

Astrophysical Dynamamos

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Astronomy 253: Plasma Astrophysics

April 21, 2014

These lecture notes are based off of Kulsrud, Cowling (1981), Beck et al. (1996), Priest (2014), Widrow (2002), Kulsrud (1999), Moffatt (1978; no, nothing to do with Doctor Who), and several other sources.

- ▶ Magnetic fields in stars, galaxies, and planets
- ▶ Cowling's anti-dynamo theorem
- ▶ Mean-field dynamo theory
- ▶ α - ω dynamos
- ▶ Cosmic ray driven dynamos
- ▶ Laboratory dynamo experiments

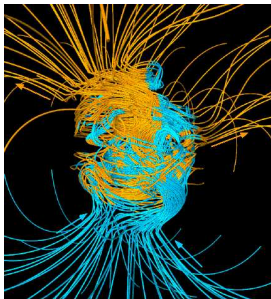
Introduction

- ▶ A *dynamo* converts kinetic energy into magnetic energy
- ▶ Motivating questions:
 - ▶ What is the origin of astrophysical magnetic fields?
 - ▶ How do magnetic field strengths grow to their observed strengths?
- ▶ Dynamo theory provides explanations for magnetic fields in the universe
- ▶ A self-exciting dynamo is not supported by external fields or currents (self-sustaining)
 - ▶ Requires an initial seed field
- ▶ Most work in astrophysics has been on *kinematic dynamos* in which the velocity field is specified
- ▶ The feedback of the magnetic field on the motions is often neglected but can be important when \mathbf{B} is strong
 - ▶ Included in *nonlinear* or *hydromagnetic dynamos*

The Terrestrial Dynamo

- ▶ 1600: William Gilbert concluded that the Earth was magnetic and proposed that Earth's core contains permanently magnetized material (e.g., lodestone)
- ▶ Earth's magnetic axis is close to its rotational axis
 - ▶ Suggests that magnetism and rotation are related
- ▶ 1919: Joseph Larmor that a dynamo gives birth to $\mathbf{B}_{\text{Earth}}$
- ▶ Decay time of Earth's magnetic field is ~ 20 kyr
 - ▶ Must be continually generated

The Terrestrial Dynamo



- ▶ Motion in Earth's liquid outer core is driven by heat transport from solid inner core (convection-driven dynamo)
- ▶ Earth's magnetic field reverses every ~ 200 kyr (but varies from 5 kyr to 50 Myr)
 - ▶ Paleomagnetism: magnetic field freezes into certain rocks when they form
 - ▶ Duration of reversals is just a few kyr

Magnetic fields of exoplanets: habitability

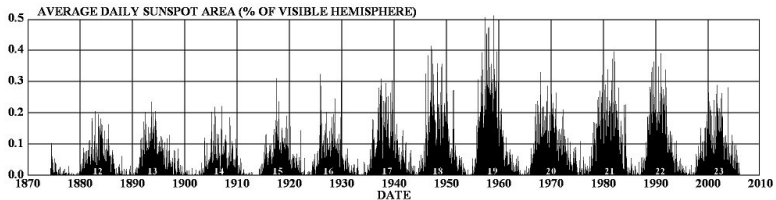
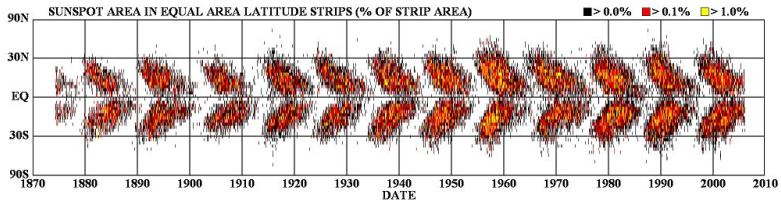
- ▶ Atmospheric escape mechanisms include:
 - ▶ Thermal escape (Jeans escape)
 - ▶ Interaction of the atmospheres of moons with the planetary magnetosphere
 - ▶ Erosion by stellar winds
- ▶ The importance of different mechanisms depend on the properties of the planet, atmosphere, and the stellar wind
 - ▶ Venus and Mars have very weak magnetic fields but different dominant escape mechanisms
- ▶ Planetary magnetic fields help prevent erosion by stellar winds
- ▶ The habitability of exoplanets depends partially on magnetic fields!

