Mass distribution of halos seeded by cosmic string loops

I. A SIMPLE MODEL

According to recent string simulations, the typical string loops formed at time t_i in the matter era have characteristic length

$$L_*(t_i) \sim 0.15 t_i. \tag{1}$$

The mass of such loops is

$$M_*(t_i) = \mu L_*(t_i), \tag{2}$$

where μ can be parametrized as

$$\mu = 1.4 \times 10^{28} G\mu \text{ g/cm} = 1.4 \times 10^{20} \mu_{-8} \text{ g/cm}. \tag{3}$$

In this section, we shall adopt a simple model in which all relevant loops formed at time t_i have length $L \sim L_*(t_i)$. Because of the crude nature of this approximation, we shall not be too fussy about numerical factors.

The loop distribution observed in the simulations is actually rather broad, extending to a large range of lengths smaller than $L_*(t_i)$. In the next section we shall discuss the effect of this broader loop distribution on the mass distribution of halos. We shall see that the simple model we adopted here is in fact reasonably accurate.

Dark matter halos around loops will reach mass comparable to the mass of the loop in about one Hubble time and will grow like $M(t) \propto t^{2/3}$ afterwards. Loops at the large end of the distribution have relatively low velocities, so it seems reasonable to use the spherical model, which gives

$$M(t; t_i) = \frac{2}{5} M_*(t_i) \left(\frac{t}{t_i}\right)^{2/3} = 0.06 \mu t_i^{1/3} t^{2/3}. \tag{4}$$

The number density of loops formed at time t_i can be estimated from energy conservation. The energy that goes into loop production per unit volume per unit time can be estimated as (see, e.g., [1], sec. 9.3.3)

$$\dot{\rho}_{loops}(t_i) = \frac{2}{3t}(1 - 2\langle v^2 \rangle) \rho_{inf}(t_i). \tag{5}$$

Here, $\langle v^2 \rangle \approx 0.34$ is the rms string velocity,

$$\rho_{inf}(t) = \frac{\zeta\mu}{t^2} \tag{6}$$

is the energy density in infinite strings, and $\zeta \approx 4$. Combining all this, we have

$$\dot{\rho}_{loops}(t_i) = 0.22 \frac{\rho_{inf}}{t_i} \approx \frac{\mu}{t_i^3}. \tag{7}$$

Assuming that a fraction κ of this energy is turned into loops of length $L_*(t_i)$, we find the following expression for the density of loops formed in a time interval $\Delta t_i \sim t_i$,

$$t_i \frac{dn}{dt_i} = \frac{\kappa t_i}{\mu L_*(t_i)} \dot{\rho}_{loops}(t_i) \approx 7\kappa t_i^{-3} \sim 1.4 t_i^{-3}. \tag{8}$$

Here, in the last step we have used the value suggested by the simulations, $\kappa \sim 0.2$. Using eq.(4) for the loop mass, we can express this as

$$M \frac{dn}{dM} = \frac{1}{3} t_i \frac{dn}{dt_i} = 1 \times 10^{-4} (\mu/M)^3 \sim 3 \times 10^{-3} \mu_{-8}^3 \left(\frac{10^{11} M_\odot}{M} \right)^3 Mpc^{-3}, \tag{9}$$

where $\mu_{-8} \equiv G\mu/10^{-8}$. The distribution (9) extends to

$$M_{max}(t) \sim 0.4 M_*(t) = 0.06\mu t = 6 \times 10^{13} \mu_{-8} (1+z)^{-3/2} M_\odot \tag{10}$$

and sharply drops to zero at larger masses.

Eq.(9) gives the number of loops per logarithmic mass interval per unit physical volume. This distribution is independent of time; the only thing that changes is the upper cutoff $M_{max}(t)$, given by Eq.(10). To compare to Avi's plot, note that he plotted the halo density $\tilde{n}(M)$ per comoving volume. This is related to (9) by an extra factor $(1+z)^{-3}$,

$$M \frac{d\tilde{n}}{dM} \sim 3 \times 10^{-3} \mu_{-8}^3 (1+z)^{-3} \left(\frac{10^{11} M_\odot}{M} \right)^3 (cMpc)^{-3}, \tag{11}$$

where $cMpc$ stands for "comoving megaparsec", that is, the comoving scale which is equal to 1 Mpc at present.

Another quantity of interest is the fraction of matter in halos per logarithmic mass interval as a function of redshift,

$$\nu_M = \frac{M^2}{\rho_m} \frac{dn}{dM}, \tag{12}$$

where

$$\rho_m = \frac{1}{6\pi G t^2} = \frac{(1+z)^3}{6\pi G t_0^2} \tag{13}$$

is the matter density. Using Eq.(9) for $n(M)$, we find

$$\nu_M(z) = 1 \times 10^{-3} \mu_{-8}^3 (1+z)^{-3} \left(\frac{10^{11} M_\odot}{M} \right)^2 . \tag{14}$$

As before, this distribution applies up to $M \sim M_{max}(z)$.

