Beyond Ideal MHD

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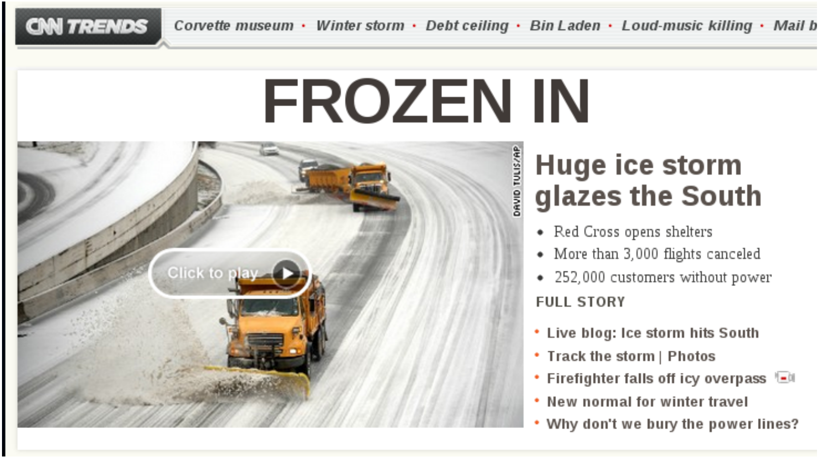
Astronomy 253: Plasma Astrophysics

February 12, 2014

These lecture notes are largely based on *Plasma Physics for Astrophysics* by Russell Kulsrud, *Lectures in Magnetohydrodynamics* by Dalton Schnack, *Ideal Magnetohydrodynamics* by Jeffrey Freidberg, and *The Physics of Fully Ionized Gases* by Lyman Spitzer.

The frozen-in condition doesn't just apply to plasmas



- Ironically, few would call this weather ideal!

Outline

- Resistivity and Viscosity
  - Dimensionless numbers/similarity scaling
- Generalized Ohm's Law
  - Hall effect
  - Biermann battery
- Anisotropic thermal conduction

How do we approach a problem when MHD is insufficient?

- Extended MHD
  - Keep the fluid approximation
  - Add terms including effects beyond MHD
  - Resistivity, viscosity, anisotropic thermal conduction, separate ion and electron temperatures, neutrals, etc.
- Kinetic theory/particle-in-cell simulations
  - Abandon the fluid approximation
  - Keep track of particles/distribution functions directly
- A hybrid approach
  - Keep some parts of the fluid approximation
  - Express other parts kinetically

Key Properties of Ideal MHD

- *Frozen-in condition:* if two parcels of plasma are attached by a field line at one time, they will continue to be attached by a field line at future times
  - Magnetic topology is preserved
- Mass, momentum, and energy are convserved
- Helicity and cross-helicity are conserved
- Only adiabatic heating/cooling
- No dissipation!
- The equations of ideal MHD have no inherent scale ("scale-free")

The ideal MHD Ohm's law

- The ideal Ohm's law is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = 0 \tag{1}$$

- By combining this equation with Faraday's law, we arrive at the induction equation for ideal MHD

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) \tag{2}$$

- When these conditions are met, the frozen-in condition is valid.

The resistive MHD Ohm's law

- The resistive Ohm's law is[1]

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \tag{3}$$

- By combining this equation with Faraday's law, we arrive at the induction equation for resistive MHD

$$\begin{aligned} \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{V} \times \mathbf{B}) - c\nabla \times (\eta \mathbf{J}) & (4) \\ &= \nabla \times (\mathbf{V} \times \mathbf{B}) - \frac{c^2}{4\pi}\nabla \times (\eta \nabla \times \mathbf{B}) & (5) \end{aligned}$$

where we allow η to vary in space.

[1] Kulsrud's definition of η differs by a factor of c.