The Solar Dynamo

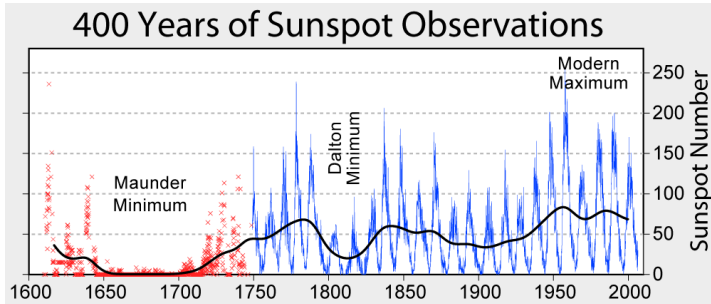
- ▶ Key ingredients
 - ▶ Differential rotation
 - ▶ Convection outer envelope surrounding a radiative core
 - ▶ Meridional flow transports flux towards/away from poles
 - ▶ Expulsion of flux/helicity through eruptions
- ▶ Key features
 - ▶ Eleven year sunspot cycle
 - ▶ Sunspots located near two belts of latitude
 - ▶ Magnetic field flips every ~ 11 years around solar maximum
 - ▶ The magnetic field is complex — not well-described as a dipole!
 - ▶ North-south asymmetry in solar cycle

Butterfly diagram

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS

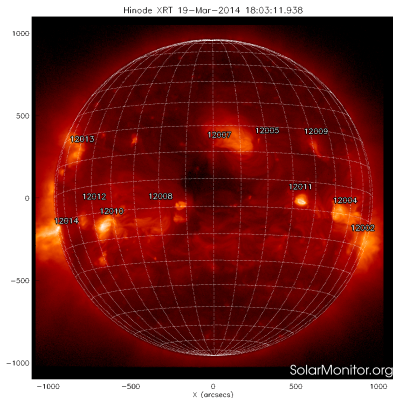
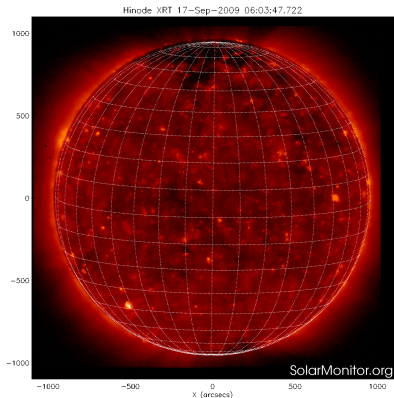


Sunspot record over centuries



- ▶ Extended Maunder minimum from ~ 1650 to ~ 1710
- ▶ Have to cross-calibrate between different observers with different definitions of a sunspot, observing tools, eyesight, etc.

We are just past a weak solar maximum after a deep solar minimum



- ▶ Forecasting the solar cycle is extremely difficult
- ▶ Are we going into another Dalton-type minimum?
- ▶ If you don't like the space weather, just wait five years

The solar cycle has important implications for interplanetary travel

- ▶ Suppose you want to plan a trip to Mars
- ▶ Is it better to go at solar minimum or solar maximum?

