Astrophysical Dynamos

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Outline

- ▶ Magnetic fields in planets, stars, and galaxies
- ▶ Cowling's anti-dynamo theorem
- ▶ Mean-field dynamo theory
- ▶ α-ω dynamos
- ▶ Cosmic ray driven dynamos
- ▶ Laboratory dynamo experiments

## Introduction

- A **dynamo** converts kinetic energy into magnetic energy
- A self-sustaining dynamo is not supported by external fields or currents
- Dynamo theory provides explanations for how magnetic fields grow
  - Dynamos often require an initial seed field
- Motivating questions
  - What is the origin of astrophysical magnetic fields?
  - What sets the magnetic field strength and structure of stars, planets, and galaxies?
- Kinematic dynamo: prescribe a velocity field that causes the magnetic field to grow
- Nonlinear dynamo: include the feedback by the Lorentz force on plasma motions
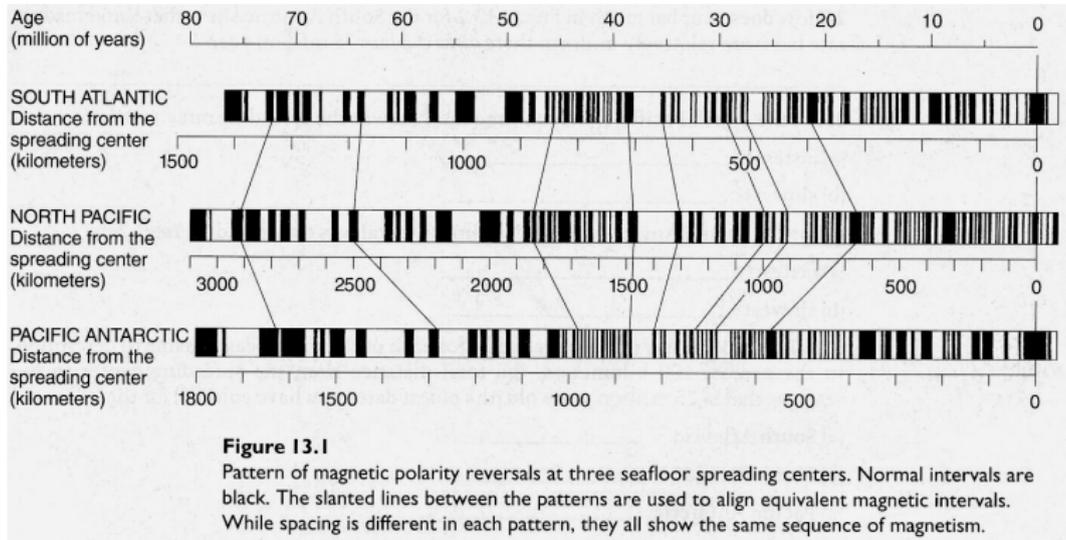
The Terrestrial Dynamo

- 1600: William Gilbert concluded that the Earth was a magnet and proposed that Earth's core contains permanently magnetized material (e.g., lodestone)
- Earth's magnetic axis is close to its rotational axis
  - Suggests that magnetism and rotation are related
  - Gilbert incorrectly guessed that magnetism caused rotation
- 1919: Joseph Larmor proposed that a dynamo gives birth to Earth's magnetic field
- Decay time of Earth's magnetic field is $\sim$20 kyr
  - Must be continually regenerated (no, not like on Doctor Who)
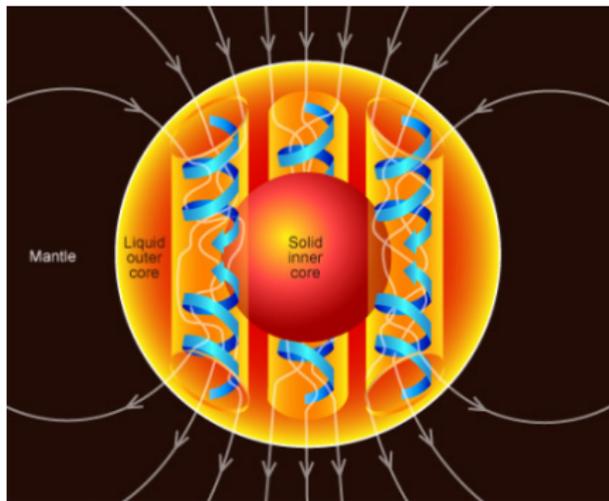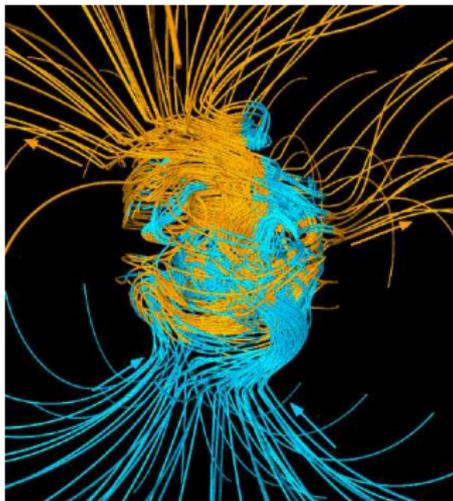
- ▶ Motion in Earth's liquid outer core is driven by heat transport from solid inner core (convection-driven dynamo)
- ▶ Quicker rotation of inner core than rest of Earth (probably)

Earth's magnetic field reverses every 5 kyr to 50 Myr

Figure 13.1
Pattern of magnetic polarity reversals at three seafloor spreading centers. Normal intervals are black. The slanted lines between the patterns are used to align equivalent magnetic intervals. While spacing is different in each pattern, they all show the same sequence of magnetism.