The induction equation with constant resistivity

- If η is constant, then the induction equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times (\mathbf{V} \times \mathbf{B})}_{\text{advection}} + \underbrace{D_\eta \nabla^2 \mathbf{B}}_{\text{diffusion}} \tag{6}$$

  where the electrical diffusivity is defined to be

$$D_\eta \equiv \frac{\eta c^2}{4\pi} \tag{7}$$

  and has units of a diffusivity: $\frac{\text{length}^2}{\text{time}}$ or cm^2 s^{-1} in cgs
- Annoyingly, conventions for η vary. Sometimes D_η is called η in Eq. 6. In SI units, typically $D_\eta \equiv \frac{\eta}{\mu_0}$.

Writing the induction equation as a diffusion equation

- If $\mathbf{V} = 0$, the resistive induction equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = D_\eta \nabla^2 \mathbf{B} \tag{8}$$

- The second order spatial derivative corresponds to diffusion
  - A fourth order spatial derivative corresponds to hyperdiffusivity
- This is a parabolic vector partial differential equation that is analogous to the heat equation
- Let's look at the x component of Eq. 8

$$\frac{\partial B_x}{\partial t} = D_\eta \left(\frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} + \frac{\partial^2 B_x}{\partial z^2} \right) \tag{9}$$

This corresponds to the diffusion of B_x in the x, y, and z directions

How do we find the resistivity?

- Spitzer resistivity results from collisions between electrons and ions and is given by

$$D_\eta \equiv \frac{\eta c^2}{4\pi} \approx \frac{0.42 \times 10^7}{T_{eV}^{3/2}} \frac{\text{cm}^2}{\text{s}} \tag{10}$$

  where T_{eV} is in eV and 10^4 K ≈ 1 eV
- The resistivity is actually anisotropic ($\eta_\perp = 1.96\eta_\parallel$) but we typically assume that η is isotropic
  - The Eighth Bronze Rule of Astrophysics: $2 \approx 1$
- We will discuss collisions and transport in detail in a few weeks
- Though derived in the 1950s, Spitzer resistivity was not tested experimentally until the 2000s!
  - Trintchouk et al. (2003); Kuritsyn et al. (2006) at MRX

Resistive diffusion time

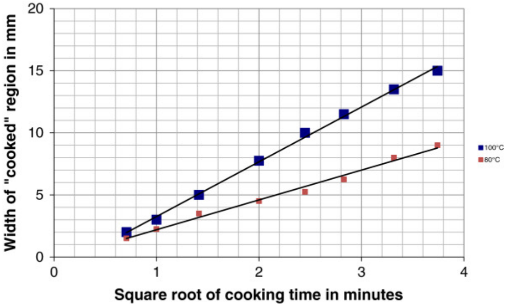
- Since we can put resistivity in units of length^2/time, we can formulate a resistive diffusion time scale:

$$t_\eta \equiv \frac{L_0^2}{D_\eta} \tag{11}$$

where L_0 is the characteristic length scale of the problem

- As with any 'normal' diffusivity, t_D depends quadratically on the length scale

Application: the physics of cooking!



- The cooking time for potato cubes depends quadratically on the length of one side
- This is another example of

$$\text{diffusion time} = \frac{(\text{length scale})^2}{\text{diffusion coefficient}} \tag{12}$$

Viscosity. . . it's such a drag!

- Viscosity transports momentum between parts of the fluid that are in relative motion
- Viscosity results from particle collisions
- The momentum equation with viscosity is given by

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{\mathbf{J} \times \mathbf{B}}{c} - \nabla p + \rho \nu \nabla^2 \mathbf{V} \tag{13}$$

Here, ν is the *kinematic viscosity*

Putting the momentum equation in dimensionless form

- Define $\mathbf{V} \equiv V_0 \tilde{\mathbf{V}}$... where V_0 is a characteristic value and $\tilde{\mathbf{V}}$ is dimensionless, and use $V_0 \equiv L_0/t_0$
- Use these definitions in the momentum equation while including only the viscosity term on the RHS

