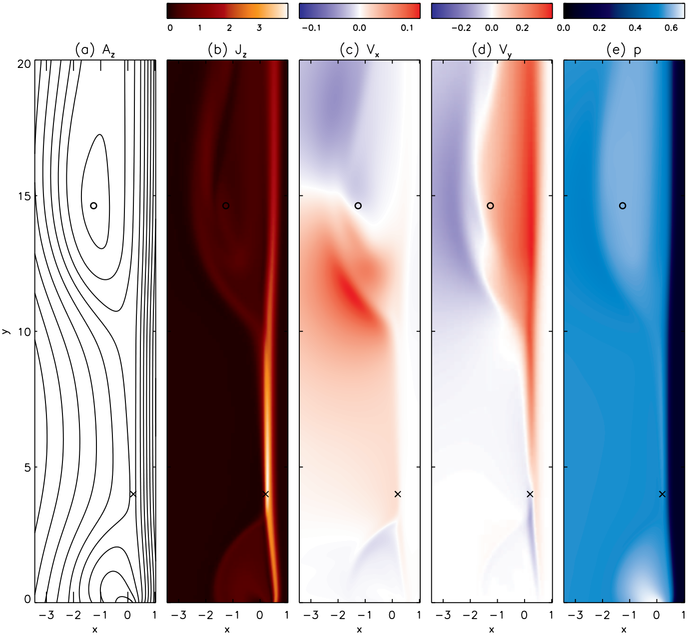
How does magnetic asymmetry impact the standard model of solar flare reconnection?

- We perform resistive MHD simulations of line-tied asymmetric reconnection using NIMROD (Murphy et al. 2012)
  - Initial X-line perturbation near wall representing photosphere for Harris-like configuration with $B_L/B_R \in \{0.125, 0.25, 0.5, 1\}$

$$B_y(x) = \frac{B_0}{1+b} \tanh\left(\frac{x}{\delta_0} - b\right) \tag{2}$$

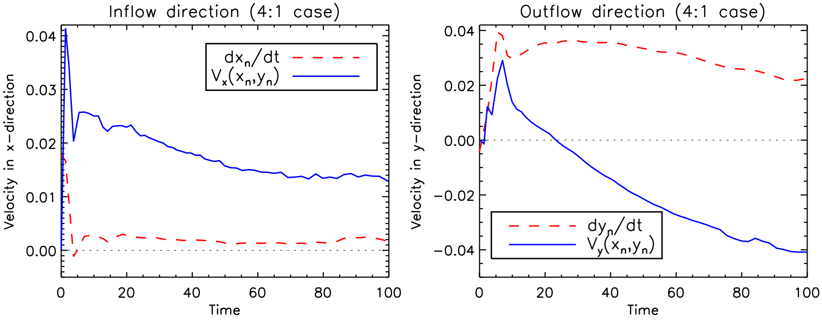
  - Caveats: β larger than reality; unphysical upper wall BC; no vertical stratification, 3D effects/guide field, collisionless effects

The X-point is low so most released energy goes up



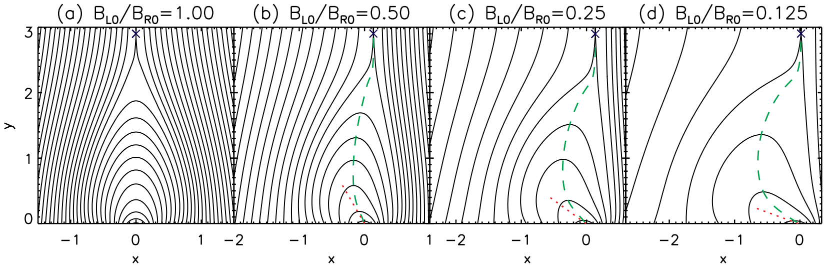
$$\frac{B_L}{B_R} = 0.25$$
$$V_{Ah} = 0.5$$

There is significant plasma flow across the X-line in both the inflow and outflow directions (see also Murphy 2010)