Stellar dynamos

- ▶ Sun has radiative core and convective outer envelope
 - ▶ Tachocline (boundary between radiative core and convective envelope) plays important role in dynamo
- ▶ M dwarf stars are entirely convective
 - ▶ Often strong magnetic activity
- ▶ B stars have a convective inner core and radiative outer envelope
 - ▶ Buoyancy can bring flux from core to surface
- ▶ Properties of dynamos depend significantly on convection profile and stellar rotation

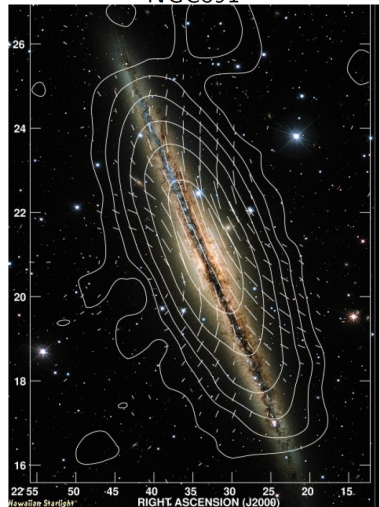
Magnetic fields in galaxies

M51



A. Fletcher et al. 2008

NGC891



M. Krause et al. 2008

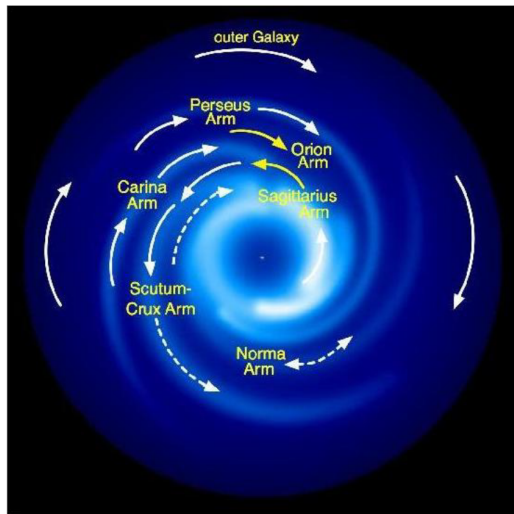
Properties of interstellar gas in disk galaxies

- ▶ The gas layer is highly flattened
- ▶ Gas contribution to galactic gravitational field is negligible
- ▶ Rotationally dominated but with supersonic turbulence
- ▶ Differential rotation
- ▶ Highly variable in density and temperature
- ▶ Ionization in cold regions is very superthermal
- ▶ Magnetized and coupled to cosmic ray component

Properties of the Galactic magnetic field

- ▶ The magnetic field is much more extended than the gas layer (~ 1.5 Mpc vs. ~ 200 pc)
 - ▶ Cosmic rays more extended than gas layer too
- ▶ **B** mostly parallel to galactic plane except near center
- ▶ **B** aligned with Galactic rotation and even spiral arms
- ▶ Large scale ($\gtrsim 3$ kpc) and small scale/ (random) components of **B**
- ▶ $B_{tot} \sim 4\text{--}5 \mu\text{G}$; irregular component is $\sim 1.5\text{--}2.0 \mu\text{G}$

There are several magnetic field reversals in the Milky Way



- ▶ Question: how are magnetic fields in our galaxy diagnosed?

What about irregular galaxies?

- ▶ Properties of ISM in irregular galaxies
 - ▶ Solid body rotation/more chaotic rotation
 - ▶ Slower rotation
 - ▶ ISM is more easily disrupted by star formation
- ▶ The magnetic fields of only a few irregular galaxies have been studied
 - ▶ \mathbf{B} is sometimes strong enough to be dynamically important
 - ▶ Regular and irregular components of \mathbf{B} with much variation between galaxies
- ▶ The α - ω dynamo does not work in absence of differential rotation
- ▶ Turbulent dynamos?

Key questions regarding the origin and growth of cosmic magnetic fields

- ▶ How did the very first (seed) magnetic fields originate?
- ▶ How do magnetic fields grow in galaxies, stars, and planets?
- ▶ How do large-scale components of the field develop?
- ▶ What allows some dynamos to be fast?
- ▶ How is magnetic flux expelled?
- ▶ What are the roles of cosmic rays, supernovae, turbulence, and magnetic reconnection in astrophysical dynamos?
- ▶ What do some dynamos exhibit spontaneous or quasicyclical field reversals?
- ▶ How do dynamos saturate?