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} = \rho \nu \nabla^2 \mathbf{V} \tag{14}$$

$$\frac{V_0}{t_0} \frac{\partial \tilde{\mathbf{V}}}{\partial \tilde{t}} + \frac{V_0^2}{L_0} \tilde{\mathbf{V}} \cdot \tilde{\nabla} \tilde{\mathbf{V}} = \frac{\nu V_0}{L_0^2} \tilde{\nabla}^2 \tilde{\mathbf{V}}$$

$$\frac{\partial \tilde{\mathbf{V}}}{\partial \tilde{t}} + \tilde{\mathbf{V}} \cdot \tilde{\nabla} \tilde{\mathbf{V}} = \frac{\nu}{V_0 L_0} \tilde{\nabla}^2 \tilde{\mathbf{V}} \tag{15}$$

where the Reynolds number is

$$\mathrm{Re} \equiv \frac{V_0 L_0}{\nu} \tag{16}$$

The Reynolds number gauges the importance of the advective term compared to the viscous term

- We can rewrite the momentum equation with only viscosity as

$$\frac{\partial \tilde{\mathbf{V}}}{\partial \tilde{t}} + \underbrace{\tilde{\mathbf{V}} \cdot \tilde{\nabla} \tilde{\mathbf{V}}}_{\text{advection}} = \underbrace{\frac{1}{\mathrm{Re}} \tilde{\nabla}^2 \tilde{\mathbf{V}}}_{\text{viscous diffusion}} \tag{17}$$

- A necessary condition for turbulence to occur is if

$$\mathrm{Re} \gg 1 \tag{18}$$

so that the viscous term is negligible on scales $\sim L_0$.

  - In astrophysics, usually $\mathrm{Re} \equiv \frac{L_0 V_0}{\nu} \ggg\ggg 1$

- The only scale in this equation is the viscous scale

Physics of cooking, part 2!

- The Reynolds number is

$$\mathrm{Re} \equiv \frac{L_0 V_0}{\nu} \tag{19}$$

- If you want to make peanut butter turbulent, you can either
  - Get a humongous vat of it (increase L_0), or
  - Stir it up really quickly (increase V_0)
- Alternatively, since viscosity is a function of temperature (as a in a plasma), you could also heat it up (decrease ν)

Putting the induction equation in dimensionless form

- The induction equation with uniform resistivity is

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + D_\eta \nabla^2 \mathbf{B} \tag{20}$$

- Again, define $\mathbf{B} \equiv B_0 \tilde{\mathbf{B}}$... with $V_0 \equiv L_0 / t_0$

$$\frac{B_0}{t_0} \frac{\partial \tilde{B}}{\partial \tilde{t}} = \frac{V_0 B_0}{L_0} \tilde{\nabla} \times \left(\tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \right) + \frac{D_\eta B_0}{L_0^2} \tilde{\nabla}^2 \tilde{\mathbf{B}} \tag{21}$$

$$\frac{\partial \tilde{B}}{\partial \tilde{t}} = \tilde{\nabla} \times \left(\tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \right) + \frac{D_\eta}{L_0 V_0} \tilde{\nabla}^2 \tilde{\mathbf{B}} \tag{22}$$

Defining the magnetic Reynolds number and Lundquist number

- We define the *magnetic Reynolds number* as

$$\mathrm{Rm} \equiv \frac{L_0 V_0}{D_\eta} \tag{23}$$

- The Alfvén speed is

$$V_A \equiv \frac{B}{\sqrt{4\pi\rho}} \tag{24}$$

- We define the *Lundquist number* as

$$S \equiv \frac{L_0 V_A}{D_\eta} \tag{25}$$

  where we use that the characteristic speed for MHD is $V_0 = V_A$

Rm and S gauge the relative importance between the advection term and the resistive diffusion term

- We can write the induction equation as

$$\frac{\partial \tilde{B}}{\partial \tilde{t}} = \tilde{\nabla} \times \left(\tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \right) + \frac{1}{\mathrm{Rm}} \tilde{\nabla}^2 \tilde{\mathbf{B}} \tag{26}$$

  - If $\mathrm{Rm} \gg 1$ then advection is more important (ideal MHD limit)
  - IF $\mathrm{Rm} \ll 1$ then diffusion is more important
- Usually in astrophysics, $\mathrm{Rm} \ggg\gg 1$
  - Example: $T \sim 10^4$ K, $L_0 \sim 1$ pc, $V \sim 1$ km/s. Then

$$\mathrm{Rm} \sim 10^{16}$$

  Interstellar plasmas are extremely highly conducting!