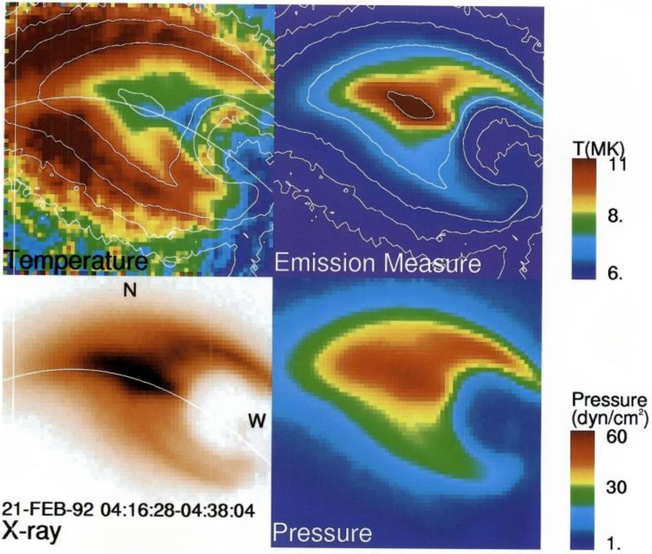
- $V_x(x_n, y_n)$ and $V_y(x_n, y_n)$ give the flow velocity at the X-line
- $\mathrm{d}x_n/\mathrm{d}t$ and $\mathrm{d}y_n/\mathrm{d}t$ give the rate of X-line motion
- For $t \gtrsim 25$, the X-line moves upward *against* the bulk flow

The flare loops develop a skewed candle flame shape



- Dashed green line: loop-top positions from simulation
- Dotted red line: analytic asymptotic approximation using potential field solution

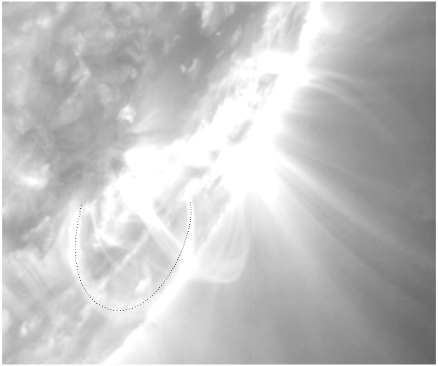
The Tsuneta (1996) flare is a famous candidate event



- Shape suggests north is weak **B** side

We fit simulated loops to multi-viewpoint observations to constrain the magnetic asymmetry

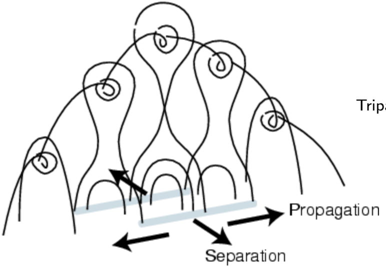
STEREO A



With D. Ranquist
and M. P. Miralles

- The most important constraints are
  - Location of looptop relative to footpoints
  - Different perspectives from *STEREO A/B* and *SDO*
- Results for two events: asymmetries between 1.5 and 4.0
- Symmetric simulations inconsistent with these observations

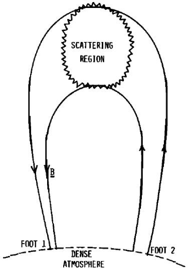
Asymmetric speeds of footpoint motion



Tripathi et al. (2006)

- The footpoints of newly reconnected loops show apparent motion away from each other as more flux is reconnected
- Equal amounts of flux reconnected from each side
  $\Rightarrow$ Weak **B** footpoint moves faster than strong **B** footpoint
- Because of the patchy distribution of flux on the photosphere, more complicated motions frequently occur

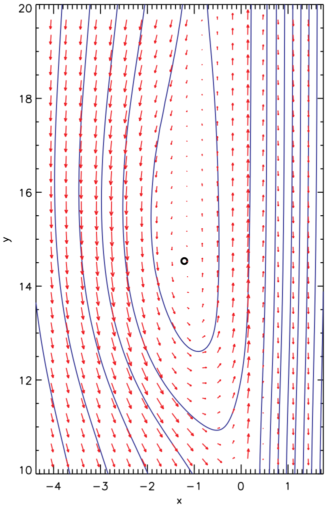
Asymmetric hard X-ray (HXR) footpoint emission



Melrose & White (1979, 1981)

- HXR emission at flare footpoints results from energetic particles impacting the chromosphere
- Magnetic mirroring is more effective on the strong **B** side
- More particles should escape on the weak **B** side, leading to greater HXR emission
- This trend is observed in $\sim$2/3 of events (Goff et al. 2004)