- ▶ Oceanic spreading occurs as new crust forms due to volcanic activity and pushes older crust away
- ▶ Magnetic minerals align with Earth's magnetic field to give a geological record magnetic field direction (paleomagnetism)

Magnetic fields of exoplanets: habitability

- Atmospheric escape mechanisms include:
  - Thermal escape (Jeans escape)
  - Interaction of the atmospheres of moons with the planetary magnetosphere
  - Erosion by stellar winds
- The importance of different mechanisms depend on the properties of the planet, atmosphere, and the stellar wind
  - Venus and Mars have weak magnetic fields but different dominant escape mechanisms
- Planetary magnetospheres reduce erosion by stellar winds
- The habitability of exoplanets depends partially on magnetic fields!

The Solar Dynamo

- Key ingredients
    - Differential rotation
    - Convective envelope surrounding a radiative core
    - Meridional flow transports flux towards/away from poles
    - Expulsion of flux/helicity through eruptions
- Key features
    - Eleven year sunspot cycle
        - Sunspots are a proxy for magnetic activity
        - Sunspots located near two belts of latitude
    - Magnetic field flips every $\sim$11 years around solar maximum
    - The magnetic field is complex — not well-described as a dipole!
    - North-south asymmetry in solar cycle

Butterfly diagram

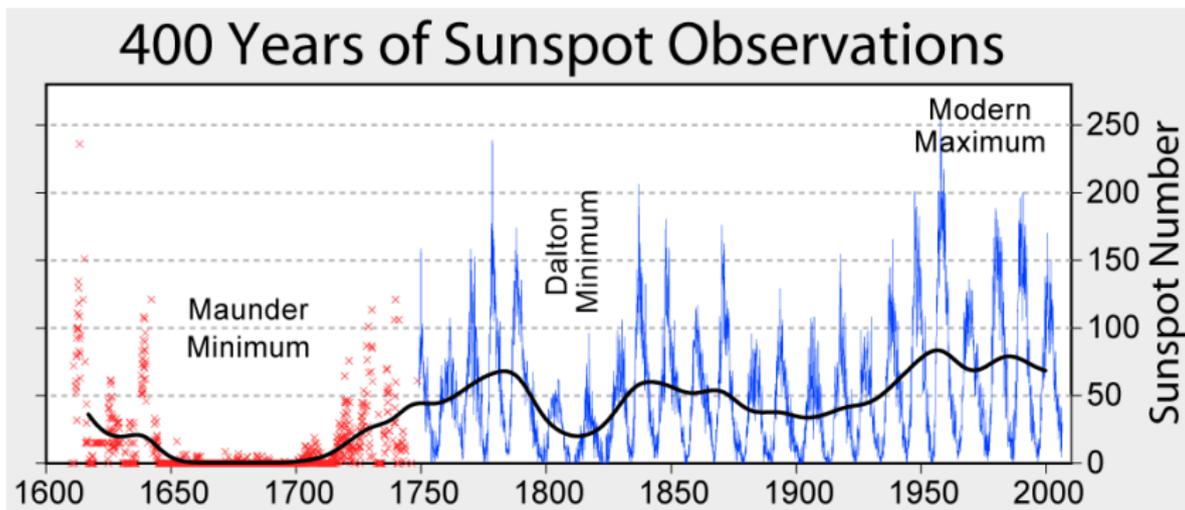DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS
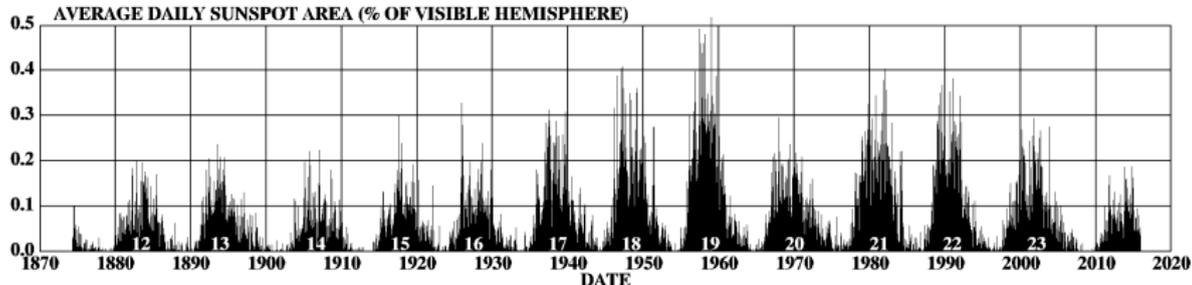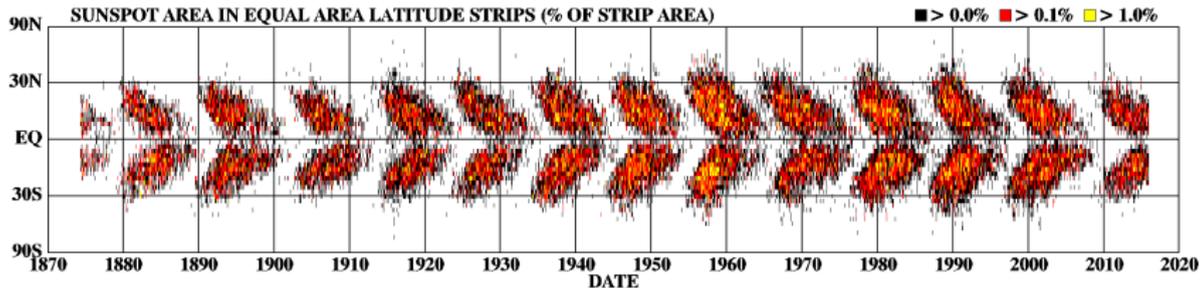
- ▶ Sunspots first emerge in two bands at mid-latitudes
- ▶ As the solar cycle proceeds, the sunspot bands become closer to the equator

