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PULSAR TIMING NOISE FROM BROWNIAN MOTION OF THE SUN

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ABSTRACT

Recently, Pulsar Timing Arrays (PTAs) reported a signal at nanohertz frequencies consistent with a stochastic gravitational wave background. Here, I show that the Brownian motion of the Sun as a result of its random gravitational interactions with the cluster of thousands of unmodeled Main-belt asteroids of diameters $\lesssim 80$ km, not included in the Solar system ephemeris, introduces correlated timing noise for pulsars with the magnitude and frequencies of the reported signal.

1. INTRODUCTION

Pulsar Timing Arrays (PTAs) monitor a population of millisecond radio pulsars (Foster & Backer 1990) in an attempt to detect correlated shifts in the arrival times of their radio pulses (Hellings & Downs 1983; Finn et al. 2009). Several PTAs reported recently the discovery of a correclated signal at frequencies of 1-10 nanohertz (Antoniadis et al. 2022; Agazie et al. 2023a; Reardon et al. 2023). The signal was interpreted as a stochastic gravitational wave background, potentially from mergers of supermassive black hole binaries throughout cosmic history (Agazie et al. 2023b; Padmanabhan & Loeb 2024).

Here, I show that the detected stochastic signal is also consistent in amplitude and frequencies with the Brownian motion of the Sun as a result of its interaction with unmodeled Main-belt asteroids of diameters $\lesssim 80$ km. In § 2, I derive the amplitude and frequencies of the resulting Doppler noise in pulsar timing, and in § 3, I suggest future prospects for characterizing this unmodeled noise.

2. BROWNIAN MOTION OF THE SUN

The Brownian motion of a massive object embedded in a gravitationally bound cluster of low-mass perturbers was studied both analytically and using N-body simulations in the context of massive black holes in star clusters (Chatterjee et al. 2002a,b; Merritt et al. 2007). Here, I apply these well-studied results to the Solar system, where the Sun is embedded in a cluster of unmodeled Main-belt asteroids (Novaković et al. 2022; Raymond & Nesvorný 2022). The many-body system of asteroids with diameters $\lesssim$ 80km represent objects that are not included in the Solar system ephemeris (Park et al. 2021) and is known to have a substantial spread in inclinations and eccentricities (Davis et al. 2002; Maeda et al. 2021).

Previous analysis of PTA data was based on the Solar system ephemeris calculated from well-studied objects in the Solar system (see Agazie et al. (2023c) and Vallisneri et al. (2020), and references therein). The Solar system ephemeris model includes 343 asteroids (Park et al. 2021), but according to the cumulative asteroid number versus diameter in Davis et al. (2002), there are many more than 343 Main belt asteroids below a mass scale of $\sim 10^{21}$ g.

Caballero et al. (2018) derived limits on individual unmodeled objects with masses as small as $\sim 10^{-11} M_\odot = 2 \times 10^{22}$ g in Keplerian orbits around the Sun. However, a cluster of thousands of unmodeled perturbers which are not included in the Solar system ephemeris (Park et al. 2021), is known to exist in the Main asteroid belt (Davis et al. 2002; Maeda et al. 2021; Novaković et al. 2022; Raymond & Nesvorný 2022). These unmodeled objects which are not part of the Solar system ephemeris (Park et al. 2021), result in a Brownian motion of the Sun and introduce a stochastic noise to the PTA signal that cannot be picked up through a fit to a Keplerian orbit of a single unmodeled object, as done by Caballero et al. (2018) or Guo et al. (2018, 2019). Below I derive the characteristic amplitude and frequencies of this unmodeled noise based on known properties of the Main asteroid belt (Davis et al. 2002; Maeda et al. 2021; Novaković et al. 2022; Raymond & Nesvorný 2022).

The motion of the Sun relative to the Solar system barycenter satisfies momentum conservation with all objects of masses m_i and velocity $\mathbf{v}_i$ in the Solar system,

$$M_\odot \mathbf{v}_\odot = \Sigma_i m_i \mathbf{v}_i \ . \tag{1}$$

The effects of the planets, minor planets, moons and massive asteroids is already included in the ephemeris model of the Solar system used by PTAs (Park et al. 2021; Agazie et al. 2023c; Vallisneri et al. 2020).

Here, I focus on the large population of unmodeled asteroids ($\lesssim$ 80km in diameter) as a source of residual noise in correlated pulsar timing (see also Fedderke et al. (2021)). Squaring both sides of equation (1), taking a statistical average (denoted hereafter by angular brackets, following the ergodic theorem) and ignoring velocity correlations among the many asteroids, I get the dispersion in random velocity fluctuations of the Sun as a result of its stochastic gravitational interactions with the cluster of unmodeled asteroids,

$$\langle |v_\odot^2| \rangle = \frac{N_a m_a^2}{M_\odot^2} \langle |v_a^2| \rangle \ , \tag{2}$$

where m_a and v_a are the characteristic mass and velocity of the unmodeled asteroids and $N_a \gtrsim 10^3$ is their total number.

As expected, equation (2) yields kinetic-energy equipartition between the central mass and its perturbers if the total mass of the perturbers equals the central mass, $N_a m_a = M_\odot$. However, for a smaller perturber population, as in the case of the Solar system, Poisson ($\sim \sqrt{N_a}$) fluctuations yield a reduced velocity dispersion for the central mass.