The outflow plasmoid develops net vorticity because the reconnection jet impacts it obliquely rather than directly



- Velocity vectors in reference frame of O-point
- Rolling motion observed in many prominence eruptions

Take away points

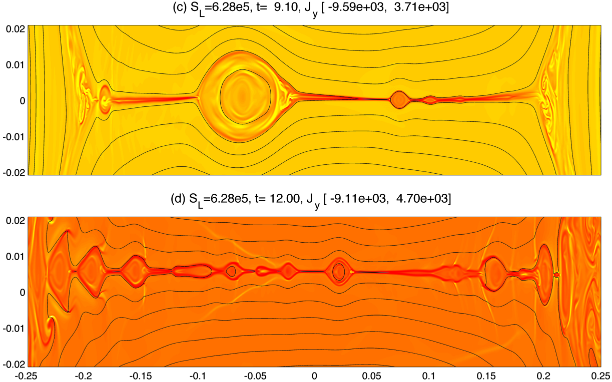
- Magnetic asymmetry leads to observational consequences during solar reconnection
  - Flare loops with skewed candle flame shape
  - Asymmetric footpoint motion and hard X-ray emission
  - Drifting of current sheet into strong field region
  - Rolling motions in rising flux rope
- Important effects not included in these simulations:
  - Realistic 3D magnetic geometry
  - Patchy distribution of photospheric flux
  - Vertical stratification of atmosphere
  - Collisionless effects

Open Questions: Solar Asymmetric Reconnection

- How can we use observation and simulation to determine the roles of 3D effects?
- What are the best ways to observationally test these falsifiable predictions?
- What connections are there to energetic particle transport?

Part II: The Plasmoid Instability During Asymmetric Inflow Magnetic Reconnection

Elongated current sheets are susceptible to the plasmoid instability (Loureiro et al. 2007)



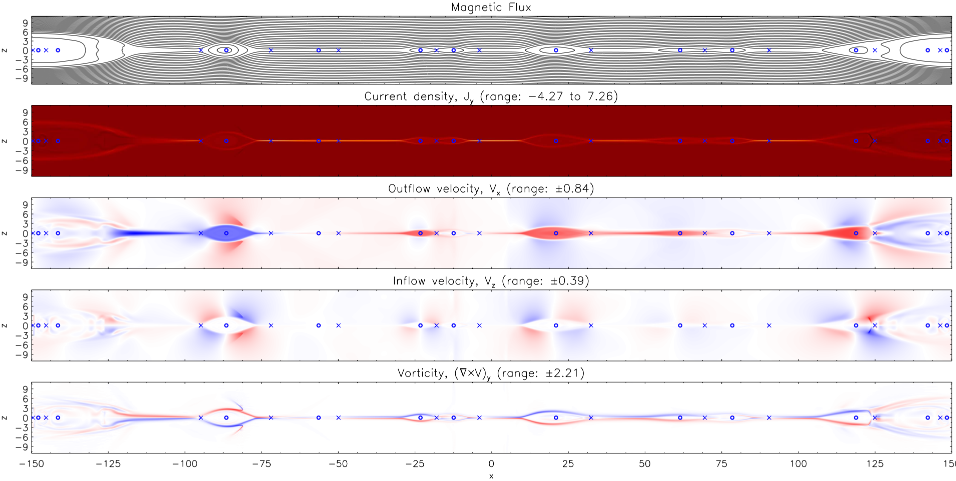
Bhattacharjee et al. (2009)
Huang et al. (2010–2013)

- The reconnection rate levels off at $\sim$0.01 for $S \gtrsim 4 \times 10^4$
- Shepherd & Cassak (2010) argue that this instability creates small enough structures for collisionless reconnection to onset
- Are CME current sheet blobs related to plasmoids? (Guo et al. 2013)

What are the dynamics of the plasmoid instability during asymmetric inflow reconnection?

