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QUANTUM-MECHANICAL SUPPRESSION OF GAS ACCRETION BY PRIMORDIAL BLACK HOLES

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ABSTRACT

The Schwarzschild radii of primordial black holes (PBHs) in the mass range of 6×10^{14} g to 4×10^{19} g match the sizes of nuclei to atoms. I discuss the resulting quantum-mechanical suppression in the accretion of matter by PBHs within gaseous astrophysical environments.

1. INTRODUCTION

Current cosmological constraints allow for the possibility that dark matter is made of primordial black holes (PBHs) in the asteroid mass range of $\sim 10^{18}$-10^{22} g (Carr & Hawking 1974; Carr & Kuhnel 2021; Green 2024; Carr & Green 2024). The accretion of atomic gas or a plasma onto PBHs was described so far (De Luca et al. 2022; De Luca & Bellomo 2023; Fowler & Anantua 2023; Curd et al. 2024) based on the hydrodynamics of a continuous fluid. Here, I point out that this formalism is invalid because of quantum-mechanical effects which dominate for asteroid-mass PBHs.

A classical black hole is the ultimate prison. However, it is difficult to fit a plump prisoner into a prison that is much smaller than the prisoner's body size. In the case of a black hole, the prison walls are represented by the event horizon, which in the non-spinning case is a sphere with the Schwarzschild radius,

$$r_{\mathrm{Sch}} = \left(\frac{2GM}{c^2}\right) = 5.3 \times 10^{-9}\ \mathrm{cm} \left(\frac{M}{3.6 \times 10^{19}\ \mathrm{g}}\right) , \tag{1}$$

where G is Newton's constant, c is the speed of light and M is the black hole mass (Schwarzschild 1916). The Schwarzschild radius equals the Bohr radius of the hydrogen atom (Bohr 1913),

$$r_{\mathrm{B}} = \frac{\hbar^2}{m_e e^2} = 5.3 \times 10^{-9}\ \mathrm{cm} , \tag{2}$$

for a black hole mass of $M = 3.6 \times 10^{19}$ g. PBHs of this mass could have been produced in the early Universe. The Schwarzschild radius equals the radius of the proton, $r_p = 8.4 \times 10^{-14}$ cm (Antognini et al. 2013) for a black hole mass of $M = 5.7 \times 10^{14}$ g. This happens to be a few times the mass of a PBH that evaporates by Hawking radiation (Hawking 1974) on a timescale comparable to the age of the Universe. Much smaller PBHs would have disappeared by now.

2. QUANTUM SUPPRESSION OF ASTROPHYSICAL ACCRETION

Black holes grow in mass by accreting matter from their astrophysical environment (Yuan & Narayan 2014; Blaes 2014). For black holes more massive than the Sun, the event horizon is large enough to be considered as a mouth that absorbs many atoms at once and so the accreted matter can be approximated as a continuous fluid. But in the regime of asteroid-mass PBHs, this assumption is no longer valid. These black holes feast on a single atom, a single proton or a single electron at a time, because their event horizon is smaller than the spatial extent of the quantum-mechanical wave function of these particles.

Consider a situation where a hydrogen atom is attracted gravitationally towards a PBH. In the naive perception of the atom as a point particle, it could reach the black hole center from a distance r over a free-fall time, $\sim (r^3/GM)^{1/2}$. However, if the horizon is smaller than the size of the atom, then this 'plump prisoner' will not be captured and most of the atom will stay predominantly outside the event horizon. Even if the PBH is massive enough to absorb the more compact proton, the extended wave function of the electron could remain outside the PBH. In that case, the PBH will acquire a positive electric charge. The electric force that binds the electron to the proton will be enhanced by the PBH gravity.

At distances much larger than the horizon, both electric and gravitational forces decline inversely with distance squared. Hence, the effective Bohr radius (marked hereafter by a tilde) of the *hydrogen+PBH* atom shrinks to a value,

$$\tilde{r}_{\mathrm{B}} = \frac{\hbar^2}{m_e(e^2 + GMm_e)} = 5.3 \times 10^{-9}\ \mathrm{cm} \times \left[1 + \left(\frac{M}{3.8 \times 10^{15}\ \mathrm{g}}\right)\right]^{-1} . \tag{3}$$

Correspondingly, the effective Rydberg energy levels will be modified to $E_n = -1\tilde{\mathrm{R}}\mathrm{y}/n^2$, with n an integer. The modified binding energy of the ground state would be,

$$-E_1 = 1\tilde{\mathrm{R}}\mathrm{y} = \frac{\hbar^2}{2m_e\tilde{r}_{\mathrm{B}}^2} = 13.6\ \mathrm{eV} \times \left[1 + \left(\frac{M}{3.8 \times 10^{15}\ \mathrm{g}}\right)\right]^{2} . \tag{4}$$

The gravitational and electric forces are of equal magnitude for a PBH mass of $M = 3.8 \times 10^{15}$ g, and gravity wins at larger masses. The atomic radius in equation (3) shrinks below the Schwarzschild radius in equation (1) for a PBH mass $M = 3.7 \times 10^{17}$ g. At this mass, the gravitational binding energy in equation (4) approaches a quarter of the electron rest-mass energy. For higher PBH masses, the Compton wavelength of the electron, $(\hbar/m_e c) = 4 \times 10^{-11}$ cm, is smaller than the Schwarzschild radius, and the ground state of the bound electron can be inside the horizon.