Recall: The Biermann battery can generate magnetic fields when none were present before

- ▶ If you assume a scalar electron pressure, your Ohm's law will be

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = -\frac{\nabla p_e}{en_e}. \quad (1)$$

If you combine this with Faraday's law, you will arrive at

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - c \frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \quad (2)$$

- ▶ Dynamos can *amplify* magnetic fields but not create them from scratch
- ▶ The Biermann battery provides a seed field that can then grow as a result of dynamo action
- ▶ Other seed field generation mechanisms have been proposed

Cowling's anti-dynamo theorem

- ▶ In dynamos, too much symmetry is a bad thing
- ▶ Cowling (1934): a steady axisymmetric field cannot be maintained by dynamo action
- ▶ Cowling's theorem applies to exact axisymmetry, but in practice successful dynamos cannot possess too much symmetry
- ▶ Suppose we start out with a magnetic field of the form

$$\mathbf{B} = B_\theta(r, z)\hat{\boldsymbol{\theta}} + \mathbf{B}_p(r, z) \quad (3)$$

where B_θ is the toroidal (azimuthal) component and \mathbf{B}_p is the poloidal (radial & axial) component

- ▶ Assume that the magnetic field is time-independent and non-uniform

The poloidal field consists of closed flux surfaces

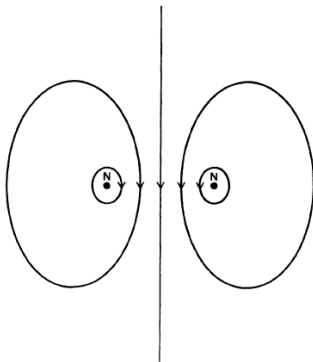


Fig. 9.2. The magnetic field lines in a meridional plane for an axisymmetric field.

- ▶ By symmetry, each meridional half-plane must contain at least one O-type neutral point in the poloidal field (denoted 'N')

Integrate Ohm's law around the closed field line

- ▶ Ohm's law is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \quad (4)$$

- ▶ The path integral along the closed field line (or null line) is

$$\begin{aligned} \oint_C \eta \mathbf{J} \cdot d\mathbf{s} &= \oint_C \mathbf{E} \cdot d\mathbf{s} + \oint_C \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot d\mathbf{s} \\ &= \int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s} + \oint_C \frac{\mathbf{V} \times \mathbf{B}}{c} \cdot d\mathbf{s} \quad (5) \end{aligned}$$

where we used Stokes' theorem.

- ▶ The first term on the RHS vanishes because we assume a steady state
- ▶ The second term on the RHS vanishes because \mathbf{B} is parallel to the path integral

Finishing the proof

- ▶ Ohm's law reduces to

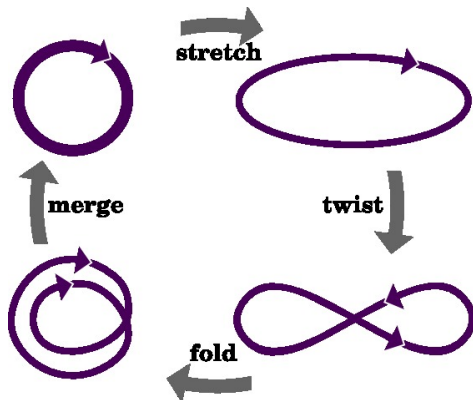
$$\oint_C \mathbf{J} \cdot d\mathbf{s} = 0 \quad (6)$$

- ▶ However, the current cannot vanish because closed flux surfaces surround N
- ▶ Therefore, this magnetic field configuration cannot sustain itself by dynamo action!
- ▶ Led to decades of skepticism that dynamo theory could work
- ▶ The difficulty with an axisymmetric field is maintaining the poloidal field that decays due to resistive diffusion along N
- ▶ Progress really began again two decades later with existence theorems for possible dynamo mechanisms (Herzenberg 1958; Backus 1958)