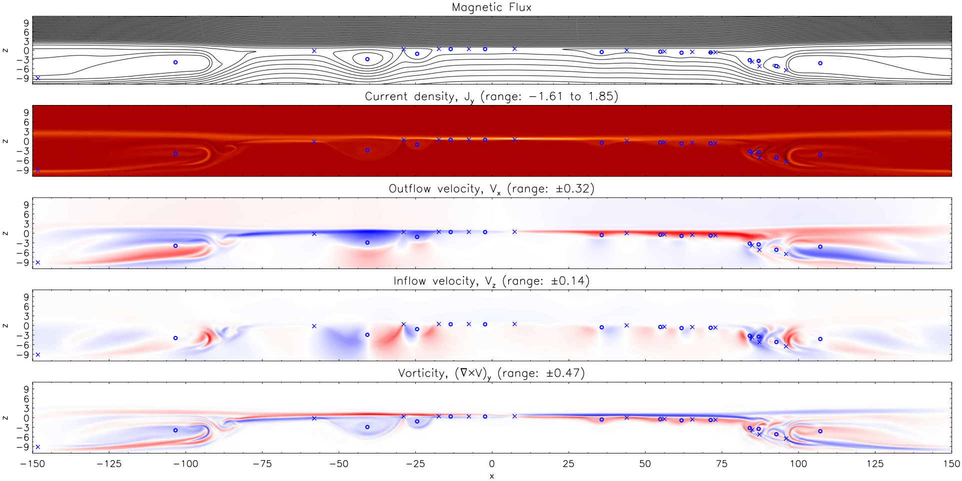
- Most simulations of the plasmoid instability assume reconnection with symmetric upstream fields
  - Simplifies computing and analysis
  - Plasmoids and outflows interact in one dimension
- In 3D, flux ropes twist and writhe and sometimes bounce off each other instead of merging
  - Asymmetric simulations offer clues to 3D dynamics
- We perform NIMROD simulations of the plasmoid instability with asymmetric magnetic fields (Murphy et al. 2013)
  - (Hybrid) Lundquist numbers up to 10^5
  - Two initial X-line perturbations along $z = 0$
  - $B_L/B_R \in \{0.125, 0.25, 0.5, 1\}$; $\beta_0 \geq 1$; periodic outflow BCs
  - Caveats: simple Harris sheet equilibrium; no guide field or 3D effects; resistive MHD

Plasmoid instability: symmetric inflow ($B_{L0}/B_{R0} = 1$)



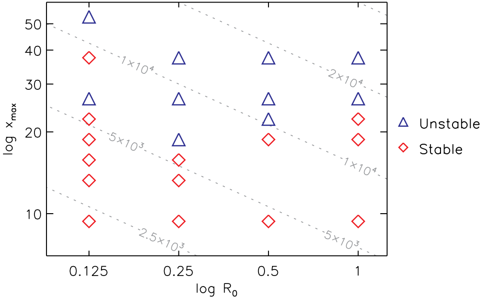
- X-points and O-points are located along symmetry axis
- X-points often located near one exit of each current sheet
- No net vorticity in islands

Plasmoid instability: asymmetric inflow ($B_{L0}/B_{R0} = 0.25$)



- Displacement between X-point and O-points along z direction
- Islands develop preferentially into weak field upstream region
- Islands have vorticity and downstream regions are turbulent

The instability onsets for smaller Lundquist numbers for moderate magnetic asymmetry



$R_0 = \frac{B_{R0}}{B_{L0}}$

$S_h = \frac{LV_{Ah}}{\eta}$

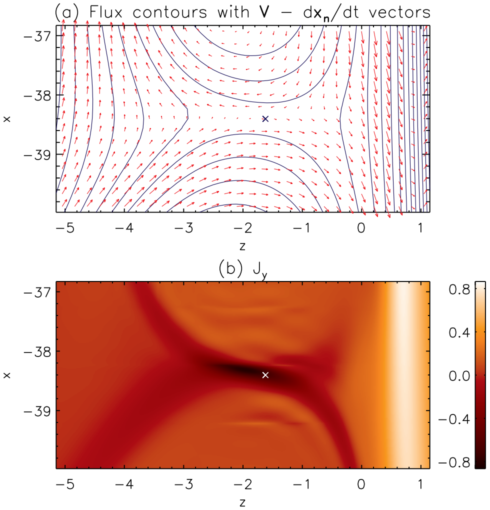
$V_{Ah} = \sqrt{\frac{B_L B_R (B_L + B_R)}{\rho_L B_R + \rho_R B_L}}$

Contours are constant hybrid Lundquist number, S_h

Note: for scaling study, we keep **B** constant on strong field side and decrease **B** on weak field side

- New X-points form for smaller domain sizes for cases with moderate magnetic asymmetry
- Can this be recovered by a linear stability analysis?