For lower mass PBHs, accretion could be significantly suppressed in common astrophysical environments, where gas particles have non-relativistic thermal speeds and the mass density is much lower than the nuclear density of neutron stars. In the rarefied environments of the interstellar or intergalactic media, a low-mass PBH would accrete one particle at a time with a significant quantum-mechnical suppression in accretion rate relative to hydrodynamic expectations.

3. DISCUSSION

To obtain reliable assessments of the accretion rate by PBHs with masses $M \lesssim 4 \times 10^{19}$ g, one must calculate the wave function of electrons and protons using the Dirac equation in the background PBH metric. For each quantum state and energy level, the overlap of the electron wave function with the volume interior to the event horizon sets a finite half-life for the electron to stay in a bound state outside the horizon. Afterwards, the electron will join the proton inside the horizon and neutralize the PBH charge.

In the quantum world, there is a finite probability per unit time for a plump atom to be captured by a small event horizon. The quantum transition to the final state of capture resembles tunneling through a barrier.

A bound electron can be captured by an atomic nucleus even without a central black hole. This well-known process, called electron capture, involves the absorption an electron from an inner atomic shell by a proton-rich nucleus of a neutral atom (Alvarez 1937). The rate for this capture is related to the overlap of the electron wave function with the volume of the atomic nucleus.

Similarly, the accretion of protons and electrons by a PBH in the asteroid mass range, is dictated by the overlap of their quantum-mechanical wave function with the volume interior to the PBH event horizon.

Ignoring Hawking radiation, an estimate for the half-life of a bound electron around a black hole with $r_{\rm Sch} < \tilde{r}_{\rm B}$ (or equivalently $M < 3.7 \times 10^{17}$ g) which already captured a proton, is

$$\tau_{1/2} \sim \frac{(\tilde{r}_{\rm B}/\tilde{v}_{\rm B})}{(r_{\rm Sch}/\tilde{r}_{\rm B})^3} = 2.2 \times 10^{-11}\ {\rm s} \left(\frac{M}{3.8 \times 10^{15}\ {\rm g}}\right)^{-3} \left[1 + \left(\frac{M}{3.8 \times 10^{15}\ {\rm g}}\right)\right]^{-5}, \quad (5)$$

where the effective electron speed is,

$$\tilde{v}_{\rm B} \sim \left[\frac{(e^2 + GMm_e)}{\hbar c}\right] c = 2.2 \times 10^8\ {\rm cm\ s^{-1}} \left[1 + \left(\frac{M}{3.8 \times 10^{15}\ {\rm g}}\right)\right]. \quad (6)$$

The accretion rate corresponding to the absorption of an electron-proton pair per $\tau_{1/2}$ is,

$$\dot{M} \equiv \frac{m_p}{\tau_{1/2}} = 7.5 \times 10^{-14}\ {\rm g\ s^{-1}} \left(\frac{M}{3.8 \times 10^{15}\ {\rm g}}\right)^3 \left[1 + \left(\frac{M}{3.8 \times 10^{15}\ {\rm g}}\right)\right]^5. \quad (7)$$

The accretion rate is too low to add significant mass to PBHs with $M \lesssim 1.4 \times 10^{17}$ g over the entire age of the Universe.

Including Hawking radiation (Hawking 1974) would further lower the accretion rate since the Hawking temperature, $T_{\rm H} = (\hbar c^3/8\pi k_B GM) = 2.7 \times 10^6\ {\rm eV}\ (M/3.8 \times 10^{15}\ {\rm g})^{-1}$, exceeds the binding energy $1\tilde{\rm R}{\rm y}$ in equation (4), and the Hawking luminosity, $L_{\rm H} = (\hbar c^6/15360\pi G^2 M^2) = 1.5 \times 10^{26}\ {\rm eV\ s^{-1}}\ (M/3.8 \times 10^{15}\ {\rm g})^{-2}$, exceeds $1\tilde{\rm R}{\rm y}/\tau_{1/2}$. The outward flux of high energy photons and electron-positron pairs could suppress accretion altogether in rarefied astrophysical environments.

At high enough plasma densities, a bound state could involve multiple electrons and protons simultaneously. When the inflow of fresh protons into the PBH exceeds $\tau_{1/2}^{-1}$, the PBH could be charged positively up to a maximum charge, $Q_{\rm max} \sim (GMm_p/e^2)e = (M/2.1 \times 10^{12}\ {\rm g})e$, at which the electric repulsion equals the gravitational attraction for external protons. At this maximum charge (which will be screened over the Debye length in the surrounding plasma), the accretion rate can be enhanced relative to equation (7) since the effective Bohr radius for the ground state of the innermost electron at $Q_{\rm max}$ shrinks below the Schwarzschild radius for PBH masses $M \gtrsim 8.6 \times 10^{15}$ g.

If we ever witness an asteroid-mass black hole in the solar system, it could serve as a testbed for quantum-gravitational physics on a subatomic scale.

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