Slow vs. fast dynamos

- ▶ A dynamo is *fast* if it can operate in the limit of low resistivity: $\eta \rightarrow 0$
 - ▶ Examples (probably) include galactic dynamos and the solar dynamo
- ▶ A dynamo is *slow* if it shuts off in the limit of low resistivity
 - ▶ Example: the terrestrial dynamo

Stretch, twist, and fold (Zel'dovich 1972)



- ▶ What type of flow might realistically produce this effect?
- ▶ 'Merge' step requires magnetic reconnection

Mean field dynamo theory

- ▶ The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (7)$$

- ▶ Assume that the magnetic field and velocity are given by a sum of slowly varying mean components and weak small scale fluctuating components

$$\mathbf{V} = \langle \mathbf{V} \rangle + \mathbf{V}' \quad (8)$$

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}' \quad (9)$$

- ▶ Valid approximation if there is a characteristic length for large-scale fluctuations that is separated from a characteristic

Mean field dynamo theory

- ▶ The evolution of the mean field is found by averaging the induction equation over the small scale fluctuations

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times [\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle] + \nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle + \eta \nabla^2 \langle \mathbf{B} \rangle \quad (10)$$

The second term on the RHS is crucial for dynamo action

- ▶ The evolution of the time-varying component is

$$\begin{aligned} \frac{\partial \mathbf{B}'}{\partial t} &= \nabla \times [\langle \mathbf{V} \rangle \times \mathbf{B}' + \mathbf{V}' \times \langle \mathbf{B} \rangle] \\ &\quad + \nabla \times [\mathbf{V}' \times \mathbf{B}' - \langle \mathbf{V}' \times \mathbf{B}' \rangle] + \eta \nabla^2 \mathbf{B}' \quad (11) \end{aligned}$$

Use this equation to find $\nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle$

- ▶ Can often make the assumption that $\langle \mathbf{V} \rangle = 0$

What is $\langle \mathbf{V}' \times \mathbf{B}' \rangle$?

- ▶ Assuming isotropic helical turbulence, this term can be written as

$$\langle \mathbf{V}' \times \mathbf{B}' \rangle = -\alpha \langle \mathbf{B} \rangle + \beta \nabla \times \langle \mathbf{B} \rangle \quad (12)$$

where

$$\alpha = \langle \mathbf{V}' \cdot \nabla \times \mathbf{V}' \rangle \frac{\tau}{3} \quad (13)$$

$$\beta = \langle \mathbf{V}' \cdot \mathbf{V}' \rangle \frac{\tau}{3} \quad (14)$$

where τ is the velocity correlation time of the turbulence

- ▶ The dynamo equation then becomes

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle) + \nabla \times (\alpha \langle \mathbf{B} \rangle) + (\beta + \eta) \nabla^2 \langle \mathbf{B} \rangle \quad (15)$$

What are α and β ?

- ▶ β represents turbulent diffusivity
 - ▶ Acts on the mean field
 - ▶ In a fast dynamo, $\beta \gg \eta$
- ▶ The α effect corresponds to a mean turbulent EMF that's parallel to \mathbf{B} : $\mathcal{E} = \alpha \langle \mathbf{B} \rangle$
 - ▶ The α effect can deform a straight magnetic field into a helix
 - ▶ Sign of α depends on what direction the velocity vector rotates in (fluid helicity)
 - ▶ For α effect to occur, need fluid helicity $\neq 0$
 - ▶ Need finite resistivity so that magnetic field and velocity fluctuations are out of phase
 - ▶ α is in dimensions of velocity

What does α do for us?