Sunspot record over centuries

400 Years of Sunspot Observations

- ▶ Extended Maunder minimum from ~1650 to ~1710
- ▶ Have to cross-calibrate between different observers with different sunspot criteria, observing tools, & eyesight

We are in the declining phase of Solar Cycle 24 after an extended minimum and weak solar maximum
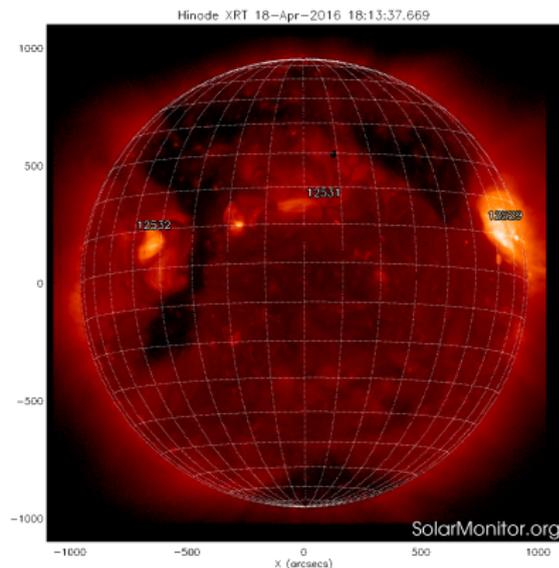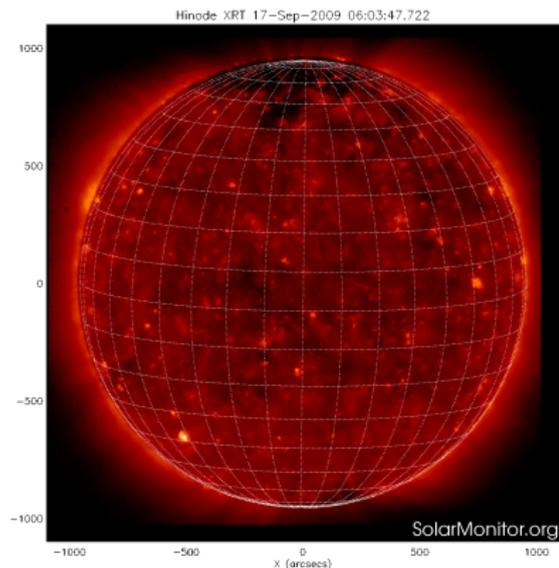
- ▶ Forecasting the solar cycle is extremely difficult
- ▶ Are we going into another Dalton-type minimum?
- ▶ If you don't like the space weather, just wait five years

The solar cycle has important implications for interplanetary travel

- The heliosphere is extended near solar maximum, which protects astronauts from interstellar cosmic rays
- However, flares and coronal mass ejections are associated with Solar Energetic Particle (SEP) events
- Suppose you want to plan a trip to Mars
- Is it better to go at solar minimum or solar maximum?

Stellar dynamos

- Sun has radiative core and convective outer envelope
  - Tachocline (boundary between radiative core and convective envelope) plays important role in dynamo
- M dwarf stars are entirely convective
  - Often strong magnetic activity
- B stars have a convective inner core and radiative outer envelope
  - Buoyancy can transport magnetic fields from core to surface
- Properties of dynamos depend significantly on convection profile and stellar rotation

Magnetic fields in galaxies

M51

A. Fletcher et al. 2008

NGC891
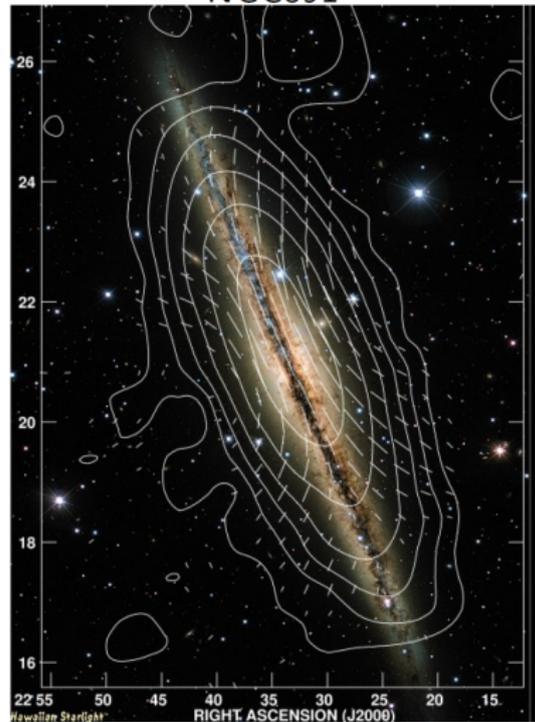
M. Krause et al. 2008

Properties of interstellar plasma in disk galaxies

- ▶ The plasma layer is highly flattened
- ▶ Gas contribution to galactic gravitational field is negligible
- ▶ Rotationally dominated but with supersonic turbulence
- ▶ Differential rotation
- ▶ Highly variable in density and temperature
- ▶ Ionization in cold regions is very superthermal
- ▶ Magnetized and coupled to cosmic ray component