Re, Rm, and S can also be expressed as the ratio of timescales

- The Reynolds number is

$$\mathrm{Re} \equiv \frac{L_0 V_0}{\nu} = \frac{\text{viscous timescale}}{\text{advection timescale}} \tag{27}$$

- The magnetic Reynolds number is

$$\mathrm{Rm} \equiv \frac{L_0 V_0}{D_\eta} = \frac{\text{resistive diffusion timescale}}{\text{advection timescale}} \tag{28}$$

- The Lundquist number is

$$S \equiv \frac{L_0 V_A}{D_\eta} = \frac{\text{resistive diffusion timescale}}{\text{Alfvén wave crossing time}} \tag{29}$$

The relative importance of viscosity vs. resistivity is given by the magnetic Prandtl number

- The magnetic Prandtl number is

$$\mathrm{Pm} \equiv \frac{\nu}{D_\eta} = \frac{\text{resistive diffusion timescale}}{\text{viscous timescale}} \qquad (30)$$

  where ν and D_η are both in units of a diffusivity: length^2/time
- Usually $\mathrm{Pm} \not\approx 1$, but people doing simulations (like me) often set $\mathrm{Pm} = 1$ for simplicity (sigh)
- In plasma turbulence, the Prandtl number determines which scale is larger: the viscous dissipation scale or the resistive dissipation scale
  - Helps determine the physics behind dissipation of energy in turbulence!

Is there flux freezing in resistive MHD?

- Short answer: no!
- Long answer: let's modify the frozen-flux derivation!
- The change of flux through a co-moving surface bounded by a contour $\mathcal{C}$ is

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -c \oint_{\mathcal{C}} \left(\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} \right) \cdot \mathrm{d}\boldsymbol{l} \tag{31}$$

- In ideal MHD, the integrand is identically zero.
- In resistive MHD, the integrand is $\eta \mathbf{J}$!
- The change in flux becomes

$$\frac{\mathrm{d}\Psi}{\mathrm{d}t} = -c \oint_{\mathcal{C}} \eta \mathbf{J} \cdot \mathrm{d}\boldsymbol{l} \tag{32}$$

This is generally $\neq 0$, so flux is not frozen-in.

Viscous and resistive heating

- Ohmic (resistive) heating is given by

$$Q_\eta = \mathbf{E} \cdot \mathbf{J} = \eta J^2 \tag{33}$$

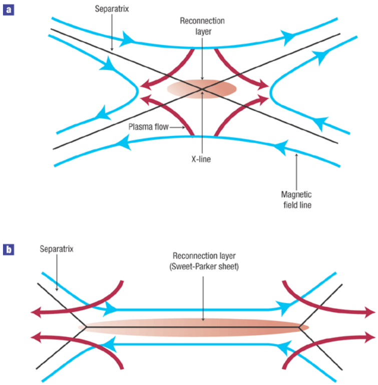
  This shows up as a source term in the energy equation
- Heating preferentially occurs in regions of very strong current, but those regions typically have small volumes
- Viscous heating is of the form

$$Q_\nu = \rho\nu\nabla\mathbf{V}^T : \nabla\mathbf{V} \tag{34}$$

Key Properties of Resistive MHD

- Magnetic topology is not preserved
- Mass, momentum, and energy are conserved
- Helicity and cross-helicity are approximately conserved
- There is Ohmic (resistive) and viscous heating
- There's dissipation!
- The scales in the problem are set by viscosity and resistivity

The resistive term allows *magnetic reconnection* to happen



- Magnetic reconnection is the breaking and rejoining of field lines in an otherwise highly conducting plasma
- Reconnection preferentially occurs in *current sheets*: regions where there are sharp changes in **B**
- Resistive diffusion is usually negligible outside of these regions

But there's more!