- ▶ Suppose we have

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\alpha \langle \mathbf{B} \rangle), \quad (16)$$

$$\langle \mathbf{B} \rangle = \hat{\mathbf{x}} \langle B_x \rangle (y, t) + \hat{\mathbf{z}} \langle B_z \rangle (y, t) \quad (17)$$

- ▶ We then have two coupled equations

$$\frac{\partial B_x}{\partial t} = \frac{\partial}{\partial y} \alpha B_z \quad (18)$$

$$\frac{\partial B_z}{\partial t} = -\frac{\partial}{\partial y} \alpha B_x \quad (19)$$

where we drop brackets for clarity

What does α do for us?

- ▶ Combining these equations gives

$$\frac{\partial^2 B_x}{\partial t^2} = -\alpha^2 \frac{\partial^2 B_x}{\partial y^2} \quad (20)$$

- ▶ Look for solutions of the form $\exp[ip t + ik_0 y]$

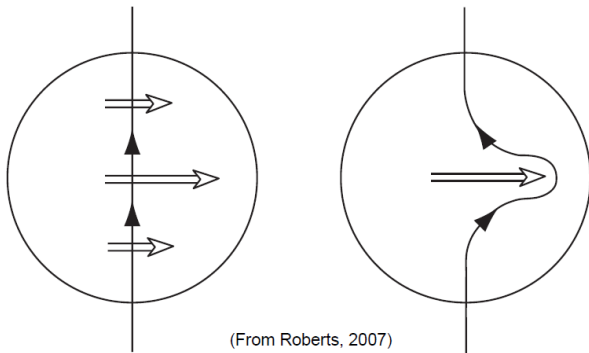
$$\begin{aligned} -p^2 B_x &= \alpha^2 k_0^2 B_x \\ p &= \pm i \alpha k_0 \end{aligned} \quad (21)$$

- ▶ This then means that exponentially growing or decaying solutions are allowed!

$$\langle B_x \rangle \propto \exp[\mp \alpha k_0 t + ik_0 y] \quad (22)$$

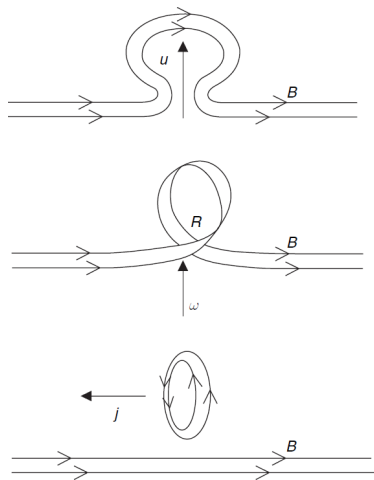
- ▶ Feedback/exponential growth from the equation for $\langle B_z \rangle$

The ω -effect occurs when poloidal magnetic field is pulled in the toroidal direction through differential rotation



- ▶ The ω -effect arises from $\nabla \times [\langle \mathbf{V} \rangle \times \langle \mathbf{B} \rangle]$ in Eq. 10
- ▶ If there's no differential rotation, there's no ω -effect

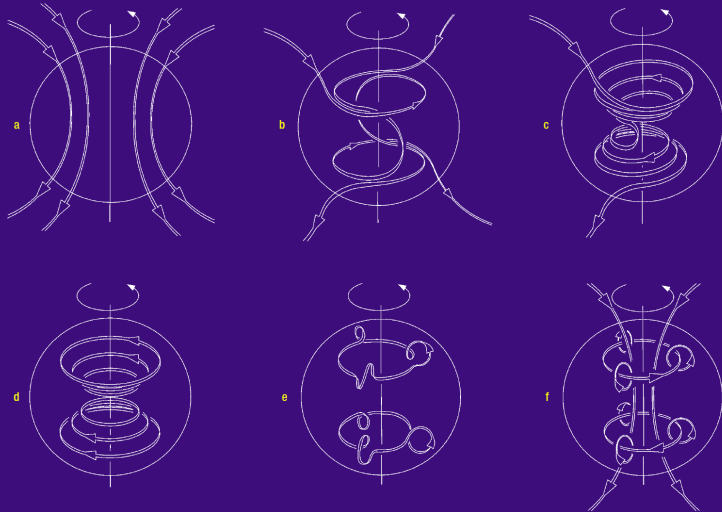
The α effect produces poloidal field from toroidal field