Properties of the Galactic magnetic field

- The magnetic field is much more extended than the gas layer ($\sim$1.5 Mpc vs. $\sim$200 pc)
  - Cosmic rays more extended than gas layer too
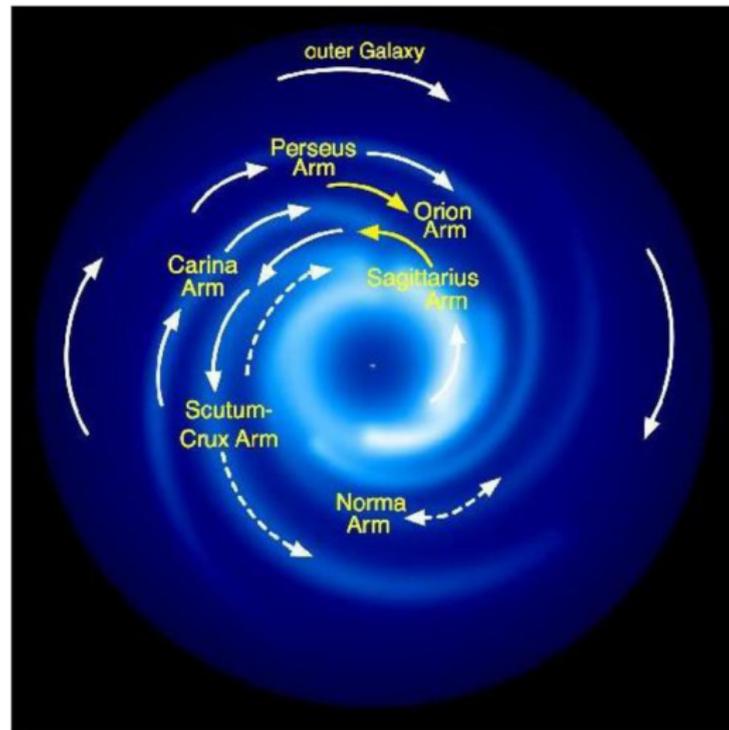- **B** mostly parallel to galactic plane except near center
- **B** aligned with Galactic rotation and even spiral arms
- Large scale ($\gtrsim$ 3 kpc) and small scale/ (random) components of **B**
- $B_{tot} \sim$ 4–5 μG; irregular component is $\sim$ 1.5–2.0 μG

- Diagnosed using Faraday rotation of polarized background sources (pulsars, quasars)

The Milky Way's magnetic field from Planck
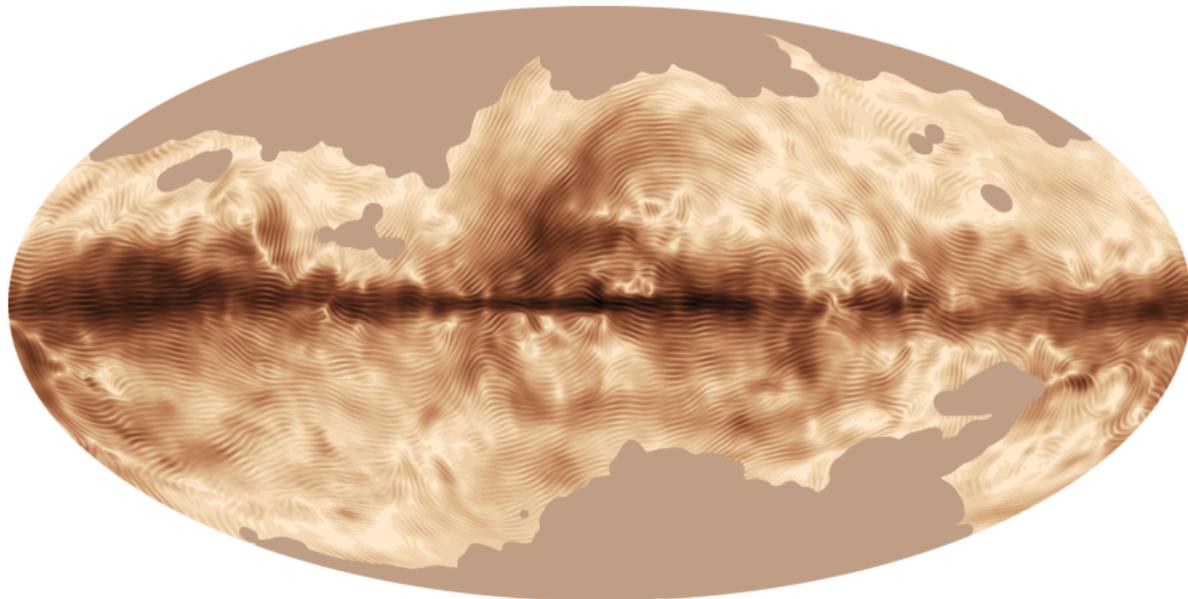
- ► Darker regions indicate stronger polarization of cosmic microwave background by dust
- ► Striations indicate direction of **B** projected on plane of sky

What about irregular galaxies?

- Properties of ISM in irregular galaxies
  - Solid body or more chaotic rotation
  - Slower rotation
  - ISM is more easily disrupted by star formation
- The magnetic fields of only a few irregular galaxies have been studied
  - **B** is sometimes strong enough to be dynamically important
  - Regular and irregular components of **B** with much variation between galaxies
- The α-ω dynamo does not work in absence of differential rotation
- Turbulent dynamos instead?

Key questions regarding the origin and growth of cosmic magnetic fields

- How did the very first (seed) magnetic fields originate?
- How do magnetic fields grow in galaxies, stars, and planets?
- How do large-scale components of the field develop?
- What allows some dynamos to be fast?
- How is magnetic flux expelled?
- What are the roles of cosmic rays, supernovae, turbulence, and magnetic reconnection in astrophysical dynamos?
- What do some dynamos exhibit spontaneous or quasicyclical field reversals?
- How do dynamos saturate?

