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SPURIOUS RADIAL MIGRATION FROM RELATIVISTIC EFFECTS IN THE MILKY-WAY DISK

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ABSTRACT

The gradient of the gravitational redshift in the potential of the Milky-Way induces an apparent spurious radial migration. I show that this effect is simply related to the local acceleration, which was measured recently by *Gaia* eDR3, implying a spectroscopic shift of $-2.4 \times 10^{-2}(r/8\ \mathrm{kpc})^{-1}$ km s^{-1} kpc^{-1}. The transverse Doppler effect yields a comparable contribution. The spurious radial velocity from both relativistic effects amounts to crossing a major portion of the Milky-Way disk during the age of the universe, and must be corrected for in any future measurement of the actual radial migration of stars.

1. INTRODUCTION

According to General Relativity, time is dilated in a gravitational potential well, ϕ, relative to a distant observer. In the weak field regime, the shift in the ticking rate of a clock leads to a spectroscopic velocity offset in the radial r direction away from the observer (Weinberg 1972),

$$\frac{\Delta v_r}{c} = -\frac{\Delta \phi}{c^2}. \tag{1}$$

The related gravitational redshift was confirmed recently on a millimeter scale using atomic clocks (Bothwell et al. 2022), and was detected previously in spectral lines emitted from compact stars, such as white dwarfs (Greenstein & Trimble 1967; Barstow et al. 2005; Falcon et al. 2012) or neutron stars (Cottam et al. 2002).

The latest data from *Gaia* eDR3 implied that the Milky Way has a nearly flat rotation curve with a local circular speed $v_c \approx 230$ km s^{-1} that declines gently outwards by an amount $(dv_c/dr) = -(1.7 \pm 0.1)$ km s^{-1} kpc^{-1} (Eilers et al. 2019; Mróz et al. 2019).

2. RESULTS

Equation (1) implies an apparent spurious radial velocity with a radial gradient,

$$\frac{\partial v_r}{\partial r} = -\frac{1}{c}\frac{\partial \phi}{\partial r} = -\frac{1}{c}\frac{v_c^2}{r}, \tag{2}$$

where $g = -(\partial\phi/\partial r) = -(v_c^2/r)$ is the local gravitational acceleration. This leads to the spurious inference of radial recession (net redshift) away from the Sun for stars closer in to the Galactic center and radial approach (net blueshift) of stars farther out than the Sun's Galactocentric distance.

Data from *Gaia* eDR3 (Gaia Collaboration et al. 2021) implies $g = -(2.32 \pm 0.16) \times 10^{-8}$ cm s^{-2}, yielding a velocity gradient,

$$\frac{\partial v_r}{\partial r} = -2.4 \times 10^{-2} \left(\frac{r}{8\ \mathrm{kpc}}\right)^{-1}\ \mathrm{km\ s^{-1}\ kpc^{-1}}. \tag{3}$$

The spurious radial velocity is significant, as it amounts to crossing a major portion of the Milky-Way disk during the age of the universe. An actual radial migration of this magnitude is inferred by other means (Frankel et al. 2018; Lian et al. 2022). The velocity amplitude is smaller than current measurement errors (Eilers et al. 2019; Mróz et al. 2019).

Time dilation is also sourced by the Lorentz factor, leading to the transverse Doppler effect, which is not accounted for in non-relativistic calculations. At small velocities, the effect is second order in the velocity over the speed of light. Owing to the virial theorem, $\langle (v/c)^2 \rangle = -(1/2)\langle(\phi/c^2)\rangle$, the transverse Doppler effect is comparable in magnitude to the gravitational redshift effect, and leads to a spurious radial velocity gradient,

$$\frac{\partial v_r}{\partial r} = \frac{1}{2}c\frac{\partial}{\partial r}\left(\frac{v_c}{c}\right)^2 = 9 \times 10^{-2}\ \mathrm{km\ s^{-1}}\frac{\partial}{\partial r}\left(\frac{v_c}{230\ \mathrm{km\ s^{-1}}}\right)^2. \tag{4}$$

3. IMPLICATIONS

The sum of the relativistic effects in equations (3) and (4) implies a spurious radial migration, that amounts to crossing a major portion of the Milky-Way disk during the age of the universe. An actual migration of this magnitude is inferred by other means (Frankel et al. 2018; Lian et al. 2022), and its proper direct measurement in the future would need to correct for the relativistic effects mentioned here.

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