Secondary merging is doubly asymmetric



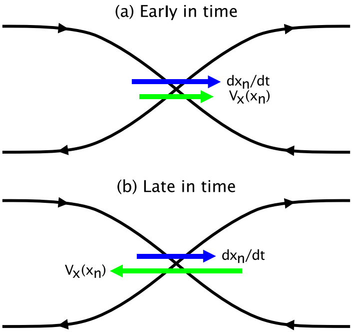
- Bottom island is much larger $\Rightarrow$ island merging is not head-on
- Flow pattern dominated by shear flow associated with island vorticity $\Rightarrow$ Partial stabilization of secondary reconnection

Open Questions: Asymmetric Plasmoid Instability

- What insights might these simulations provide for the 3D plasmoid instability?
  - Will merging between 'flux ropes' be less efficient?
- How do reconnection sites interact in 3D?
- Why do new X-lines form for smaller domain sizes with moderate magnetic asymmetry?
- How do asymmetry and 3D effects modify statistical models of islands? (Fermo/Drake, Uzdensky, Huang...)
- What mistakes are we making by using 2D simulations to interpret fundamentally 3D behavior?

Part III: What does it mean for a magnetic null point to move?

What does it mean for a magnetic null point to move?



Murphy (2010)

- In these simulations, the nulls move at velocities different from the plasma flow velocity: $\frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t} \neq \mathbf{V}(\mathbf{x}_n)$
  - Gap between flow stagnation point and magnetic field null
  - Plasma flow and X-line motion often in different directions
- To understand this, we derive an exact expression describing the motion of an isolated null point
  - We consider isolated null points because null lines and null planes are structurally unstable in 3D

Definitions

- The time-dependent position of an isolated null point is

$$\mathbf{x}_n(t) \tag{3}$$

- The null point's velocity is:

$$\mathbf{U} \equiv \frac{\mathrm{d}\mathbf{x}_n}{\mathrm{d}t} \tag{4}$$

- The Jacobian matrix of the magnetic field at the null point is

$$\mathbf{M} \equiv \begin{pmatrix} \partial_x B_x & \partial_y B_x & \partial_z B_x \\ \partial_x B_y & \partial_y B_y & \partial_z B_y \\ \partial_x B_z & \partial_y B_z & \partial_z B_z \end{pmatrix}_{\mathbf{x}_n} \tag{5}$$

The local magnetic field structure near the null is given by $\mathbf{B} = \mathbf{M}\mathbf{r}$ where $\mathbf{r}$ is the position vector relative to the null.

We derive an expression for the motion of a null point in an arbitrary time-varying vector field with smooth derivatives

- First we take the derivative of the magnetic field following the motion of the magnetic field null,

$$\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} + \left.(\mathbf{U}\cdot\nabla)\,\mathbf{B}\right|_{\mathbf{x}_n} = 0 \qquad (6)$$

  The RHS equals zero because the magnetic field will not change from zero as we follow the null point.

- By solving for **U** in Eq. 6, we arrive at the exact relation

$$\mathbf{U} = -\mathbf{M}^{-1}\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} \qquad (7)$$

  - Independent of Maxwell's equations
  - Assumes C^1 continuity of **B** about $\mathbf{x}_n$
  - Unique null point velocity when **M** is non-singular

We use Faraday's law to get an expression for the motion of a null point that remains independent of Ohm's law

- Faraday's law is given exactly by

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{8}$$

- By applying Faraday's law to Eq. 7, we arrive at

$$\mathbf{U} = \mathbf{M}^{-1} \; \nabla \times \mathbf{E}\big|_{\mathbf{x}_n} \tag{9}$$

In resistive MHD, null point motion results from a combination of advection by the bulk plasma flow and resistive diffusion of the magnetic field

- Next, we apply the resistive MHD Ohm's law,

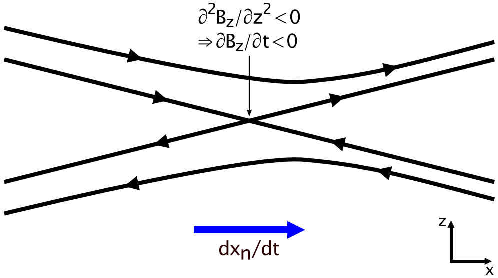
$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \tag{10}$$

  where we assume the resistivity to be uniform.
- The expression for the rate of motion of a null point becomes

$$\mathbf{U} = \mathbf{V} - \eta\, \mathbf{M}^{-1}\, \nabla^2 \mathbf{B} \tag{11}$$

  where all quantities are evaluated at the magnetic null point. The terms on the RHS represent null point motion by
  - Bulk plasma flow
  - Resistive diffusion of the magnetic field