Recall: The Biermann battery can generate magnetic fields when none were present before

- Ohm's law with a scalar electron pressure is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = -\frac{\nabla p_e}{e n_e}. \tag{1}$$

Combining this with Faraday's law yields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - c \frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \tag{2}$$

- Dynamos can **amplify** magnetic fields but not create them from scratch
- The Biermann battery provides a seed field that can then grow as a result of dynamo action
- Other seed field generation mechanisms are being investigated

Cowling's anti-dynamo theorem

- In dynamos, too much symmetry is a bad thing
- Successful dynamos cannot possess too much symmetry
- Cowling (1934): a steady axisymmetric field cannot be maintained by dynamo action
- Suppose we start out with a magnetic field of the form

$$\mathbf{B} = B_\theta(r, z)\hat{\boldsymbol{\theta}} + \mathbf{B}_p(r, z) \tag{3}$$

  where B_θ is the toroidal (azimuthal) component and $\mathbf{B}_p$ is the poloidal (radial & axial) component
- Assume that the magnetic field is time-independent and non-uniform
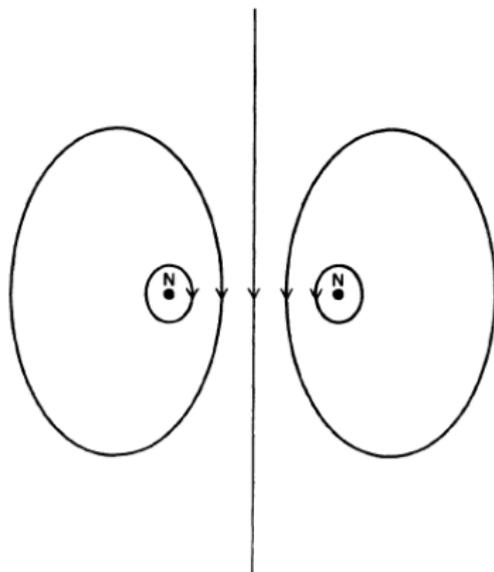
The poloidal field consists of closed flux surfaces

Fig. 9.2. The magnetic field lines in a meridional plane for an axisymmetric field.

- By symmetry, each meridional half-plane must contain at least one **O-point** in the poloidal field (denoted 'N')
- There may be an axisymmetric toroidal field component

- Ohm's law is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \tag{4}$$

- The path integral along the closed field line or null line is

$$
\begin{aligned}
\oint_{\mathcal{C}} \eta \mathbf{J} \cdot \mathrm{d}\mathbf{s} &= \oint_{\mathcal{C}} \mathbf{E} \cdot \mathrm{d}\mathbf{s} + \oint_{\mathcal{C}} \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot \mathrm{d}\mathbf{s} \\
&= \int_{\mathcal{S}} (\nabla \times \mathbf{E}) \cdot \mathrm{d}\mathbf{s} + \oint_{\mathcal{C}} \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot \mathrm{d}\mathbf{s} \tag{5}
\end{aligned}
$$

  where we used Stokes' theorem.

- The first term on the RHS vanishes because we assume a steady state

- The second term on the RHS vanishes because $\mathbf{B}$ is either parallel to the path integral or zero

Finishing the proof

- Ohm's law reduces to

$$\oint_{\mathcal{C}} \mathbf{J} \cdot \mathrm{d}\mathbf{s} = 0 \qquad (6)$$

- However, the current cannot vanish because closed flux surfaces surround N
- Therefore, this magnetic field configuration cannot sustain itself by dynamo action!
- The difficulty with an axisymmetric field is maintaining the poloidal field that decays due to resistive diffusion
- Led to decades of skepticism that dynamo theory could work
- Progress really began again two decades later with existence theorems for possible dynamo mechanisms (Herzenberg 1958; Backus 1958)

Slow vs. fast dynamos

- A dynamo is **fast** if it can operate in the limit of low resistivity: $\eta \to 0$
  - Examples (probably) include galactic dynamos and the solar dynamo
- A dynamo is **slow** if it shuts off in the limit of low resistivity
  - Example: the terrestrial dynamo

Stretch, twist, and fold (Zel'dovich 1972)
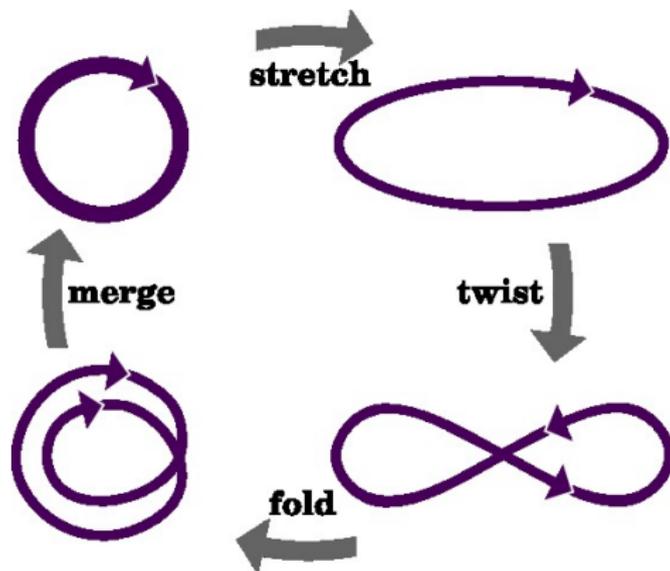
- What type of flow might realistically produce this effect?
- 'Merge' step requires magnetic reconnection

Mean field dynamo theory

- The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \qquad (7)$$

- Represent $\mathbf{B}$ and $\mathbf{V}$ each as a sum of a slowly-varying mean field component and a weak small-scale fluctuation component

$$\mathbf{V} = \langle \mathbf{V} \rangle + \mathbf{V}' \qquad (8)$$
$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}' \qquad (9)$$

  Valid if there is a separation of scales between the large-scale and small-scale fluctuations

Mean field dynamo theory

▶ The evolution of the mean field is found by averaging the induction equation over the small scale fluctuations