(From Roberts, 2007)

- ▶ The α -effect arises from $\nabla \times \langle \mathbf{V}' \times \mathbf{B}' \rangle$ in Eq. 11

The α - ω dynamo (from Love 1999)



2: The α - ω dynamo mechanism. Conventional geodynamo theory presupposes (a) an initial, primarily dipolar, poloidal magnetic field. The ω -effect consists of differential rotation, (b) and (c), wrapping the magnetic field around the rotational axis, thereby (d) creating a quadrupolar toroidal magnetic field. Symmetry is broken, and dynamo action maintained, by the α -effect, whereby helical upwelling (e) creates loops of magnetic field. These loops coalesce (f) to reinforce the original dipolar field, thus closing the dynamo cycle.

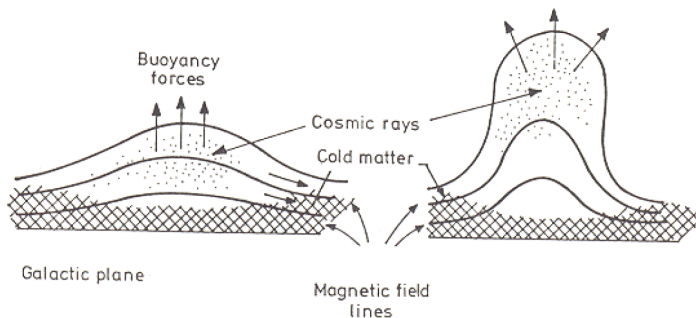
Criticisms of mean field dynamo theory

- ▶ Is the *ad hoc* separation of scales valid?
- ▶ Neglects backreaction of the fluctuating components
 - ▶ Requires including Lorentz force in equations
 - ▶ If turbulent motions on small-scales are suppressed, this may shut off the dynamo
 - ▶ Quenching of dynamo can be approximating by have α decrease in an *ad hoc* manner when approaching saturated value
- ▶ MHD turbulence is anisotropic
 - ▶ However, can take α and β to be tensors

Numerical simulations of dynamos require fewer assumptions

- ▶ No need for separation of variables ansatz
- ▶ Can use full MHD equations, or full mean-field dynamo equation
- ▶ Can include feedback when the magnetic field becomes strong
- ▶ Can include additional physics for a more realistic ISM
 - ▶ Density waves, feedback from star formation on ISM, etc.
 - ▶ Cosmic rays
- ▶ Caveats
 - ▶ Resistivity much larger than reality
 - ▶ Limited separation between small and large scales

Parker instability



- ▶ If an initially uniform magnetic field supported against gravity becomes wavy, plasma will flow along field lines into the lower dips
- ▶ The \sim massless components of the ISM (cosmic rays, magnetic fields) become buoyantly unstable and can rise to form loops
- ▶ Allows escape of magnetic field from galaxies into the halo

Cosmic-ray driven dynamo

- ▶ Magnetic fields, gas, and cosmic rays are in rough energy equipartition in the ISM
 - ▶ Suggests that these different components are coupled
- ▶ Parker (1992) proposed a dynamo in which cosmic ray buoyancy instabilities play an important role
 - ▶ Numerous loops form and bulge into the galactic halo
 - ▶ Reconnection frees these loops from the initially toroidal field
 - ▶ Differential rotation, the Coriolis effect, and other instabilities then act on the poloidal loops and toroidal field
- ▶ Growth rate found to be comparable to a rotation time