Murphy (2010): 1D X-line retreat via resistive diffusion



- B_z is negative above and below the X-line
- Diffusion of B_z leads to the current X-line position having negative B_z at a slightly later time
- The X-line moves to the right as a result of diffusion of the normal component of the magnetic field

$$\frac{\mathrm{d}x_n}{\mathrm{d}t} = V_x(x_n) - \eta \left[\frac{\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2}}{\frac{\partial B_z}{\partial x}} \right]_{x_n} \qquad (12)$$

What does it mean for a magnetic null point to move?

- The velocity of a null point depends intrinsically on *local* plasma parameters evaluated at the null
- Global dynamics help set the local conditions
- A unique null point velocity exists if $\mathbf{M}$ is non-singular
- Nulls are not objects and cannot be pushed by, e.g., pressure gradient forces
  - Indirect coupling between the momentum equation and the combined Faraday/Ohm's law
  - Plasma not permanently affixed to nulls in non-ideal cases
- Our expression provides a further constraint on the structures of asymmetric diffusion regions (Cassak & Shay 2007)
- How do we connect this local expression into global models?

Appearance and disappearance of null points

- In resistive MHD, nulls must diffuse in and out of existence
  - Not accounted for in bifurcation theory/topological analysis
- At instant of formation or disappearance, a null is degenerate so **M** is singular
- For a null to exist at $\mathbf{r}_\star$ at a slightly later time $t_\star$, we need

$$\mathbf{M}\mathbf{r}_\star + t_\star \left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n} = 0 \qquad (13)$$

  where $\mathbf{B}(\mathbf{r}_\star) = \mathbf{M}\mathbf{r}_\star$ and $t_\star$ is the amount of time for the field at $\mathbf{r}_\star$ to disappear
  - $\mathbf{M}^{-1}$ exists $\Rightarrow$ unique velocity: $\mathbf{U} = \mathbf{r}_\star / t_\star = -\mathbf{M}^{-1} \left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n}$
  - Nulls bifurcate in directions along which $\mathbf{B}$ and $\left.\frac{\partial \mathbf{B}}{\partial t}\right|_{\mathbf{x}_n}$ are oppositely directed (find all solutions $\mathbf{r}_\star / t_\star$)
  - If no such directions exist, then the degenerate null disappears

Can we perform a similar local analysis to describe the motion of separators?

- A separator is a magnetic field line connecting two null points
  - These are often important locations for reconnection.
- Suppose that there is non-ideal behavior only along one segment of a separator.
- At a slightly later time, the field line in the ideally evolving region will in general no longer be the separator, even though the evolution was locally ideal.
- Therefore, it is not possible to find an exact expression describing separator motion based solely on local parameters.
- However, a global approach could lead to an exact expression by taking into account connectivity changes along the separator as well as motion of its endpoints.

Open Questions: Null Point Motion

- How do we connect this local expression to global models?
  - X-line retreat in magnetotail
  - Solar flares
- How do we connect this into models of the structure of diffusion regions? (e.g., Cassak & Shay 2007)
- Can we derive an expression for how the Jacobian evolves in time in non-ideal cases? (cf. Hornig & Schindler 1996)
- What mathematical tools can we use to derive an expression for the motion of separators?

Conclusions

- Magnetic asymmetry during solar eruptions lead to observational consequences
  - Flare loops have a skewed candle flame shape
  - Asymmetric footpoint motion and hard X-ray emission
  - Drifting of current sheet into strong field region
  - Rolling motions in rising flux rope
- Magnetic asymmetry qualitatively changes the dynamics of the plasmoid instability
  - Islands develop into weak field upstream region
  - Jets impact islands obliquely $\Rightarrow$ net vorticity
  - Secondary merging is less efficient
- We derive an exact expression to describe the motion of magnetic field lines
  - The motion of magnetic null point depends on parameters evaluated at the null
  - Null point motion in resistive MHD is caused by bulk plasma flow and diffusion of the component of $\mathbf{B}$ orthogonal to the motion