The Magnetorotational Instability

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These slides are based off of Balbus & Hawley (1991), Hawley & Balbus (1991), Balbus (2003), Kulsrud §7.4, a 2009 presentation by E. Knobloch, and Ji & Balbus (2013)

Outline

- Accretion problem
- Linear properties of MRI
- Nonlinear properties of MRI
- Laboratory astrophysics studies of MRI
- Open questions

The accretion problem

- Angular momentum is strictly conserved
- Infalling matter has too much angular momentum to be accreted directly $\rightarrow$ formation of accretion disk
- Examples include:
  - Protostellar/protoplanetary disks
  - Roche lobe overflow
  - Disks surrounding black holes in galactic nuclei
- Key problem: how is angular momentum transported outward so that the accretion process occurs?

Properties of accretion disks

- Keplerian flow profile: the angular velocity is

$$\Omega(r) \propto R^{-3/2} \tag{1}$$

  so that

$$\frac{\mathrm{d}\Omega}{\mathrm{d}R} < 0 \tag{2}$$

- Protoplanetary disks: $T \sim 10$ K, $\frac{n_e}{n} \lesssim 10^{-10}$, $B \sim 1$ G (?), $n \sim 10^{10}$–10^{12} cm^{-3}, $L \sim$ 10s of AU
- Supermassive black hole accretion disks: $T \sim 10^8$ K, $\frac{n_e}{n} \sim 1$, $B = ?$, $n \gtrsim 10^{12}$ cm^{-3}, $R_s \sim 2$ AU for $M \sim 10^8 M_\odot$
- Structure of accretion disks impacted by local radiation field, radiative transfer effects, etc.

Is molecular viscosity sufficient to drive accretion?

- In terms of specific angular momentum $L = RV_\theta$

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) L = \underbrace{\frac{1}{R} \frac{\mathrm{d}}{\mathrm{d}R} \left(R^3 \rho \nu \frac{\mathrm{d}\Omega}{\mathrm{d}R} \right)}_{\text{viscous torque}} \tag{3}$$

- From dimensional analysis and using $L \sim R^2 \Omega$, the accretion time is

$$\frac{\rho L}{\tau_\nu} \sim \rho \nu \Omega \quad \Longrightarrow \quad \tau_\nu \sim \frac{R^2}{\nu} \tag{4}$$

- If you plug in values for a protostellar disk, it would take longer than a Hubble time for a star to form!

If you don't understand it, invoke turbulence!

- Shakura and Sunyaev (1973) postulated that shear-driven hydrodynamic turbulence could lead to an enhanced viscosity
- They parameterized the effective viscosity as

$$\nu = \alpha H c_s \tag{5}$$

  where H is the disk thickness and c_s is the speed of sound
- The coefficient α is a dimensionless parameter that is between $\sim$0.1 and $\sim$1 to match observations

But what sets α?

- Accretion disks are expected to be hydrodynamically stable according to the Rayleigh stability criterion,

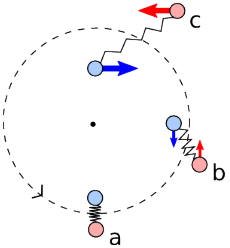
$$\frac{\partial \left(R^2\Omega\right)}{\partial R^2} > 0 \tag{6}$$

- This criterion applies to axisymmetric disk perturbations
  - Are any non-axisymmetric/finite amplitude modes unstable?
  - Recent laboratory experiments further support HD stability
- Many mechanisms have been investigated and found to be insufficient
  - Turbulence driven by shear flow is not sufficient
  - Shear instabilities, barotropic/baroclinic instabilities, sound waves, shocks, finite amplitude instabilities
- Key MHD alternative: the magnetorotational instability (MRI)

The magnetorotational instability (MRI)

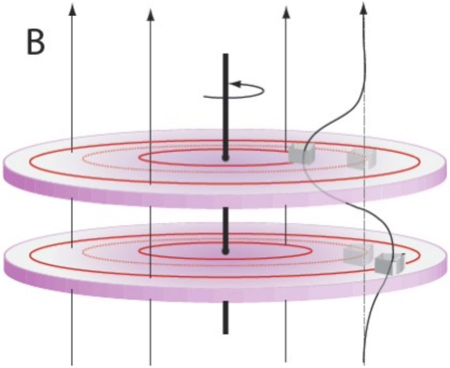
- Originally discovered by Velikov (1959) and Chandrasekhar (1960)
- Occasionally revisited over the next few decades
  - Applications to geodynamo and stellar differential rotation
- Importance for accretion disks not recognized until Balbus & Hawley (1991)
- Leading mechanism for driving turbulence and momentum transport in accretion disks
- Also applied to supernovae, the ISM, etc.

