in our body together far greater than gravity, but even the gravitational self-force of our body on itself overwhelms the cosmic influence. Only on very large scales does the cosmic gravitational force dominate the scene. This also implies that we cannot observe the cosmic expansion with a local laboratory experiment; in order to notice the expansion we need to observe sources which are spread over the vast scales of millions of light years.

The space-time of an expanding, homogeneous and isotropic, flat Universe can be described very simply. Because the cosmological principle, we can establish a unique time coordinate throughout space by distributing clocks which are all synchronized throughout the Universe, so that each clock would measure the same time t since the Big Bang. The space-time (4–dimensional) line element ds, commonly defined to vanish for a photon, is described by the Friedmann-Robertson-Walker (FRW) metric,

$$ds^2 = c^2 dt^2 - d\ell^2, \tag{1.1}$$

where c is the speed of light and $d\ell$ is the spatial line-element. The cosmic expansion can be incorporated through a scale factor $a(t)$ which multiplies the fixed (x, y, z) coordinates tagging the clocks which are themselves "comoving" with the cosmic expansion. For a flat space,

$$d\ell^2 = a(t)^2(dx^2 + dy^2 + dz^2) = a^2(t)(dr^2 + r^2 d\Omega), \tag{1.2}$$

where $d\Omega = d\theta^2 + \sin^2\theta d\phi^2$ with (r, θ, ϕ) being the spherical coordinates centered on the observer, and $(x, y, z) = (r\cos\theta, r\sin\theta\cos\phi, r\sin\theta\sin\phi)$

A source located at a separation $R = a(t)r$ from us would move at a velocity $v = dR/dt = \dot{a}r = (\dot{a}/a)R$, where $\dot{a} = da/dt$. Here r is a time-independent tag, denoting the present-day distance of the source (when $a(t) \equiv 1$). Defining $H = \dot{a}/a$ which is constant in space, we recover the Hubble expansion law $v = HR$.

Edwin Hubble measured the expansion of the Universe using the Doppler effect. We are all familiar with the same effect for sound waves: when a moving car sounds its horn, the pitch (frequency) we hear is different if the car is approaching us or receding away. Similarly, the wavelength of light depends on the velocity of the source relative to us. As the Universe expands, a light source will move away from us and its Doppler effect will change with time. The Doppler formula for a nearby source of light (with a recession speed much smaller than the speed of light) gives

$$\frac{\Delta\nu}{\nu} \approx -\frac{\Delta v}{c} = -\left(\frac{\dot{a}}{a}\right)\left(\frac{R}{c}\right) = -\frac{(\dot{a}\Delta t)}{a} = -\frac{\Delta a}{a}, \tag{1.3}$$

with the solution, $\nu \propto a^{-1}$. Correspondingly, the wavelength scales as $\lambda = (c/\nu) \propto a$. We could have anticipated this outcome since a wavelength can be used as a measure of distance and should therefore be stretched as the Universe expands. This holds also for the de Broglie wavelength $\lambda_{\rm dB} = (h/p) \propto a$, characterizing the quantum-mechanical wavefunction of a massive particle with momentum p (where h is Planck's constant). Consequently, the kinetic energy of a non-relativistic particle scales as $(p^2/2m_p) \propto a^{-2}$, and so in the absence of heat exchange with other systems the temperature of a gas of non-relativistic protons and electrons would cool faster ($\propto a^{-2}$) than the CMB temperature ($h\nu \propto a^{-1}$) as the Universe expands and a increases. The redshift z is defined through the factor $(1 + z)$ by which the photon wavelength was stretched (or its frequency reduced) between its emission and observation times. If we define $a = 1$ today, then $a = 1/(1 + z)$ at earlier times. Higher redshifts correspond to a higher recession speed of the source relative to us (ultimately approaching the speed of light when the redshift goes to infinity), which in turn implies a larger distance (ultimately approaching our horizon, which is the distance traveled by light since the Big Bang) and an earlier emission time of the source in order for the photons to reach us today.

We see high-redshift sources as they looked at early cosmic times. Observational cosmology is like archaeology – the deeper we look into space the more ancient the clues about our history are (see Figure 1.3).[iii] But there is a limit to how far back we can see: we can image the Universe only if it is transparent. Earlier than 400,000 years after the Big Bang, the cosmic gas was sufficiently hot to be fully ionized, and the Universe was opaque due to scattering by the dense fog of free electrons that filled it. Thus, telescopes cannot be used to image the infant Universe at earlier times (at redshifts $> 10^3$). The earliest possible image of the Universe can be seen in the cosmic microwave background, the thermal radiation left over from the transition to transparency (Figure 1.1). The first galaxies are believed to have formed long after that.

