Simulation and Analysis of Magnetic Reconnection in an Experimental Geometry

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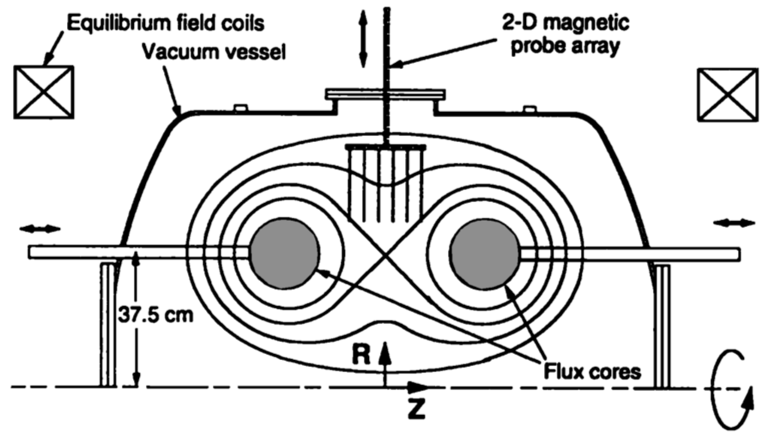
Motivation

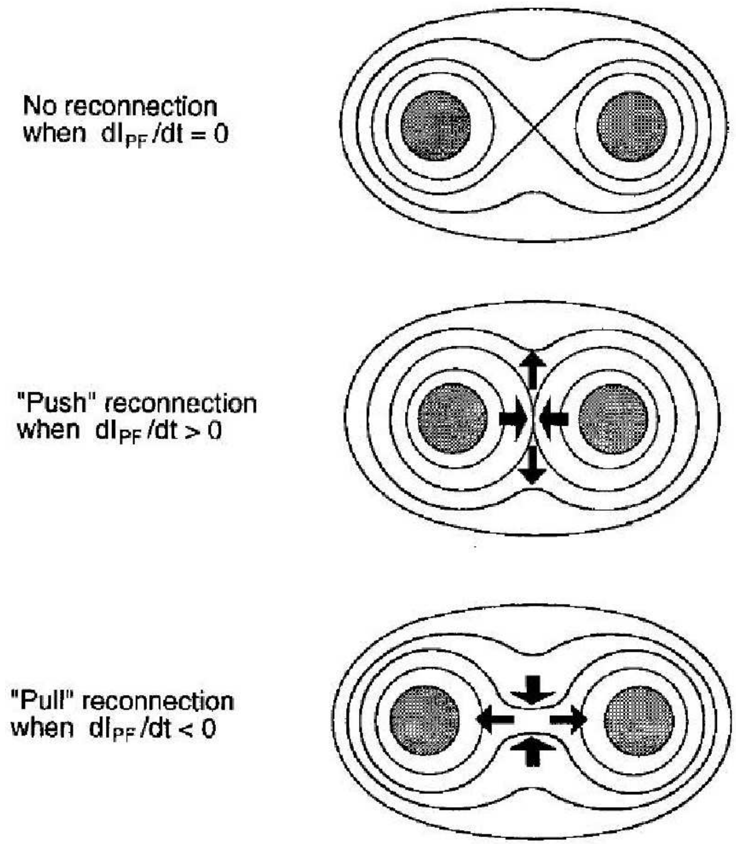
- Magnetic reconnection, or the breaking and rejoining of magnetic field lines, is an inherently multiscale process
  - Reconnection at small scales feeds back on dynamics on global scales
  - Global scales then help regulate what happens at small scales
- Two-fluid effects can facilitate fast reconnection, but are not commonly included in global simulations
- Objectives of this computational and analytical study include
  - Extended MHD simulations of reconnection in an experimental geometry
  - A model describing reconnection with asymmetry in the outflow direction