An analogy for the magnetorotational instability



- Imagine there are two space wombats in nearby Keplerian orbits who are each holding one end of a spring
- The inner space wombat is moving faster than the outer one
- The inner space wombat gets pulled back while the outer one gets pulled forward
- Angular momentum gets transported outward

MRI in 3D



- In reality, a magnetic field takes the place of a spring

Key properties of MRI

- Linearly unstable in ideal MHD (from normal mode analysis)
- Inherently local (insensitive to global BCs)
- Triggered by a weak poloidal magnetic field (B_r, B_z)
- Unstable in a regime that is Rayleigh stable
- Grows on a dynamical timescale

Deriving the MRI

- Now we follow Balbus (2003) to sketch the derivation of the MRI
- If you displace a plasma element in the orbital plane by $\boldsymbol{\xi}$ the induction equation gives

$$\delta\mathbf{B} = ikB\boldsymbol{\xi} \tag{7}$$

- The tension force is spring-like (proportional to displacement)

$$\frac{ikB}{4\pi\rho}\delta\mathbf{B} = -(\mathbf{k}\cdot\mathbf{V}_A)^2\boldsymbol{\xi} \tag{8}$$

Deriving the MRI

- Focus on a small patch of the disk at R_0 rotating at an angular velocity of $\Omega(R_0)$
- Drop terms associated with curvature that are not associated with rotation
- Need to take into account a Coriolis force $-2\mathbf{\Omega}_0 \times \mathbf{V}$ and a centrifugal force $R\Omega_0^2$ when in a rotating reference frame
- To leading order in $x \equiv R - R_0$, the difference between centrifugal and gravitational forces in the corotating frame is

$$R\Omega^2(R_0) - R\Omega^2(R) = -x\frac{\mathrm{d}\Omega^2}{\mathrm{d}R} \tag{9}$$

Deriving the MRI

- For pressure-free displacements with vertical wavenumber, the equations of motion become

$$\frac{\partial^2 x}{\partial t^2} - 2\Omega \frac{\partial y}{\partial t} = -\left(\frac{\mathrm{d}\Omega^2}{\mathrm{d} \ln R} + (\mathbf{k} \cdot \mathbf{V}_A)^2 \right) x \qquad (10)$$

$$\frac{\partial^2 y}{\partial t^2} + 2\Omega \frac{\partial x}{\partial t} = -(\mathbf{k} \cdot \mathbf{V}_A)^2 y \qquad (11)$$

where x and y are the radial and azimuthal displacements

Deriving the MRI

- Assuming a time dependence of $e^{i\omega t}$ yields a dispersion relation of

$$\omega^4 - \omega^2 \left[\kappa^2 + 2(\mathbf{k} \cdot \mathbf{V}_A)^2\right] + (\mathbf{k} \cdot \mathbf{V}_A)^2 \left[(\mathbf{k} \cdot \mathbf{V}_A)^2 + \frac{\mathrm{d}\Omega^2}{\mathrm{d} \ln R}\right] = 0 \tag{12}$$

- The epicyclic frequency κ is the rate at which a point mass disturbed in the plane of its orbit would oscillate about its average radial location
  - $\kappa^2 < 0 \Longrightarrow$ instability according to the Rayleigh criterion

Deriving the MRI

- Setting $\omega^2 = 0$ shows that the MRI is unstable when

$$\frac{\mathrm{d}\Omega^2}{\mathrm{d}r} \geq 0 \tag{13}$$

for wavenumbers satisfying

$$k^2 V_A^2 + \frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln R} < 0 \tag{14}$$

- This is satisfied in a Keplerian flow profile, so accretion disks are linearly unstable to the MRI!
- The instability criterion is most easily met when B is small!
- The growth rate of the fastest growing mode is

$$|\omega_{\mathrm{max}}| = \frac{1}{2}\left|\frac{\mathrm{d}\Omega}{\mathrm{d}\ln R}\right| \tag{15}$$