The expansion history of the Universe is captured by the scale factor $a(t)$. We can write a simple equation for the evolution of $a(t)$ based on the behavior of a small region of space. For that purpose we need to incorporate the fact that in Einstein's theory of gravity, not only does mass density ρ gravitate but pressure p does too. In a homogeneous and isotropic Universe, the quantity $\rho_{\rm grav} = (\rho + 3p/c^2)$ plays the role of the gravitating mass density ρ of Newtonian gravity. There are several examples to consider. For a radiation fluid,[iv] $p_{\rm rad}/c^2 = \frac{1}{3}\rho_{\rm rad}$, implying that $\rho_{\rm grav} = 2\rho_{\rm rad}$.

On the other hand, if the vacuum has a nonzero energy density that is constant in space and time, the so-called **cosmological constant**, then the pressure of the vacuum is negative because by opening up a new volume increment ΔV one *gains* an energy $\rho_{\rm vac}c^2\Delta V$ – instead of losing it, as is the case for normal fluids that expand into more space. In thermodynamics, pressure is derived from the deficit in energy per unit of new volume, which in this case gives $p_{\rm vac}/c^2 = -\rho_{\rm vac}$. This in turn leads to another reversal of signs, $\rho_{\rm grav} = (\rho_{\rm vac} + 3p_{\rm vac}/c^2) = -2\rho_{\rm vac}$, which may be interpreted as repulsive gravity! This surprising result gives rise to the phenomenon of accelerated cosmic expansion, which characterized the early period of cosmic inflation as well as the latest six billions years of cosmic history.

As the Universe expands and the scale factor increases, the matter mass density declines inversely with volume, $\rho_{\rm matter} \propto a^{-3}$, whereas the radiation energy

[iii]Cosmology and archaeology share another similarity: both are *observational*, rather than *experimental*, sciences. As such, we are forced to interpret the complicated physics of actual systems rather than design elegant experiments that can answer targeted questions. Although simplified models can be built in the laboratory (or even inside computers), the primary challenge of cosmology is figuring out how to extract useful information from real and complex systems that cannot be artificially altered.

[iv]The momentum of each photon is $\frac{1}{c}$ of its energy. The pressure is defined as the momentum flux along one dimension out of three, and is therefore given by $\frac{1}{3}\rho_{\rm rad}c^2$, where $\rho_{\rm rad}$ is the equivalent mass density of the radiation.