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times [\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle] + \nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle + \eta \nabla^2 \langle \mathbf{B} \rangle \quad (10)$$

The second term on the RHS is crucial for dynamo action

▶ The evolution of the time-varying component is

$$\begin{aligned} \frac{\partial \mathbf{B}'}{\partial t} &= \nabla \times \left[\langle \mathbf{V} \rangle \times \mathbf{B}' + \mathbf{V}' \times \langle \mathbf{B} \rangle \right] \\ &\quad + \nabla \times \left[\mathbf{V}' \times \mathbf{B}' - \langle \mathbf{V}' \times \mathbf{B}' \rangle \right] + \eta \nabla^2 \mathbf{B}' \quad (11) \end{aligned}$$

Use this equation to find $\nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle$

▶ Can sometimes choose a reference frame with $\langle \mathbf{V} \rangle = 0$

What is $\langle \mathbf{V}' \times \mathbf{B}' \rangle$?

▶ Assuming isotropic turbulence with nonzero velocity helicity, this term can be written as

$$\langle \mathbf{V}' \times \mathbf{B}' \rangle = -\alpha \langle \mathbf{B} \rangle + \beta \nabla \times \langle \mathbf{B} \rangle \tag{12}$$

where

$$\alpha = \langle \mathbf{V}' \cdot \nabla \times \mathbf{V}' \rangle \frac{\tau}{3} \tag{13}$$

$$\beta = \langle \mathbf{V}' \cdot \mathbf{V}' \rangle \frac{\tau}{3} \tag{14}$$

where τ is the velocity correlation time of the turbulence

▶ The dynamo equation then becomes

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle) + \nabla \times (\alpha \langle \mathbf{B} \rangle) + (\beta + \eta) \nabla^2 \langle \mathbf{B} \rangle \tag{15}$$

What are α and β?

- β represents turbulent diffusivity
  - Acts on the mean field
  - In a fast dynamo, $\beta \gg \eta$
- The α effect corresponds to a mean turbulent EMF that's parallel to **B**: $\mathcal{E} = \alpha \langle \mathbf{B} \rangle$
  - The α effect can deform a straight magnetic field into a helix
  - Sign of α depends on what direction the velocity vector rotates in (fluid helicity)
  - For α effect to occur, need fluid helicity $\neq 0$
  - Need finite resistivity so that magnetic field and velocity fluctuations are out of phase
  - α is in dimensions of velocity

What does α do?

- Suppose we have

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\alpha \langle \mathbf{B} \rangle), \qquad (16)$$

$$\langle \mathbf{B} \rangle = \hat{\mathbf{x}} \langle B_x \rangle (y, t) + \hat{\mathbf{z}} \langle B_z \rangle (y, t) \qquad (17)$$

- We then have two coupled equations

$$\frac{\partial B_x}{\partial t} = \frac{\partial}{\partial y} \alpha B_z \qquad (18)$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial y} \alpha B_x \qquad (19)$$

where we drop brackets for clarity

What does α do?

- Combining these equations gives

$$\frac{\partial^2 B_x}{\partial t^2} = -\alpha^2 \frac{\partial^2 B_x}{\partial y^2} \tag{20}$$
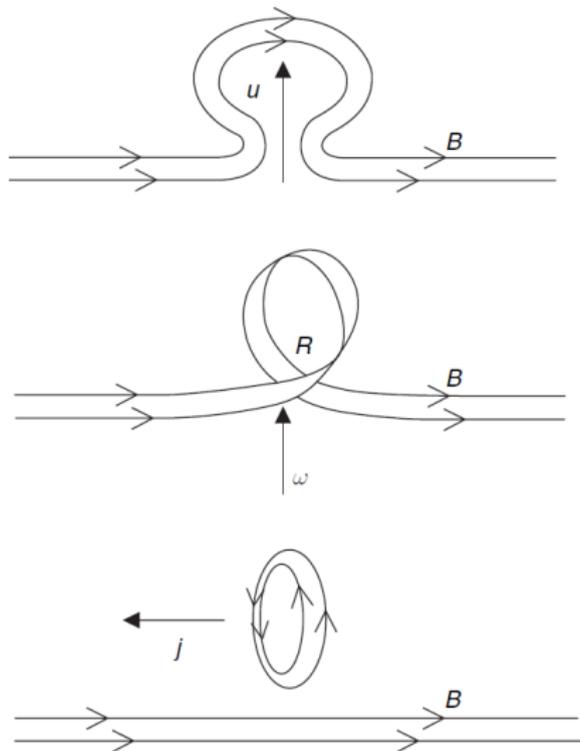
- Look for solutions of the form $e^{ipt + ik_0 y}$

$$
\begin{aligned}
-p^2 B_x &= \alpha^2 k_0^2 B_x \\
p &= \pm i\alpha k_0
\end{aligned} \tag{21}
$$

- Exponentially growing or decaying solutions are allowed!

$$\langle B_x \rangle \propto \exp\left[\mp \alpha k_0 t + ik_0 y\right] \tag{22}$$

- Feedback/exponential growth from the equation for $\langle B_z \rangle$

(From Roberts, 2007)

- The α-effect arises from $\nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle$ in Eq. 11

The ω-effect occurs when poloidal magnetic field is pulled in the toroidal direction through differential rotation