The nonlinear evolution of the MRI is studied using numerical simulations

- Local simulations often use a shearing box approximation
  - Look at a very small region in the disk with shear flow
- Global simulations can investigate effects of disk structure and boundary conditions on nonlinear MRI

Shearing box simulation from Hawley & Balbus (1991)

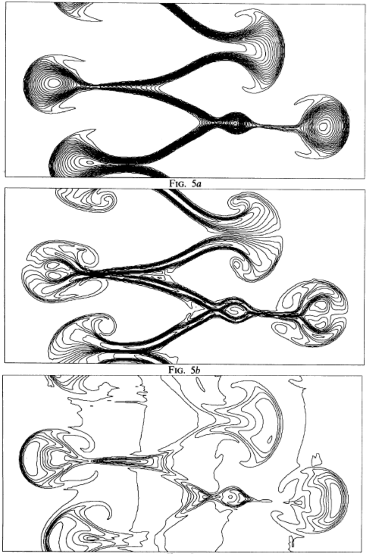
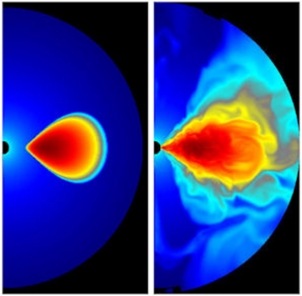


FIG. 5c

FIG. 5.—Contour plots of (*a*) the poloidal magnetic field lines, (*b*) toroidal field, and (*c*) angular momentum at 3.3 orbits in the $\beta_z = 4000$, $a = 1$ high-resolution simulation (Model 3b). There are 20 linearly spaced contours. The angular momentum values run from 9.86 to 10.14; the Keplerian value of the angular momentum at the center of the grid is 10. The toroidal field has a maximum energy density of 2×10^{-7}. At this time the *z*-length scale of the most prominent structures has been determined primarily by the wavelength of the fastest growing mode, $k_z = 2$.

Global simulation of an accretion disk around a Kerr black hole (from Hawley)

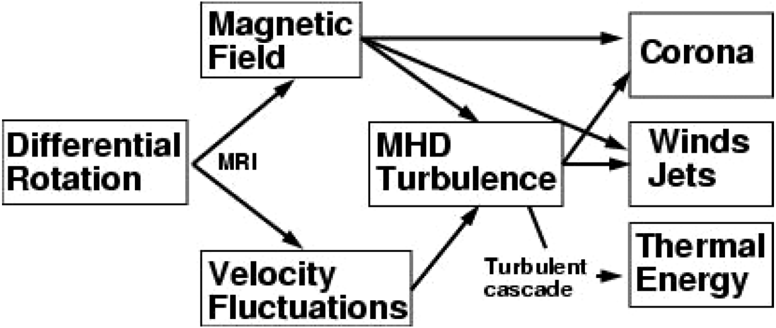


- Contours are logarithmic in density
- MRI develops on orbital timescale
- Distortion of torus
- Development of corona and wind

What causes the MRI to saturate?

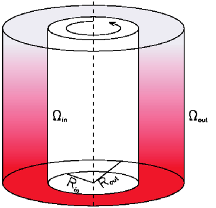
- The nonlinear saturation of the MRI is key
  - Saturation level determines level of turbulence
  - Level of turbulence determines angular momentum transport
- The dynamics of saturation are under active investigation
- What are the interconnected roles of:
  - Magnetic reconnection?
  - Dynamo?
  - Turbulence?
  - Winds/jets?
  - Helicity transport?
  - Radiative transfer/photoionization?
  - Space wombats?

Energy flow in accreting systems



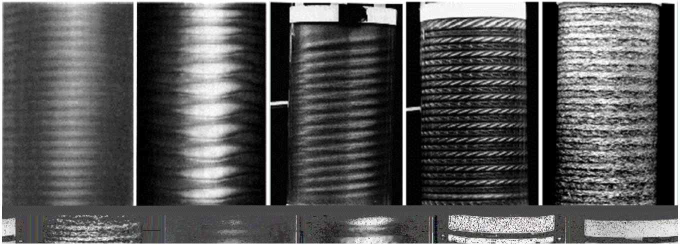
- Where do reconnection & dynamo show up in this?