- The resistive MHD Ohm's law is

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} \tag{35}$$

- However, there are additional terms that contribute to the electric field!
- The *generalized Ohm's law* is derived from the electron equation of motion

The electron equation of motion

- The electron equation of motion is

$$n_e m_e \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V}_e = -e n_e \left(\mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c} \right) - \nabla \cdot \mathbf{P}_e + \mathbf{p}_{ie} \qquad (36)$$

  where
  - $\mathbf{P}_e$ is the electron pressure tensor
  - $\mathbf{p}_{ie}$ represents the exchange of momentum between ions and electrons due to collisions (this leads to resistivity)
- To derive the generalized Ohm's law, solve for $\mathbf{E}$

The generalized Ohm's law

- The generalized Ohm's law is given by

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = \eta \mathbf{J} + \underbrace{\frac{\mathbf{J} \times \mathbf{B}}{e n_e c}}_{\text{Hall}} - \underbrace{\frac{\nabla \cdot \mathbf{P_e}}{n_e e c}}_{\text{elec. pressure}} + \underbrace{\frac{m_e}{n_e e^2} \frac{\mathrm{d}\mathbf{J}}{\mathrm{d}t}}_{\text{elec. inertia}} \quad (37)$$

- The frozen-in condition can be broken by
  - The resistive term
  - The divergence of the electron pressure tensor term
  - Electron inertia
- These additional terms introduce new physics into the system at short length scales

Each of the terms in the generalized Ohm's law enters in at a different scale

- The Hall and electron pressure terms enter in at the ion inertial length,

$$d_i \equiv \frac{c}{\omega_{pi}} = \sqrt{\frac{c^2 m_i}{4\pi n_i Z^2 e^2}}, \tag{38}$$

  which is the characteristic length scale for ions to be accelerated by electromagnetic forces in a plasma
- The electron inertia term enters in at the electron inertial length

$$d_e \equiv \frac{c}{\omega_{pe}} = \sqrt{\frac{c^2 m_e}{4\pi n_e e^2}}. \tag{39}$$

- The electron inertia term is usually negligible, so we can typically assume massless electrons (since $d_i \approx 43 d_e$)

What are typical ion and electron inertial lengths in astrophysics?

- ISM: $n \sim 1\ \mathrm{cm}^{-3} \Rightarrow d_i \sim 200\ \mathrm{km},\ d_e \sim 5\ \mathrm{km}$
- Solar corona: $n \sim 10^9\ \mathrm{cm}^{-3} \Rightarrow d_i \sim 7\ \mathrm{m},\ d_e \sim 20\ \mathrm{cm}$
- Solar wind at 1 AU: $n \sim 10\ \mathrm{cm}^{-3} \Rightarrow d_i \sim 70\ \mathrm{km},\ d_e \sim 2\ \mathrm{km}$
- That's ridiculous! These scales are tiny! Why should astrophysicists care about them at all?
  - These are comparable to dissipation length scales in MHD turbulence!
  - When current sheets thin down to these scales, reconnection becomes explosive and fast!
  - The Hall effect can modify the magnetorotational instability in protoplanetary/accretion disks
  - The Earth's magnetosphere has $\sim$ISM/solar wind densities, and 200 km is actually not ridiculously small!

Consequences of the Hall term

- In Hall MHD, the Ohm's law becomes

$$\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c} = \frac{\mathbf{J} \times \mathbf{B}}{e n_e c} \tag{40}$$

- The magnetic field becomes frozen into the electron fluid rather than the bulk plasma flow:

$$\mathbf{E} + \frac{\mathbf{V}_e \times \mathbf{B}}{c} = 0. \tag{41}$$

- The Hall term introduces dispersive *whistler* waves
  - Higher frequency waves go faster (unlike sound, Alfvén waves)

Situations where the Hall term may be important

- Planetary magnetospheres
- Dissipation scales in MHD turbulence
- Collisionless (fast) magnetic reconnection
- Accretion disks/protoplanetary disks
- Laboratory plasma experiments
- Hall thrusters (ion propulsion)
- Neutron star atmospheres/magnetospheres
- Our class's final exam

How do you get off-diagonal terms in the electron pressure tensor?