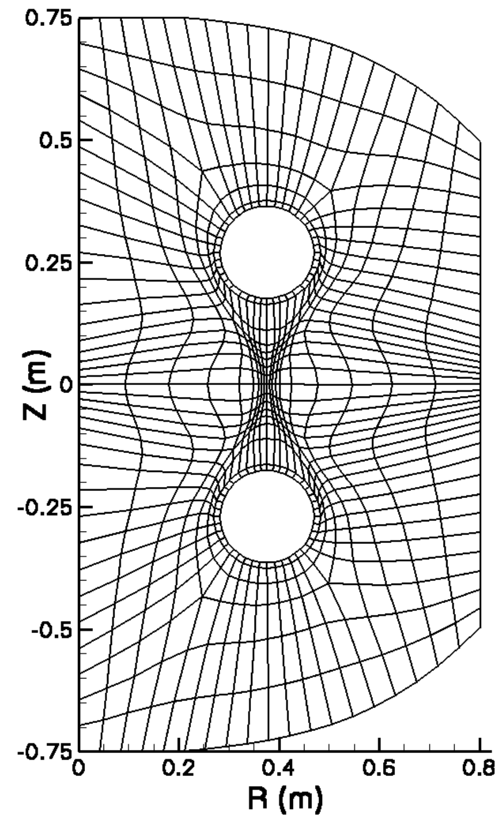
The Magnetic Reconnection Experiment



- The Magnetic Reconnection Experiment (MRX) at PPPL is a laboratory astrophysics experiment designed to study controlled axisymmetric magnetic reconnection
- Plasma parameters: $T \sim 5 - 20$ eV, $B \sim 200 - 500$ G, $S \equiv \mu_0 L V_A/\eta \sim 250 - 2500$, and $n \sim 0.1 - 1 \times 10^{14}$ cm^{-3}
- MRX is a good candidate for computational study because, unlike astrophysical plasmas, spatial scale separation is moderate and experimental plasma parameters are numerically tractable

MRX Experimental Setup





- *Left:* By changing the currents in the flux cores, two distinct modes of reconnection can be induced in MRX (Yamada et al. 1997). Our simulations of MRX investigate both of these modes of operation (Murphy & Sovinec 2008).
- *Right:* A sample finite element grid used for simulations of two-fluid pull reconnection in MRX.

NIMROD's Non-Ideal Hall MHD Model

NIMROD solves the equations of extended MHD cast in a single fluid form. The model below is used in simulations of MRX to study the interplay between local and global effects in the reconnection process.

(1) $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne}\mathbf{J} \times \mathbf{B} - \frac{1}{ne}\nabla p_e \right) + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$

(2) $$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$

(3) $$\nabla \cdot \mathbf{B} = 0$$

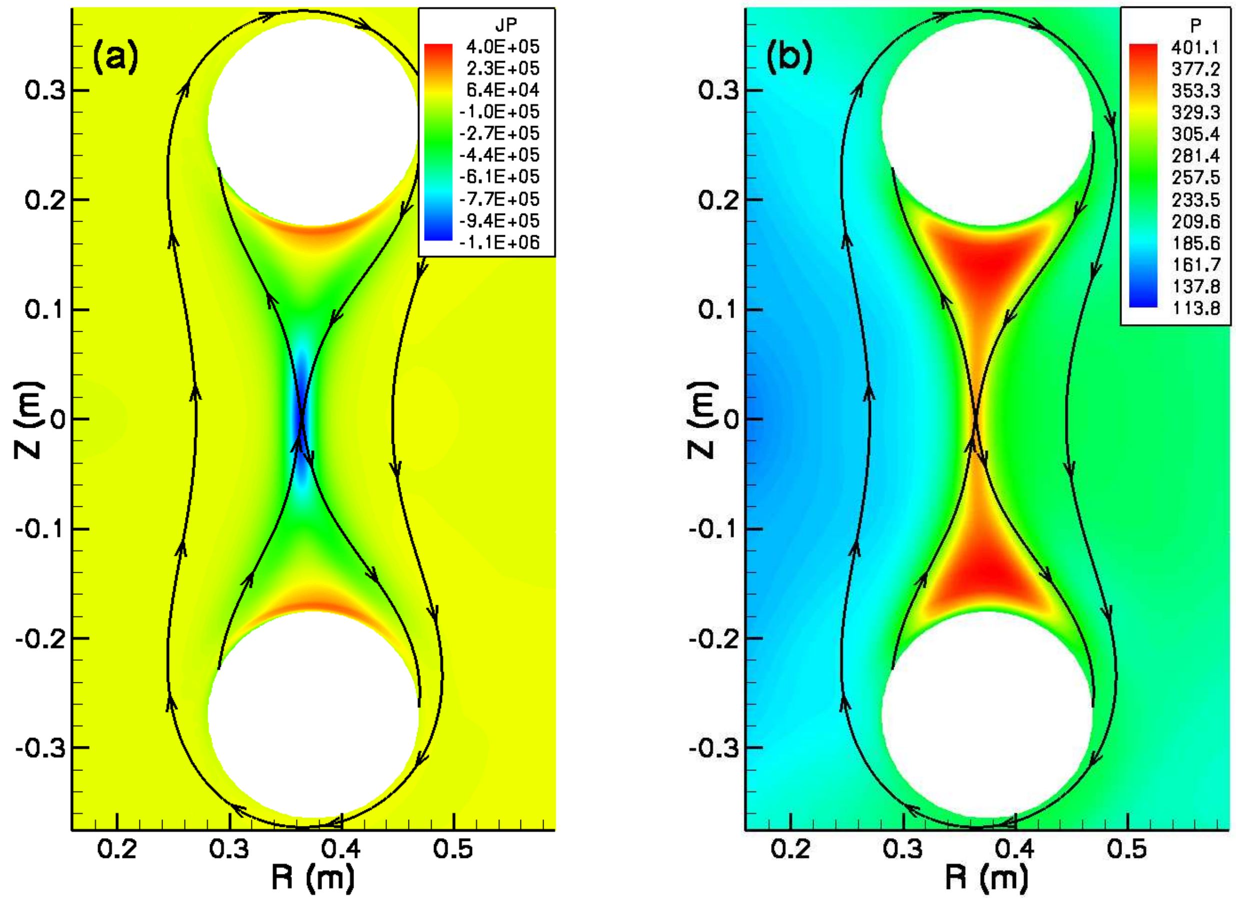
(4) $$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \rho \nu \nabla \mathbf{V}$$