(From Roberts, 2007)
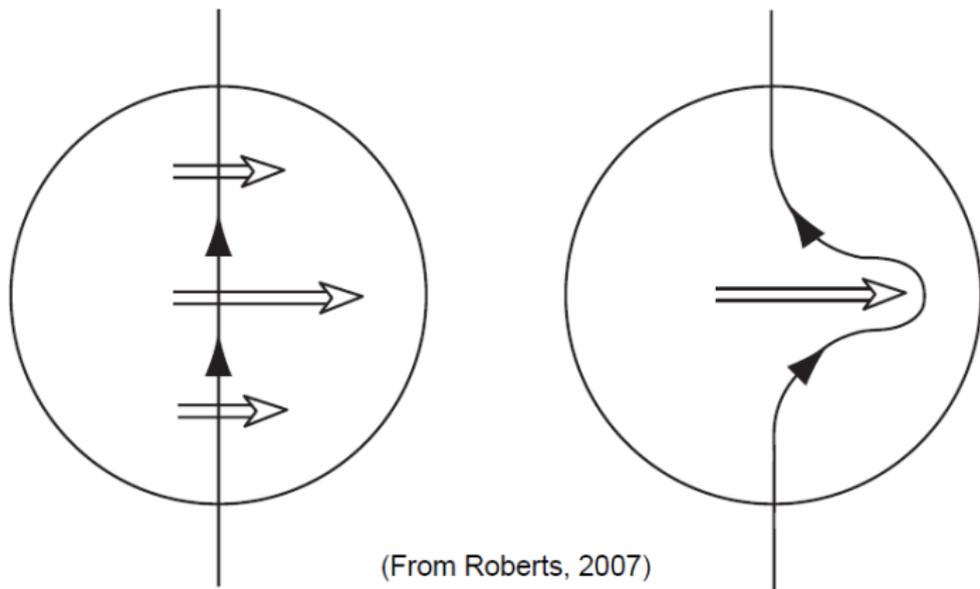
- The ω-effect arises from $\nabla \times [\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle]$ in Eq. 10
- If there's no differential rotation, there's no ω-effect

The α-ω dynamo (from Love 1999)
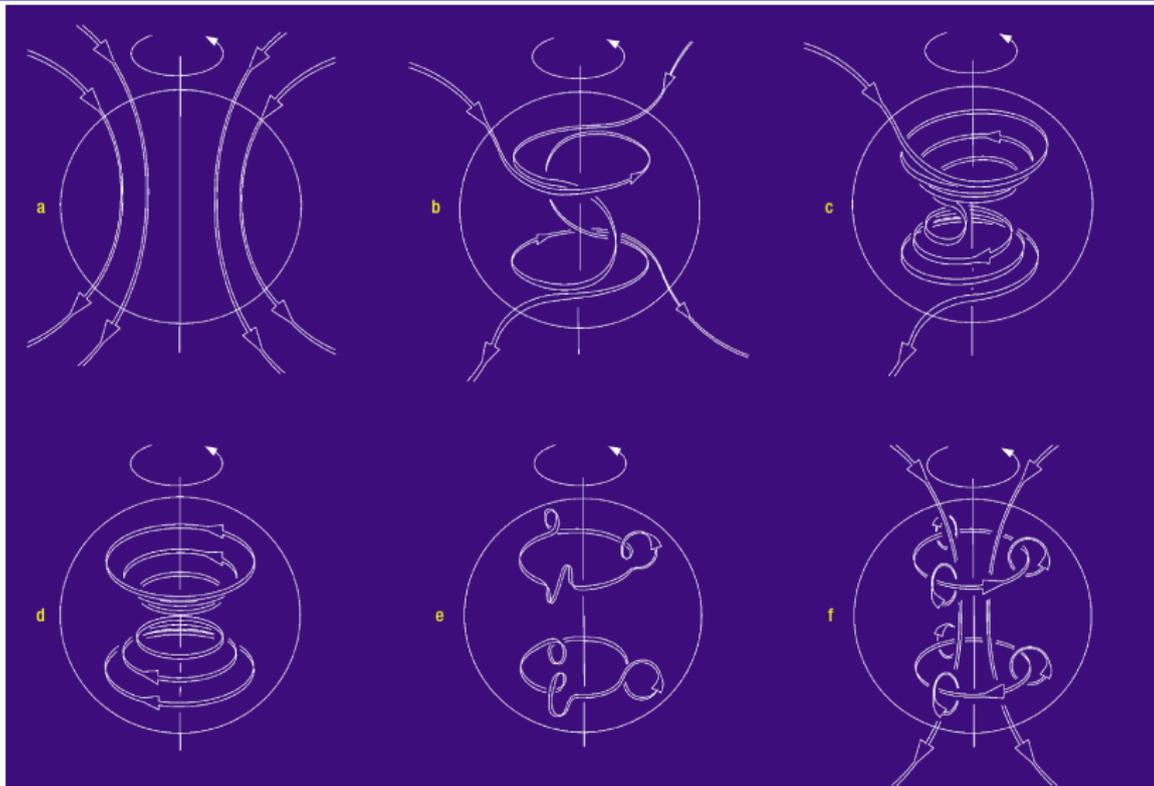
2: The α-ω dynamo mechanism. Conventional geodynamo theory presupposes (a) an initial, primarily dipolar, poloidal magnetic field. The ω-effect consists of differential rotation, (b) and (c), wrapping the magnetic field around the rotational axis, thereby (d) creating a quadrupolar toroidal magnetic field. Symmetry is broken, and dynamo action maintained, by the α-effect, whereby helical upwelling (e) creates loops of magnetic field. These loops coalesce (f) to reinforce the original dipolar field, thus closing the dynamo cycle.