- Rapid variation of **E** or other fields can lead to bunching of electrons in the phases of their gyration about magnetic field lines
  - This results in more electrons around one particular phase in their orbits than at other phases
- These *agyrotropies* are captured by off-diagonal terms in the electron pressure tensor

How did the magnetic field arise in the early universe?

- Evolutionary models of magnetic fields in galaxies must be able to explain coherent $\sim\mu$G fields by $z \sim 2$
- Dynamos require a *seed field* to exist before it can be amplified
- The seed field can be small, but must have been above some minimum value
  - Estimates vary!
- Most terms in the generalized Ohm's law rely on magnetic fields already being present for stronger fields to be generated
  - But not all!

Open questions about primordial magnetic fields[2]

- How did the first magnetic fields in the universe originate?
- Were they created during the Big Bang, or did they develop afterward?
- How coherent were the first magnetic fields? Over what length scales?
- Are galactic magnetic fields top-down or bottom-up phenomena?
- Is there a connection between the creation of the first fields and the formation of large-scale structure?
- Did primordial magnetic fields affect galaxy formation?
- Did early fields play a role in magnetic braking/angular momentum transport in Pop III stars?

[2] From Widrow (2002)

The Biermann battery is a promising mechanism for generating seed magnetic fields in the early universe

- If you assume a scalar electron pressure, your Ohm's law will be

$$\mathbf{E} + \frac{\mathbf{V} \times \mathbf{B}}{c} = -\frac{\nabla p_e}{e n_e}. \tag{42}$$

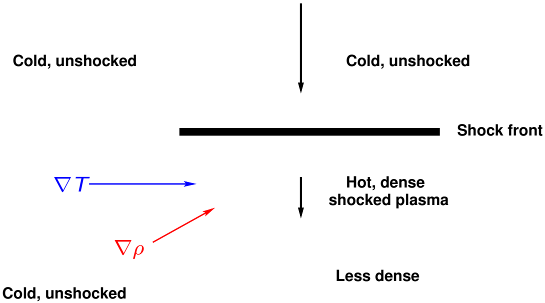
If you combine this with Faraday's law, you will arrive at

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) - c\frac{\nabla n_e \times \nabla p_e}{n_e^2 e} \tag{43}$$

- The Biermann battery term can generate magnetic fields when none were present before!

How do we generate conditions under which the Biermann battery can operate?

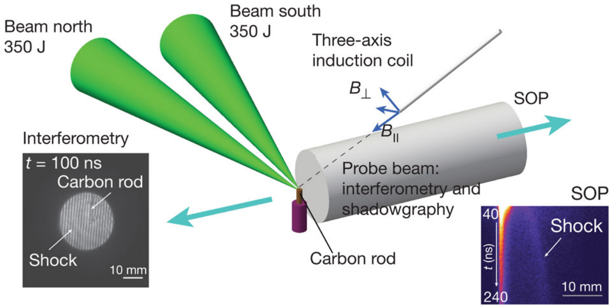
- The Biermann battery requires that ∇n_e & ∇p_e not be parallel
- Vorticity ($\boldsymbol{\omega} \equiv \nabla \times \mathbf{V}$) can generate such conditions
- Kulsrud's example: a shock of limited extent propagates into a cold medium



How does the Biermann battery fit into the big picture?

- The Biermann battery can yield $B \sim 10^{-18}$ G
  - This is plus or minus a few orders of magnitude
- Galactic magnetic fields are $\sim 10^{-6}$ G
- Dynamo theory must be able to explain magnetic field amplification of $\sim$12 orders of magnitude within a few Gyr
- There are other candidate mechanisms for magnetic field formation (e.g., Naoz & Narayan 2013)
- Key difficulty: lack of observational constraints!