Main-belt asteroids with a typical mass $m \lesssim 10^{-12} M_\odot$ and a characteristic speed ~ 20 km s^{-1}, yield a Brownian motion of the Sun with a velocity dispersion of order,

$$\delta v \equiv \langle |v_\odot^2| \rangle^{1/2} = 3.2 \times 10^{-5} \text{ cm s}^{-1} \left(\frac{N_a}{10^3} \right)^{1/2} \left(\frac{m_a}{10^{21} \text{ g}} \right) \left(\frac{\langle |v_a^2| \rangle^{1/2}}{20 \text{ km s}^{-1}} \right) \ . \tag{3}$$

For semi-major axes in the range of 2-4 au, the characteristic frequency of the perturbations would be of order the typical orbital frequency of the unmodeled asteroids, $f \sim (3 \text{ yr})^{-1} = 10^{-8}$ Hz.

The unmodeled random walk of the Sun's velocity vector in 3D is characterized by a correlation time of $f^{-1} \sim 3$ yr. It leads to Doppler-shift fluctuations in pulsar timing that is correlated among different pulsars. The resulting Doppler vector fluctuations with a stochastically varying orientation, could mascarade as a stochastic gravitational wave background of a characteristic strain,

$$h_c \sim \left(\frac{\delta v}{c} \right) \sim 10^{-15} \left(\frac{N_a}{10^3} \right)^{1/2} \left(\frac{m_a}{10^{21} \text{ g}} \right) \left(\frac{\langle |v_a^2| \rangle^{1/2}}{20 \text{ km s}^{-1}} \right) \ , \tag{4}$$

which is similar in amplitude to the gravitational wave signal reported by PTAs (Antoniadis et al. 2022; Agazie et al. 2023a; Reardon et al. 2023).

Each individual asteroid induces by itself a much smaller Solar velocity fluctuation of order (Binney & Tremaine 1987),

$$\frac{\Delta v_\odot}{c} \sim \left(\frac{2Gm_a}{b_a v_a c}\right) \sim 5\times 10^{-17}\left(\frac{m_a}{10^{21}\ \mathrm{g}}\right)\left(\frac{b_a}{3\ \mathrm{au}}\right)^{-1} , \tag{5}$$

where b_a is its distance of closest approach to the Sun (perihelion).

At any given time, the distribution of thousands of unmodeled asteroids would be randomly lopsided towards one hemisphere around the Sun, with a Poisson excess of $\sim (N_a/2)^{1/2}$ asteroids. Indeed, this enhancement yields the larger cumulative signal expressed in equation (4). The direction and amplitude of the Poisson fluctuations accelerating the Sun would change randomly over asteroid orbital times in the range, $\sim$ 10–3 yr, corresponding to the frequency range of $f \sim$ 3–10 nHz.

3. DISCUSSION

I have shown that an unmodeled population of thousands of Main belt asteroids of diameters $\lesssim$ 80km, would result in a Brownian motion of the Sun relative to the Solar system barycenter with a velocity dispersion given by equation (3). This results in an unmodeled noise for correlated timing residual of pulsars at the characteristic amplitude (4) and frequencies reported by PTAs (Antoniadis et al. 2022; Agazie et al. 2023a; Reardon et al. 2023).

When squaring the right-hand-side of equation (1), we ignored correlations between the velocities of different unmodeled asteroids. Any such correlations as a result of the phase-space clustering of asteroids due to planets, as in Jupiter's clusters of Trojans, Greeks and Hildas, would enhance the expected noise. In that case, the mass of a perturber will be that of a cluster of asteroids rather than a single asteroid. For example, there are more than 100 Jupiter's Trojans with diameters in the range 40-80km exceeding 10^{22}g in total mass, and there are many known families in the Main asteroid belt (such as Flora, Eunomia, Eos, Hungaria, Karin, Koronis, Phocaea, Vesta and Themis; see Lemaitre (2005)) in addition to the Trojans of Earth and Mars. The additional effect of these clusters is an enhancement of the noise amplitude in equation (3), which can be accounted for by using the cluster mass and number in place of m_a and N_a. This enhancement cannot exceed an extra factor of $\sqrt{N_a}$ relative to the indpendent asteroids account of equation (3), as this extra factor saturates the upper limit associated with placing all asteroids in a single cluster.

The PTA reports on a stochastic gravitational wave background (Antoniadis et al. 2022; Agazie et al. 2023a; Reardon et al. 2023) used the Hellings & Downs (1983) angular correlations among pulsars, based on the quadrupolar nature of gravitational waves. The Doppler noise considered here is expected to produce dipolar angular correlations among pulsars. However, the 3D random walk of a dipole sourced by a torodial configuration of asteroids around the Sun may be difficult to distinguish from a quadrupolar random walk sourced by a stochastic gravitational wave background, given the small number of independent correlation times (~ 5 periods of ~ 3 years) available during 15 years of observations.

An increase in the number of Main-belt asteroids included in the Solar system ephemeris (Park et al. 2021) and future observations of missed asteroids with the Legacy Survey of Space and Time (LSST) of the Vera C. Rubin Observatory in Chile (Schwamb et al. 2023), would allow better modeling of the noise they introduce to PTAs.

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