Laboratory astrophysics experiments on HD/MHD stability



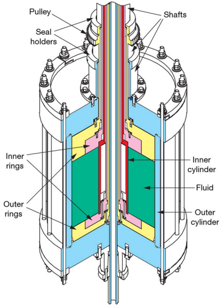
- Hydrodynamic experiments (e.g., liquid water)
  - Couette flow between inner cylinder rotating at Ω_1 and outer cylinder at Ω_2
- Liquid metal experiments
  - Can pick metals/temperatures with properties similar to water
- Plasma Couette experiments
  - Need novel techniques to establish quasi-Keplerian flow while confining the plasma

Laboratory experiments on hydrodynamic stability



- Sharply contrasting results
  - Princeton group: quasi-Keplerian flow profiles are robustly stable and HD turbulence is not sufficient to drive accretion
    - Uses multiple spinning rings at endcaps to reduce Ekman circulation
  - Maryland group: significant turbulent transport at similar Re
    - Uses long cylinder to reduce endcap effects
- More experiments are needed to explain this difference

Liquid gallium experiments



- Very similar setup to liquid
- Incompressible MHD
- Observations of MRI are ambiguous
  - Expected level of instability close to current noise level

University of Wisconsin Plasma Couette Experiment

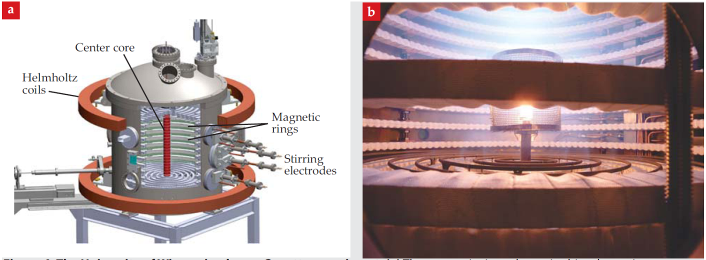


Figure 4. The University of Wisconsin plasma Couette experiment. (a) The magnetic rings shown in this schematic are responsible for confining the plasma. The edge of the plasma is stirred by applying an electric field across the magnetic field; viscous forces then ensure that the entire plasma flows. (Adapted from ref. 18.) **(b)** This photograph shows the glowing plasma, which is made by hot electrons emitted from the central core. The containment vessel is about 1 m in diameter. (Courtesy of Cami Collins and Cary Forest.)

- Uses alternating magnetic rings to keep plasma away from wall
- Electrodes stir plasma using $\mathbf{E} \times \mathbf{B}$ drift, viscous forces
- Still under development, but getting close!

How do we combine theory, simulation, observation, and experiment?

- Theory:
  - Provides information on linear properties of instability (growth rate, mode structure)
  - Provides understanding of basic physics
  - Can put simulation output (e.g., α) back into global models
  - Limited information about nonlinear instability/saturation
- Simulations:
  - Allow nonlinear investigation of instability
  - Provide insight into saturation mechanism
  - Estimate value of α
  - Show expected roles of reconnection and dynamo
  - Limited to relatively modest Re, other parameters

How do we combine theory, simulation, observation, and experiment?

- Observations:
  - Provide key constraints on plasma parameters/disk structure
  - Tests of theories and simulations
  - Difficult to determine fine-scale structure
- Experiment:
  - Provides insight into basic physics of MRI
  - Allows validation of theory and simulation
  - Works at relatively modest plasma parameters
  - Boundary conditions very different than astrophysics

Open questions about the MRI

- At what level does the MRI saturate?
- What is the nature of the turbulence resulting from the MRI?
- What is the global nature of the MRI?
- How do the Hall effect and kinetic effects modify the MRI?
- How does the MRI occur in weakly ionized plasmas?
- What is the role of the MRI in other astrophysical phenomena? (e.g., supernovae)
- How do radiation and relativity affect the MRI?

Summary

- Angular momentum transport is essential to understanding accretion
- Viscosity is not sufficient so turbulence driven by instabilities is thought to drive transport
- HD instabilities might play a role but Keplerian flow profiles are stable to the Rayleigh criterion
- MRI is the leading mechanism to drive turbulence in accretion disks
- Don't give springs to space wombats!