The Biermann battery in laser-produced plasmas as an analog for magnetic field generation in the early universe (Gregory et al. 2012)



- Shoot two lasers at an inanimate carbon rod
- Resulting shock waves produce vorticity
- Vorticity leads to magnetic field generation

Anisotropic thermal conduction

- Ideal MHD assumes an adiabatic equation of state
  - No additional heating/cooling or thermal diffusion
- In real plasmas, charged particles are much more free to propagate along field lines than across them
- There is fast thermal transport along field lines
- Thermal transport across field lines is suppressed
- Confinement of fusion plasmas requires closed flux surfaces
  - When the magnetic field becomes stochastic, heat is able to rapidly escape to the wall

How do we include anisotropic thermal conduction in the energy equation?

- The energy equation can be written as

$$\frac{\partial}{\partial t}(\rho\varepsilon) + \nabla\cdot(\rho\varepsilon\mathbf{V}) = \underbrace{-p\nabla\cdot\mathbf{V}}_{\text{compression}} - \underbrace{\nabla\cdot\mathbf{q}}_{\text{heat flux}} + \underbrace{Q-\Lambda}_{\text{heating/cooling}} \tag{44}$$

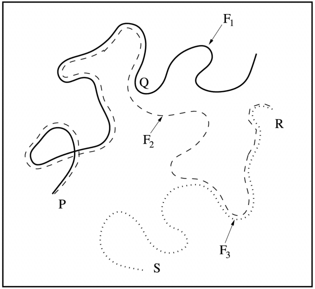
  where ε is the energy per unit mass so that $\rho\varepsilon = p/(\gamma-1)$ is the energy per unit volume and $\rho\varepsilon\mathbf{V}$ is the internal energy flux
- Heating can come from resistive/viscous heating, etc.
- The perpendicular and parallel thermal heat flux vectors are

$$\mathbf{q}_{\parallel} = -\kappa_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla T \tag{45}$$

$$\mathbf{q}_{\perp} = -\kappa_{\perp}\left(\mathbf{I}-\hat{\mathbf{b}}\hat{\mathbf{b}}\right)\cdot\nabla T \tag{46}$$

  where $\kappa_{\parallel} \gg \kappa_{\perp}$

How does stochasticity in the field modify thermal conduction?



- Electrons may jump from one field line to a neighboring one
- In a chaotic system, the field lines exponentially separate
- This results in a net effective thermal diffusion and may be important in galaxy clusters

Plasmas can have different ion and electron temperatures

- Some heating mechanisms primarily affect either ions or electrons
- There will be separate energy equations for ions and electrons
  - Different heating terms, heat flux vectors, etc.
- Equilibration occurs through collisions
- Important in collisionless or marginally collisional plasmas like the solar wind, supernova remnants, etc.

There are additional forms of viscosity that we have not covered

- These can be included in the viscous stress tensor

$$\rho \left(\frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{\mathbf{J} \times \mathbf{B}}{c} - \nabla p + \nabla \cdot \mathbf{\Pi} \qquad (47)$$

- For example, *gyroviscosity* results from electron gyration about magnetic field lines
- These are fundamentally important for magnetically confined fusion plasmas
- These viscosities are important on dissipation scales in plasma turbulence in the solar wind, ISM, and elsewhere

Summary

- Resistivity leads to diffusion of **B**
- Viscosity leads to diffusion of **V**
- Re, Rm, S, and Pm are dimensionless numbers that gauge the importance of viscosity and resistivity
- The generalized Ohm's law includes the Hall effect, the divergence of the electron pressure tensor, and electron inertia
  - These additional terms become important on short length scales and introduce new waves into the system
- The Biermann battery may be the source of seed magnetic fields in the early Universe
- Thermal conduction in plasmas is much faster along magnetic field lines than across them