(5) $$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n$$

(6) $$\frac{n}{\gamma - 1} \left(\frac{\partial T}{\partial t} + \mathbf{V} \cdot \nabla T \right) = -\frac{p}{2} \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q} + Q$$

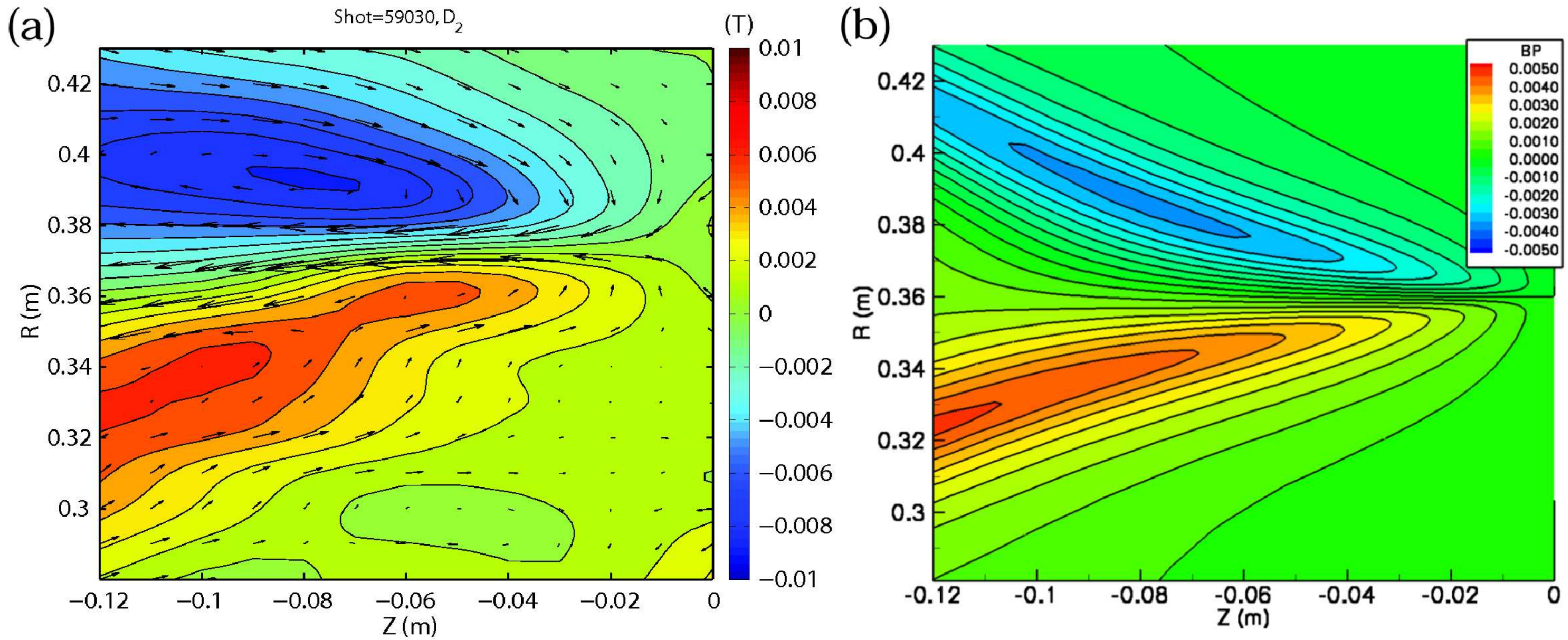
- Two-fluid effects are included via the Hall term and the electron pressure gradient in the generalized Ohm's law (blue). The terms in red are included for numerical purposes.