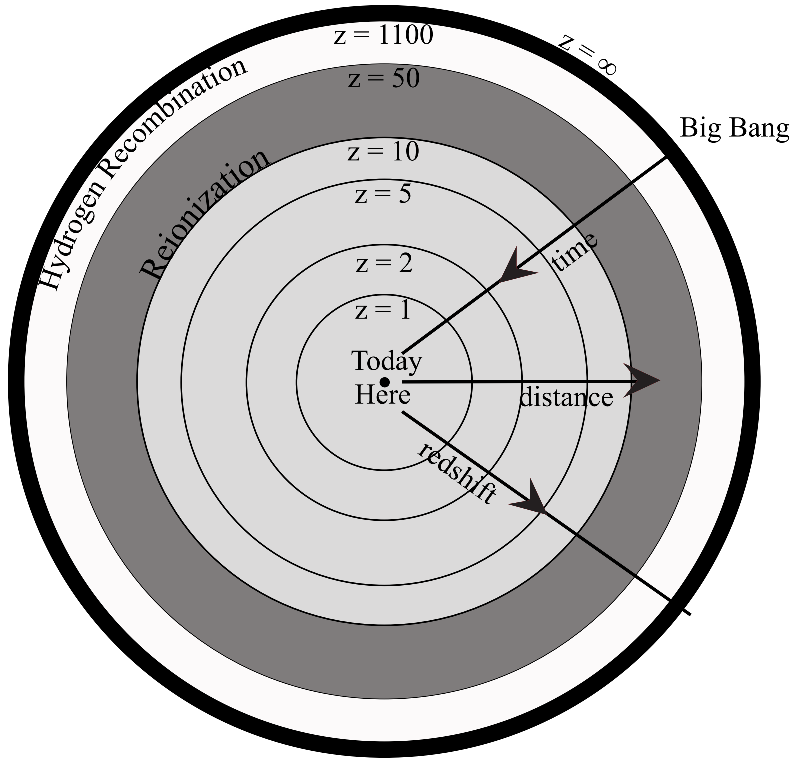

Figure 1.3 Cosmic archaeology of the observable volume of the Universe, in comoving coordinates (which factor out the cosmic expansion). The outermost observable boundary ($z = \infty$) marks the comoving distance that light has traveled since the Big Bang. Future observatories aim to map most of the observable volume of our Universe, and improve dramatically the statistical information we have about the density fluctuations within it. Existing data on the CMB probes mainly a very thin shell at the hydrogen recombination epoch ($z \sim 10^3$, beyond which the Universe is opaque), and current large-scale galaxy surveys map only a small region near us at the center of the diagram. The formation epoch of the first galaxies that culminated with hydrogen reionization at a redshift $z \sim 10$ is shaded dark gray. Note that the comoving volume out to any of these redshifts scales as the distance cubed.

density (which includes the CMB and three species of relativistic neutrinos) decreases as $\rho_{\rm rad}c^2 \propto a^{-4}$, because not only is the density of photons diluted as a^{-3}, but the energy per photon $h\nu = hc/\lambda$ (where h is Planck's constant) declines as a^{-1}. Today $\rho_{\rm matter}$ is larger than $\rho_{\rm rad}$ (assuming massless neutrinos) by a factor of $\sim 3,300$, but at $(1+z) \sim 3,300$ the two were equal, and at even higher redshifts the radiation dominated. Since a stable vacuum does not get diluted with cosmic expansion, the present-day $\rho_{\rm vac}$ remained a constant and dominated over $\rho_{\rm matter}$ and $\rho_{\rm rad}$ only at late times (whereas the unstable "false vacuum" that dominated during inflation has decayed when inflation ended).

In this book, we will primarily be concerned with the "cosmic dawn," or the era in which the first galaxies formed at $z \sim 6$–30. At these early times, the cosmological constant is very small compared to the matter densities and can generally be ignored.

1.3 MILESTONES IN COSMIC EVOLUTION

The gravitating mass, $M_{\rm grav} = \rho_{\rm grav}V$, enclosed by a spherical shell of radius $a(t)$ and volume $V = \frac{4\pi}{3}a^3$, induces an acceleration

$$\frac{d^2a}{dt^2} = -\frac{GM_{\rm grav}}{a^2}. \tag{1.4}$$

Since $\rho_{\rm grav} = \rho + 3p/c^2$, we need to know how pressure evolves with the expansion factor $a(t)$. This is obtained from the thermodynamic relation mentioned above between the change in the internal energy $d(\rho c^2 V)$ and the pdV work done by the pressure, $d(\rho c^2 V) = -pdV$. This relation implies $-3pa\dot{a}/c^2 = a^2\dot{\rho} + 3\rho a\dot{a}$, where a dot denotes a time derivative. Multiplying equation (1.4) by $\dot{a}$ and making use of this relation yields our familiar result

$$E = \frac{1}{2}\dot{a}^2 - \frac{GM}{a}, \tag{1.5}$$

where E is a constant of integration and $M \equiv \rho V$. As discussed before, the spherical shell will expand forever (being gravitationally unbound) if $E \geq 0$, but will eventually collapse (being gravitationally bound) if $E < 0$. Making use of the Hubble parameter, $H = \dot{a}/a$, equation (1.5) can be re-written as

$$\frac{E}{\dot{a}^2/2} = 1 - \Omega, \tag{1.6}$$

where $\Omega = \rho/\rho_c$, with

$$\rho_c = \frac{3H^2}{8\pi G} = 9.2 \times 10^{-30}\,\frac{\rm g}{\rm cm^3}\left(\frac{H}{70\ {\rm km\ s^{-1}Mpc^{-1}}}\right)^2. \tag{1.7}$$

With Ω_m, Ω_Λ, and Ω_r denoting the present contributions to Ω from *matter* (including cold dark matter as well as a contribution Ω_b from ordinary matter of protons and neutrons, or "baryons"), *vacuum density* (cosmological constant), and *radiation*, respectively, a flat universe with $E = 0$ satisfies

$$\frac{H(t)}{H_0} = \left[\frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{\Omega_r}{a^4}\right]^{1/2}, \tag{1.8}$$