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GRAVITATIONAL WAVE ACCELERATION TO RELATIVISTIC ENERGIES

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ABSTRACT

A charged particle can be accelerated to arbitrarily high energies by maintaining a permanent resonance with the phase of a planar gravitational wave propagating along a uniform magnetic field. The Doppler-shifted cyclotron autoresonance could potentially result in electromagnetic afterglows near gravitational-wave sources.

1. INTRODUCTION

Consider a relativistic charged particle with a mass m, a charge q, a velocity vector $\mathbf{v} = \beta c$ and a Lorentz factor $\gamma = (1-\beta^2)^{-1/2}$, that gyrates at the cyclotron frequency, $\Omega_c = (qB/mc^2)$ in a uniform magnetic field along the z-axis, $\mathbf{B} = B\hat{z}$. If the cyclotron frequency resonates with the Doppler shifted frequency of a wave propagating along the magnetic field direction, the particle would witness steady acceleration at a fixed phase of the wave crest.

For a transverse wave propagating at the speed of light, c, the Doppler-shifted cyclotron resonance is given by,

$$\Omega_c = D\omega, \tag{1}$$

where $D \equiv \gamma(1-\beta_z)$ is the Doppler factor along the z-axis (Rybicki & Lightman 1986), and ω is the wave frequency in the background frame of reference.

The relativistic equation of motion of the charged particle in the presence of either an electromagnetic wave (Loeb & Friedland 1986; Loeb et al. 1987) or a gravitational wave (Servin et al. 2001) propagating along the z-axis, admits the same identity,

$$\frac{d\gamma}{dt} = \frac{d(\gamma\beta_z)}{dt}. \tag{2}$$

This implies the remarkable result that the Doppler factor $D = \gamma(1-\beta_z)$ is a constant of motion, guaranteeing that the resonance condition in equation (1) will be maintained at all times if it is satisfied initially, even as the particle gains energy.

Under resonance, the Lorentz factor of a relativistic particle with $\gamma \gg 1$ grows steadily over time,

$$\frac{d\gamma}{dt} = \Omega_c\alpha, \tag{3}$$

where for an electromagnetic wave (Loeb & Friedland 1986):

$$\alpha_{\rm EM} = \sqrt{\frac{2}{\gamma}} \times \left(\frac{qA}{mc^2}\right), \tag{4}$$

with $\mathbf{A}$ being the vector potential. For a gravitational wave (Servin et al. 2001),

$$\alpha_{\rm GW} = 2h, \tag{5}$$

with h being the dimensionless wave amplitude.

The electromagnetic wave case serves as the basis for a novel acceleration scheme, the so-called "autoresenance laser accelerator" (Loeb & Friedland 1986). Quasi-neutrality is perfectly maintained in a symmetric electron-positron plasma (Loeb et al. 1987).

2. AUTORESONANCE GRAVITATIONAL-WAVE ACCELERATION

Equations (3) and (5) imply that a plane-parallel gravitational wave of a constant amplitude h propagating along a constant magnetic field $\mathbf{B}$, would accelerate charged particles at a constant rate to arbitrarily high energies.

However, for astrophysical sources of gravitational waves, the wave amplitude h declines inversely with distance from the source. Given a magnetic field B oriented radially from the source over a coherence length ℓ, equation (3) implies that the particle's Lorentz factor would reach a maximum value of,

$$\gamma_{\max} \sim 1 + 2h\Omega_c \left(\frac{\ell}{c}\right). \tag{6}$$

Since $2h \lesssim (R_{\rm Sch}/\ell)$ (Shapiro & Teukolsky 1986), we get

$$\gamma_{\max} \lesssim 1 + \Omega_c t_{\rm Sch}, \tag{7}$$

where $t_{\rm Sch} = (R_{\rm Sch}/c) = 10^{-4}$ s $\times$ $(M/10M_\odot)$ is the light crossing-time for the Schwarzschild radius $R_{\rm Sch} = (2GM/c^2) = 30$ km $\times$ $(M/10M_\odot)$, of a gravitational wave source of total mass M.

The cyclotron frequency for electrons is $\Omega_{c,e} = 180$ Hz $\times$ $(B/10\mu{\rm G})$ (whereas for protons, Ω_c is smaller by the particle mass-ratio of 1.836×10^3), can resonate with the frequency of gravitational waves generated in the final coalescence phase of binaries composed of stellar-mass black holes or neutron stars, which produce $\omega \sim 10$–10^4 Hz (Cahillane & Mansell 2022; Vajente 2022).

Under favorable conditions, the cyclotron resonance could boost the energies of relativistic electrons or protons in the plasma surrounding compact gravitational-wave sources. If the magnetic field originates from the source - as expected in the case of neutron star mergers, its dipole amplitude would decline inversely with distance cubed, $B \propto \ell^{-3}$, out to the scale where the interstellar magnetic field will dominate.

3. ASTROPHYSICAL IMPLICATIONS

The autoresonant acceleration by gravitational waves can heat relativistic electrons or protons in the vicinity of mergers of compact objects. This, in turn, could trigger synchrotron emission by the accelerated electrons that would result in an electromagnetic counterpart to the gravitational wave signal.

In principle, the cyclotron autoresonance could potentially lead to electromagnetic afterglows of the type reported for the black hole merger GW150914 (Loeb 2016; D'Orazio & Loeb 2018) or the neutron star merger GW170817/GRB170817A (Mooley et al. 2022). Detailed modeling is needed for the expected electromagnetic counterpart in specific environments.

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