Simulations of pull reconnection show global pressure effects are important



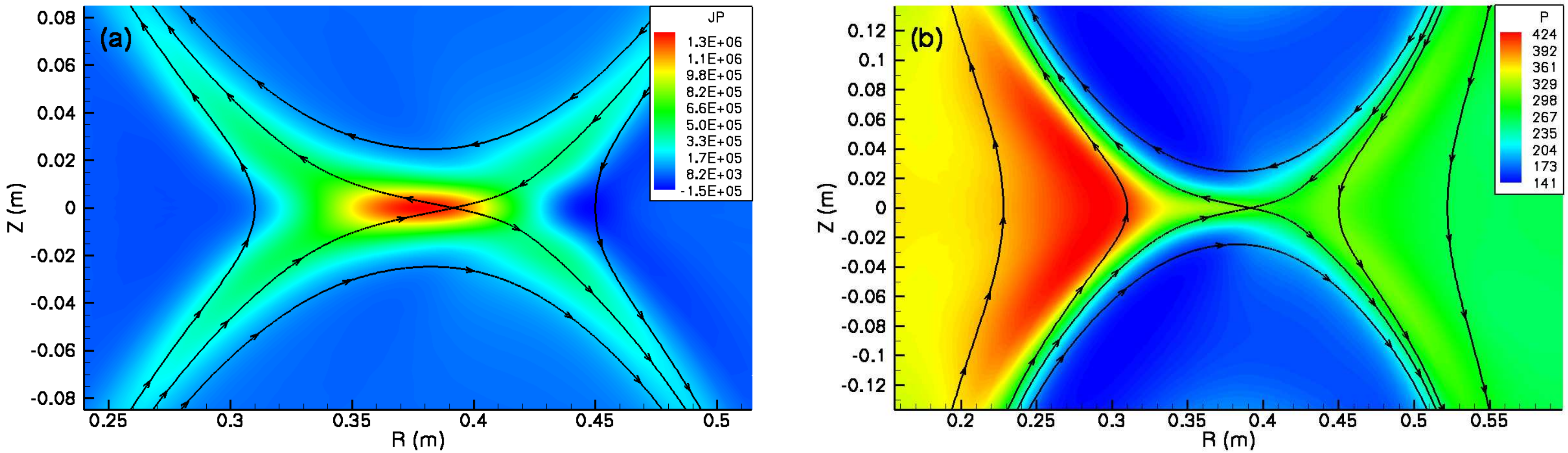
- Toroidal current density contours and plasma pressure contours (resistive MHD)
- The reconnection layer length is limited by the flux core separation and the outflow is greatly slowed due to high downstream pressure
- Density is quickly depleted on the inboard side due to the low available volume
- Higher pressure on the outboard side of the reconnection layer results in a radially inward motion of the X-point

The quadrupole shape compares favorably with experiment



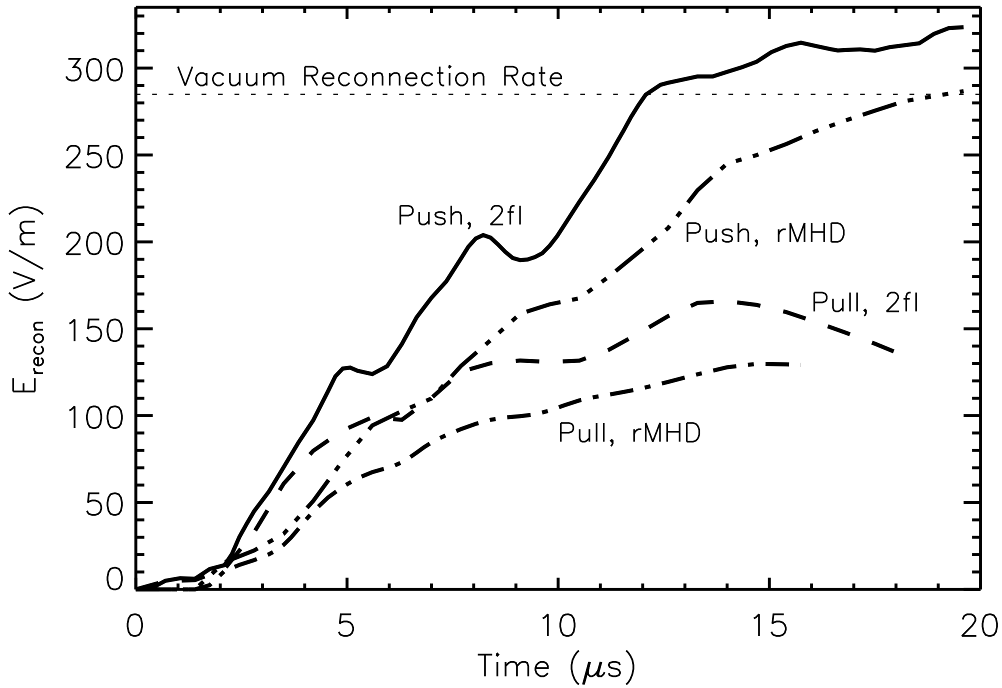
- In both experiment (left) and in our two-fluid simulations (right), the outboard quadrupole lobe peaks closer to the X-point than the inboard quadrupole lobe
- The higher density on the outboard side corresponds to weaker two-fluid effects
- Here, the asymmetry in density results from toroidicity
  - See also Pritchett (2008) for fully kinetic simulations of asymmetric reconnection in linear geometry