In this analysis we disregarded the fact that smaller halos can be incorporated into larger halos. This is probably justified in the regime where the loop-seeded halos are significantly more massive than the halos formed by inflationary perturbations.

II. A MORE REALISTIC MODEL

The density of loops with length in the interval dl formed in time interval dt_i can be expressed as

$$\frac{1}{\xi_i^3} f(l/\xi_i) \frac{dl}{l} \frac{dt_i}{\xi_i}. \tag{15}$$

Here, $\xi_i = \xi(t_i)$,

$$\xi(t) = \gamma t \tag{16}$$

is the characteristic scale of the long string network, and γ is related to ζ in Eq.(6) as $\zeta = \gamma^{-2}$. In a matter-dominated universe, $\gamma = 0.56$ (this is from Ringeval et.al.).

The density of these loops at times $t > t_i$ is then

$$n(l, t; t_i) dl dt_i = \frac{1}{l \gamma^4 t_i^4} f\left(\frac{l}{\gamma t_i} \right) \left(\frac{t_i}{t} \right)^2 dl dt_i. \tag{17}$$

The mass of the halo around a loop of length l that formed at time t_i is given by Eq.(4). Expressing l in terms of M from that equation,

$$l = \frac{5}{2} \frac{M}{\mu} \left(\frac{t_i}{t} \right)^{2/3}, \tag{18}$$

we can find the density of halos with masses between M and $M + dM$ around loops formed in the interval dt_i at times $t > t_i$,

$$\frac{1}{\gamma^4 t_i^4} f\left[\frac{5}{2} \frac{M}{\gamma \mu t_i} \left(\frac{t_i}{t} \right)^{2/3} \right] \left(\frac{t_i}{t} \right)^2 \frac{dM}{M} dt_i. \tag{19}$$

Integrating over t_i up to t, we obtain the total density of halos per logarithmic mass interval,

$$M \frac{dn}{dM}(t) = \frac{1}{\gamma^4 t^2} \int_0^t \frac{dt_i}{t_i^2} f\left(\frac{5}{2} \frac{M}{\gamma \mu t_i^{1/3} t^{2/3}} \right). \tag{20}$$

Introducing a new variable

$$x = \frac{5}{2}\frac{M}{\gamma\mu t_i^{1/3}t^{2/3}}, \tag{21}$$

we can express this as

$$M\frac{dn}{dM}(t) = \frac{24}{125}\frac{\mu^3}{\gamma M^3}\int_{x_M(t)}^{\infty} f(x)x^2dx, \tag{22}$$

where

$$x_M(t) = \frac{5}{2}\frac{M}{\gamma\mu t}. \tag{23}$$

To estimate the intergral, note that we used the same definition of $f(x)$ as in [1]. Then, according to Eqs.(9.3.24),(9.3.17),

$$\int_0^{\infty} f(x)dx = \frac{2}{3}(1-2\langle v^2\rangle)\gamma \approx 0.2\gamma. \tag{24}$$

On the other hand, the normalized loop production function $\tilde{f}(x)$ found from the simulations satisfies

$$\int_0^{\infty} \tilde{f}(x)dx = 1. \tag{25}$$

Hence,

$$f(x) = 0.2\gamma\tilde{f}(x), \tag{26}$$

and we have

$$M\frac{dn}{dM}(t) \approx 0.04\frac{\mu^3}{M^3}\int_{x_M(t)}^{\infty} \tilde{f}(x)x^2dx. \tag{27}$$

The loop production function $\tilde{f}(x)$ found in the simulations can be approximated as

$$\tilde{f}(x) = \frac{A}{x}\ln(x/x_0), \tag{28}$$

where $A = 0.01$ and $x_0 = 10^{-7}$. This expression applies for $x_0 < x < x_{max}$, where

$$x_{max} = L_*(t)/\xi(t) = 0.27. \tag{29}$$

Beyond this range, we can set $\tilde{f}(x) \approx 0$. Then

$$\int_{x_M(t)}^{\infty} \tilde{f}(x)x^2dx = \frac{A}{2}x_{max}^2\left[\ln\left(\frac{x_{max}}{x_0}\right) - \frac{1}{2}\right] - \frac{A}{2}x_M^2(t)\left[\ln\left(\frac{x_M(t)}{x_0}\right) - \frac{1}{2}\right]. \tag{30}$$

For $M \ll M_{max}(t)$, we have $x_M(t) \ll x_{max}$, and the lower limit of integration in (30) can be replaced by zero. This gives

$$\int_0^{\infty} \tilde{f}(x)x^2dx = \frac{A}{2}x_{max}^2\left[\ln\left(\frac{x_{max}}{x_0}\right) - \frac{1}{2}\right] \approx 4\times 10^{-3}. \tag{31}$$

Substituting this in (27), we find

$$M\frac{dn}{dM}(t) \approx 2 \times 10^{-4}\frac{\mu^3}{M^3}, \tag{32}$$

in a good agreement with Eq.(9) that we obtained using our simple model.

The curves shown in the plots were obtained using dn/dM from Eqs.(27),(30).

[1] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and Other Topological Defects (Cambridge University Press, Cambridge, 2000).