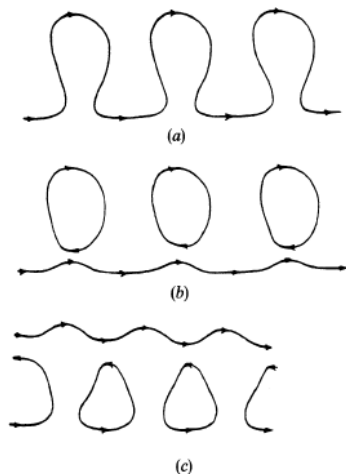
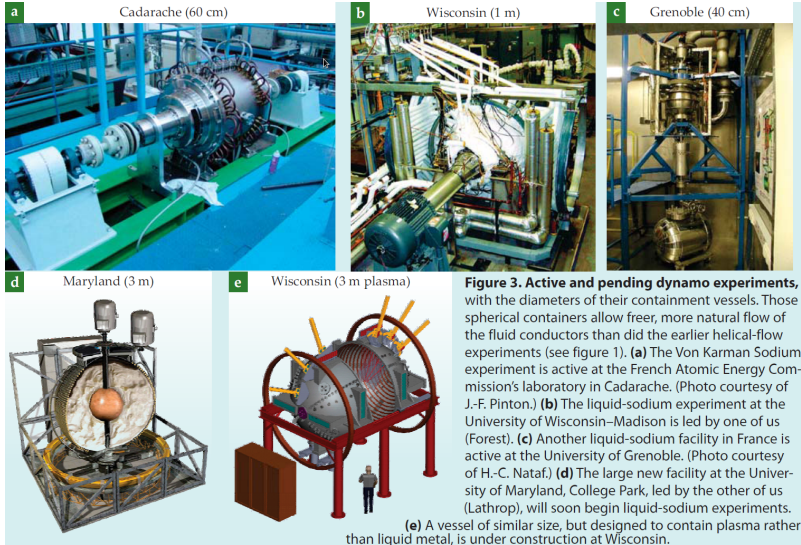


FIG. 3.—Schematic of some magnetic lobes (a) which reconnect across their own bases, (b) thereby freeing themselves from connection into the gaseous disk. In (c) the reconnection takes place between adjacent lobes, thereby freeing a band of flux from connection into the disk and leaving behind some free flux loops that are rooted in the gaseous disk but which are relatively free to be rotated about a vertical axis.

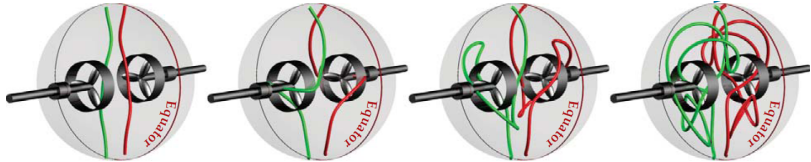
Laboratory dynamo experiments

- ▶ Liquid metal dynamo experiments
 - ▶ Stir up seed field with, e.g., propellers
 - ▶ Reynolds numbers not too far from the terrestrial dynamo
- ▶ Plasma dynamo experiments
 - ▶ Dynamo action occurs in many toroidal confined plasmas (e.g., reversed field pinches)
 - ▶ New spherical plasma dynamo experiment at Wisconsin under construction

Laboratory dynamo experiments



Laboratory dynamo experiments



- ▶ Seed field is stretched, twisted, and folded to grow into a stronger field
- ▶ Excessive turbulence makes sustained dynamo action difficult to achieve

Summary

- ▶ Dynamos convert kinetic energy to magnetic energy
- ▶ Dynamos require a seed field to operate (e.g., the Biermann battery)
- ▶ Dynamo action cannot sustain an axisymmetric magnetic field
- ▶ Mean field dynamos can explain many observations but require many assumptions
- ▶ A self-consistent theory requires the feedback by the Lorentz force on the flow
 - ▶ Not included in kinematic dynamos; need nonlinear dynamo theory
- ▶ Cosmic ray driven dynamos may be important in galaxies
- ▶ Laboratory experiments provide insight into planetary and astrophysical dynamos