Global pressure differences are also important in push reconnection



- Due to the same volume effects as in pull reconnection, the inboard downstream region quickly develops high pressure, pushing the X-point to higher radii (resistive MHD simulation)
- The position of the X-point near the outboard side of the reconnection layer allows a stronger inward-directed tension force to overcome the steeper pressure gradient
- This is an example of reconnection with asymmetry in the outflow direction

In this case, geometry influences the reconnection rate more than two-fluid effects

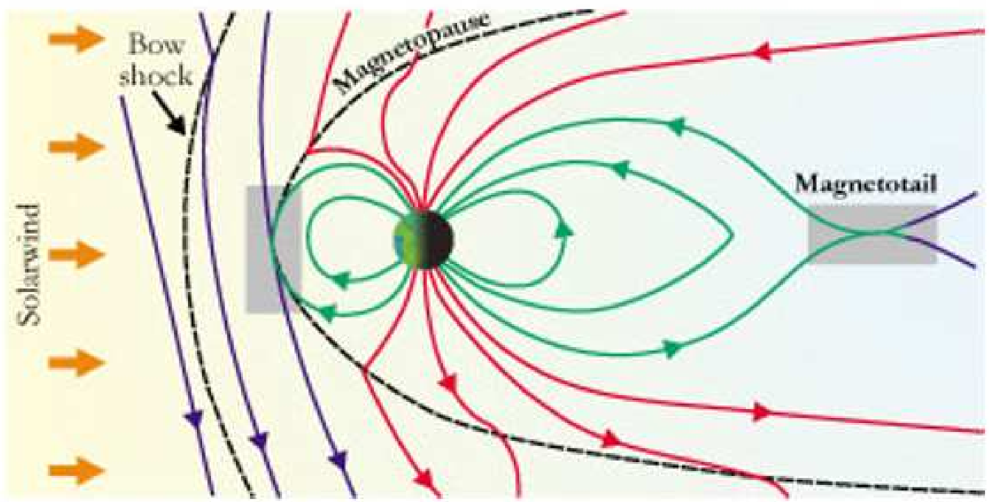


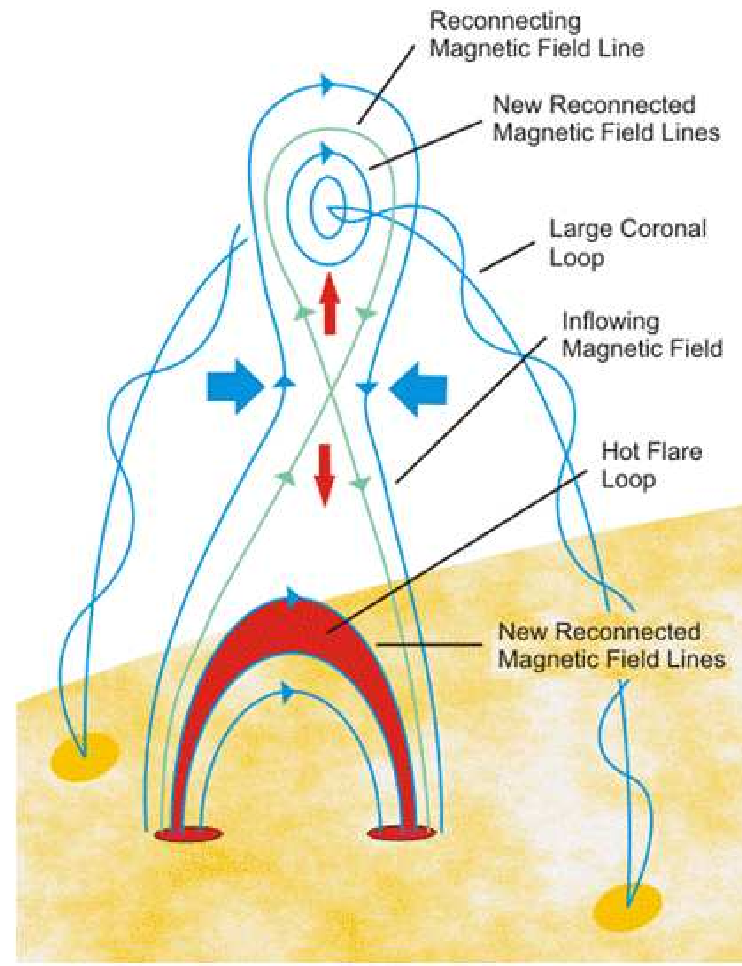
- For a given physical model, push reconnection is always quicker than pull reconnection due to the effects of downstream pressure and geometry
  - Only outflow from one side of the reconnection layer is blocked during push reconnection, whereas both sides are blocked during pull reconnection
- For a given mode of operation, two-fluid reconnection is always quicker than resistive MHD reconnection
