Avi Loeb's notes ; Nov. 8, 2024

Plasma dispersion relation gives the photon a mass:

$\omega^2 = k^2c^2 + \omega_p^2$ equivalent to: $E^2 = p^2c^2 + m_\gamma^2 c^4$

where $E = \hbar\omega$; $m_\gamma^2 = \frac{\hbar^2}{c^4}\omega_p^2$; $\omega_p^2 = \frac{4\pi n e^2}{m}$

The gravitational coupling replaces: $e^2 \to Gm^2$

which yield for gravitons:

$$m_g^2 = \frac{\hbar^2}{c^4} \times \frac{4\pi n G m^{\cancel{2}}}{\cancel{m}} = \frac{\hbar^2}{c^4} \times 4\pi G \rho$$

For a density parameter $\Omega = \rho/(3H_0^2/8\pi G)$, we get:

$$m_g^2 = \frac{\hbar^2}{c^4} \times \frac{3}{2} H_0^2 \Omega = \frac{\hbar^2}{2c^2}\left(\frac{3H_0^2\Omega}{c^2}\right) \qquad (1)$$

This would apply to any constituent: matter or Λ.

The definition of Λ yields:

$$\Lambda \equiv \frac{3H_0^2}{c^2}\Omega_\Lambda$$

$$(1) \Rightarrow \quad m_g^2 = \frac{\hbar^2}{2c^2}\Lambda$$

$$\Rightarrow \quad \Lambda = \frac{2m_g^2c^2}{\hbar^2} \quad \text{very close to: } \frac{3}{2}\frac{m_g^2c^2}{\hbar^2}$$

Writing: $\lambda_g = \frac{h}{m_g c} = 2\pi\frac{\hbar}{m_g c}$ gives:

$$\lambda_g = \ell_H\left(\frac{2\pi^2}{\Omega_\Lambda}\right)^{1/2} = 5.3\,\ell_H \quad \text{where } \ell_H = \frac{c}{H_0} = 10^{28}\,\mathrm{cm}$$

The Compton wavelength of the graviton is 5 times larger than the horizon of the observable universe and therefore can be neglected.