- Is the *ad hoc* separation of scales valid?
- Neglects backreaction of the fluctuating components
  - Requires including Lorentz force in equations
  - If turbulent motions on small-scales are suppressed, this may shut off the dynamo
  - Quenching of dynamo can be approximated by havung α decrease in an *ad hoc* manner when approaching saturated value
- MHD turbulence is anisotropic
  - However, can take α and β to be tensors

Numerical simulations of dynamos require fewer assumptions

- No need for separation of variables assumption
- Can use full MHD equations
- Can include feedback when the magnetic field becomes strong
- Can include additional physics for a more realistic ISM
    - Density waves, feedback from star formation on ISM, etc.
    - Cosmic rays
- Caveats
    - Resistivity much larger than reality
    - Limited separation between small and large scales

Parker instability
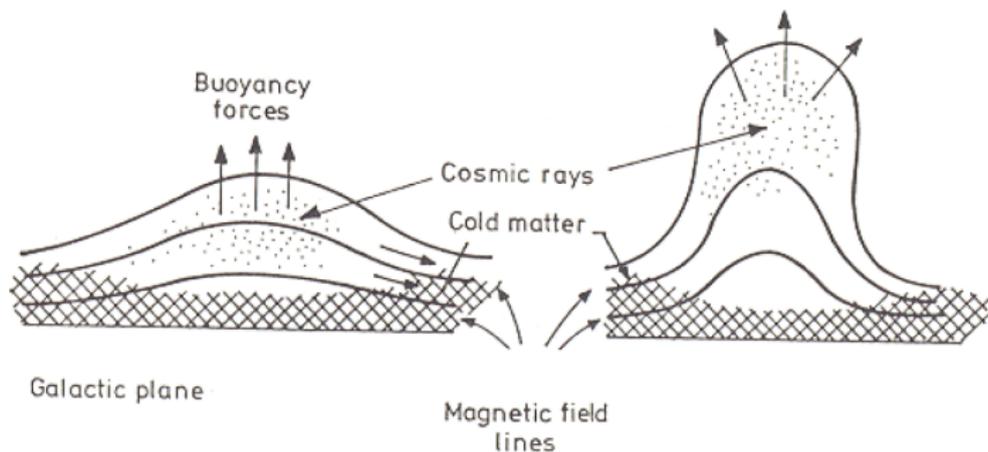
- If an initially uniform magnetic field supported against gravity becomes wavy, plasma will flow along field lines into lower dips
- The $\sim$ massless components of the ISM (cosmic rays, magnetic fields) become buoyantly unstable and can rise to form loops
- Allows escape of magnetic field from galaxies into the halo

Cosmic-ray driven dynamo

- Magnetic fields, gas, and cosmic rays are in rough energy equipartition in the ISM
  - Suggests that these different components are coupled
- Parker (1992) proposed a dynamo in which cosmic ray buoyancy instabilities play an important role
  - Numerous loops form and bulge into the galactic halo
  - Reconnection frees these loops from the initially toroidal field
  - Differential rotation, the Coriolis effect, and other instabilities then act on the poloidal loops and toroidal field
- Growth rate found to be comparable to a rotation time

- Liquid metal dynamo experiments
  - Stir up seed field with, e.g., propellors
  - Reynolds numbers not too far from the terrestrial dynamo
- Plasma dynamo experiments
  - Dynamo action occurs in many toroidal confined plasmas (e.g., reversed field pinches)
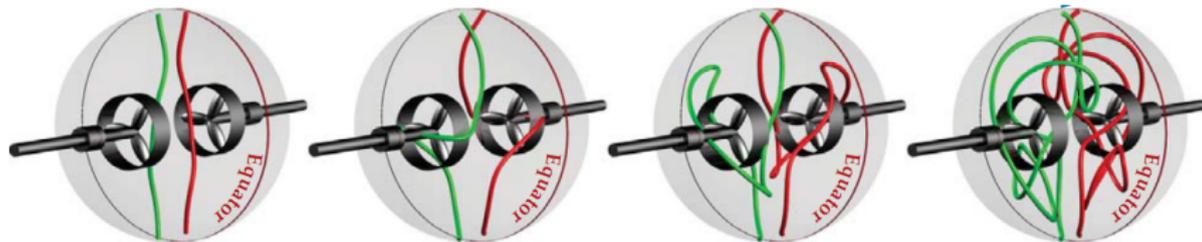 - New spherical plasma dynamo experiment at Wisconsin under construction

Laboratory dynamo experiments

a — Cadarache (60 cm)

b — Wisconsin (1 m)

c — Grenoble (40 cm)

d — Maryland (3 m)

e — Wisconsin (3 m plasma)

Figure 3. Active and pending dynamo experiments, with the diameters of their containment vessels. Those spherical containers allow freer, more natural flow of the fluid conductors than did the earlier helical-flow experiments (see figure 1). **(a)** The Von Karman Sodium experiment is active at the French Atomic Energy Commission's laboratory in Cadarache. (Photo courtesy of J.-F. Pinton.) **(b)** The liquid-sodium experiment at the University of Wisconsin–Madison is led by one of us (Forest). **(c)** Another liquid-sodium facility in France is active at the University of Grenoble. (Photo courtesy of H.-C. Nataf.) **(d)** The large new facility at the University of Maryland, College Park, led by the other of us (Lathrop), will soon begin liquid-sodium experiments. **(e)** A vessel of similar size, but designed to contain plasma rather than liquid metal, is under construction at Wisconsin.

- Seed field is stretched, twisted, and folded to grow into a stronger field
- Excessive turbulence from propellors makes sustained dynamo action difficult to achieve

Summary

- Dynamos convert kinetic energy to magnetic energy
  - Require a seed field to operate (Biermann battery?)
  - Require departure from symmetry to operate
- Dynamo theory provides explanations for the growth of astrophysical magnetic fields
  - Must explain observed properties of galactic, stellar, and planetary magnetic fields
- Mean field theory decomposes **B** into large-scale and small-scale components to allow analytical progress
- Cosmic-ray driven dynamos may operate in some galaxies
- Laboratory experiments yield insight into planetary and astrophysical dynamos