- The reconnection rate in this setup depends more on geometric mode of operation than it does on the inclusion of two-fluid effects in our model

Reconnection with asymmetry in the outflow direction

- Reconnection in physically realistic scenarios will often have asymmetry in the outflow direction
  - Reconnection in the near-Earth magnetotail
  - Coronal mass ejections and some solar flares
  - Magnetically channeled disks in the winds of massive stars
  - Reconnection during turbulence or reconnection with multiple competing magnetic islands
  - Laboratory merging of toroidal plasma configurations (including MRX)
- Cassak & Shay (2007) extend the Sweet-Parker model to describe reconnection between plasmas with different upstream magnetic field strengths and/or different densities
- We perform a similar analysis for reconnection with asymmetric downstream pressure

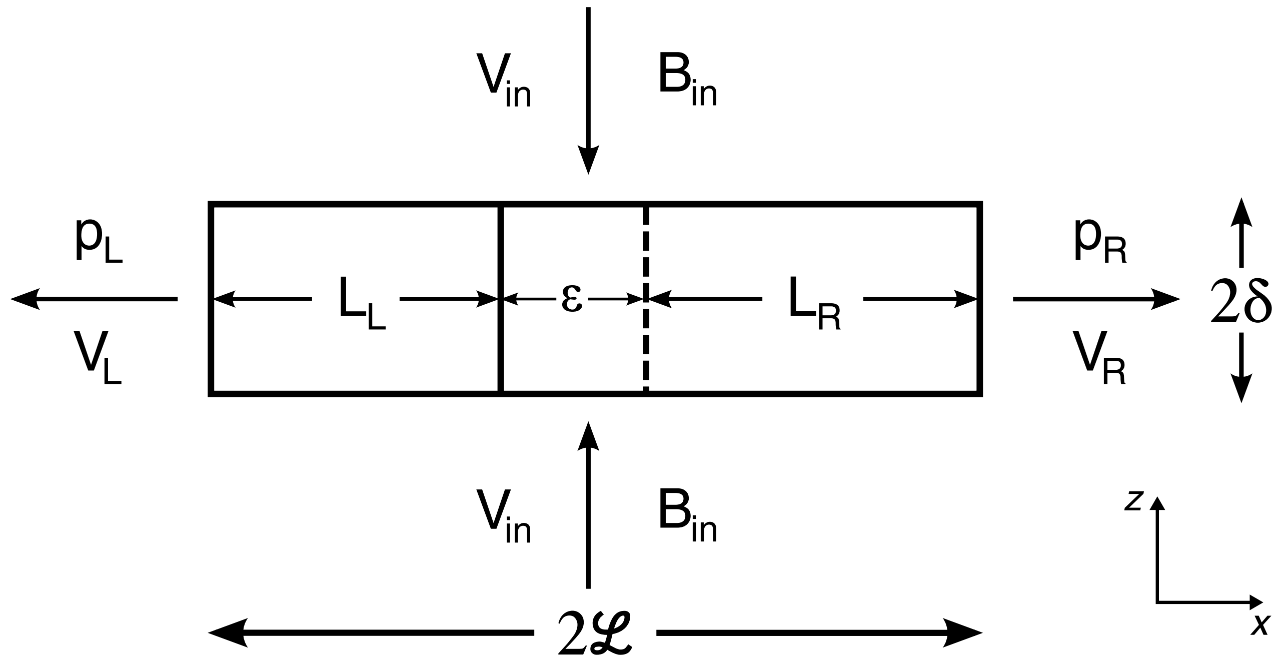
Reconnection with asymmetry in the outflow direction





- The asymmetry in the outflow direction in the Earth's magnetotail (left) and in coronal mass ejections (right) involves one outflow jet propagating into a higher density medium

Developing a model for asymmetric reconnection



- The above figure represents a current sheet with asymmetric downstream pressure ($p_L > p_R$).
- The current sheet length is given by $2\mathcal{L} \equiv L_L + \epsilon + L_R$.
- The solid vertical bar represents the flow stagnation point, and the dashed bar represents the magnetic field null.
- We assume the reconnection process is externally driven in such a way that the current sheet position remains static.

Equations of steady-state MHD in integral form

- With the help of Gauss' and Stokes' theorems, the equations of steady-state resistive MHD can be written as

$$\oint_S d\mathbf{S} \cdot (\rho \mathbf{V}) = 0 \tag{7}$$

$$\oint_S d\mathbf{S} \cdot \left[\rho \mathbf{V}\mathbf{V} + \left(p + \frac{B^2}{2\mu_0} \right) \hat{\mathbf{I}} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} \right] = 0 \tag{8}$$

$$\oint_S d\mathbf{S} \cdot \left[\left(\frac{\rho V^2}{2} + \frac{\gamma p}{\gamma - 1} \right) \mathbf{V} + \left(\frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right) \right] = 0 \tag{9}$$

$$\oint_S d\mathbf{S} \times \mathbf{E} = 0 \tag{10}$$

- In order, these represent conservation of mass, momentum, energy, and flux
- These relations are valid for any closed volume in steady-state MHD

Finding scaling relations for asymmetric outflow

- We evaluate the surface integrals given in Eqs. 7–10 over the entire reconnection layer volume
- Conservation of mass gives

$$2\rho_{in}V_{in}\mathcal{L} \sim \rho_L V_L \delta + \rho_R V_R \delta \quad (11)$$

- Conservation of momentum in the outflow direction gives

$$\rho_L V_L^2 + p_L \sim \rho_R V_R^2 + p_R \quad (12)$$

- Ignoring upstream kinetic energy and downstream magnetic energy and defining $\alpha \equiv \gamma/(\gamma - 1)$, conservation of energy gives

$$2V_{in}\mathcal{L}\left(\alpha p_{in} + \frac{B_{in}^2}{\mu_0}\right) \sim V_L\delta\left(\alpha p_L + \frac{\rho_L V_L^2}{2}\right) + V_R\delta\left(\alpha p_R + \frac{\rho_R V_R^2}{2}\right) \quad (13)$$

Deriving the outflow velocities in the incompressible case

- By using conservation of mass to eliminate $2\mathcal{L}V_{in}$ and conservation of momentum to eliminate V_R in the relationship for conservation of energy, we arrive at the cubic relationship in V_L^2

$$0 \sim V_L^4 + C_{2L}V_L^2 + C_{0L}. \tag{14}$$

with the coefficients C_{2L} and C_{0L} given by

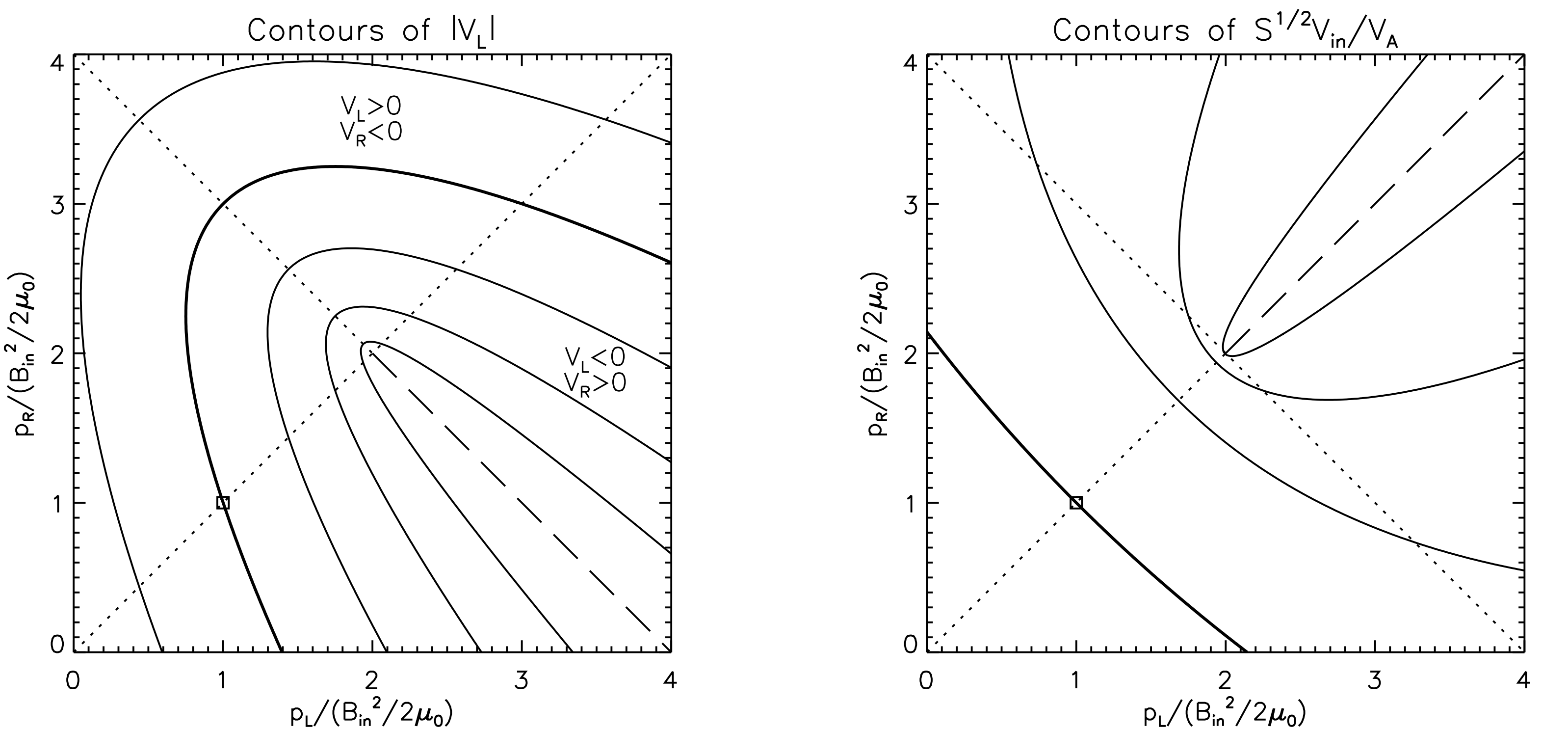
$$C_{2L} \equiv \frac{p_L - p_R}{\rho}, \tag{15}$$

$$C_{0L} \equiv 4p_{in}\left(\frac{p_L + p_R}{\rho^2}\right) - \left(\frac{2p_{in}}{\rho}\right)^2 - \left(\frac{p_L + p_R}{\rho}\right)^2 + 4V_A^2\left(\frac{p_L + p_R - 2p_{in}}{\rho}\right) - 4V_A^4. \tag{16}$$

- Then V_R can be found from conservation of momentum,

$$V_R^2 \sim V_L^2 + \left(\frac{p_L - p_R}{\rho}\right)$$

Solution plots for $|V_L|$ and $S^{1/2}V_{in}/V_A$ (incompressible case)



- *Left:* Solution contours for the magnitude of the leftward-directed outflow velocity $|V_L|$ as a function of p_L and p_R with $p_{in} = 0$. Contours are separated by 0.25V_A. The dashed line represents when $V_L = 0$.
- *Right:* Solution contours for the normalized reconnection rate $S^{1/2}V_{in}/V_A$. Contours are separated by 0.25. The dashed line represents when $S^{1/2}V_{in}/V_A = 0$.
- The reconnection rate is greatly affected only when outflow from both sides of the current sheet is blocked. The current sheet width δ is longer for greater downstream pressure.

Conclusions

- Extended MHD simulations of the Magnetic Reconnection Experiment (MRX) allow an investigation of how small scales interact with large scales in a realistic geometry
  - Much of the interplay and feedback between small and large scales is due to pressure gradients that develop as a result of the reconnection process
  - Depending on the mode of operation, reconnection in MRX is asymmetric in either the inflow or outflow direction
- Magnetic reconnection with asymmetry in the outflow direction occurs in many situations in nature and the laboratory
  - We develop a model that describes reconnection with asymmetric downstream pressure
  - Reconnection will be greatly slowed only when outflow from both ends of the reconnection layer is blocked