Research Notes

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I here explain how I obtain the mass density profiles and BH masses for a sample of 57 galaxies.

1 Surface Brightness Profiles

I use the Lauer *et al* 2005 sample, who parameterized their V band surface brightness profiles with "Nuker-law" fits, which are of the form:

$$I(R) = 2^{(\beta-\gamma)/\alpha} I_b \left(\frac{R_b}{R}\right)^{\gamma} \left[1 + \left(\frac{R}{R_b}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}. \tag{1}$$

As explained in my previous notes, it is self-consistent to "circularize" these galaxies. I discuss strategies of circularization in my previous notes, and the one that I here adopt is to take the geometric mean of the semi-major (R) and semi-minor (R_{smi}) values at a specific position within the projected galaxy image:

$$R_{circ} \equiv \frac{R + R_{mi}}{2} = \frac{R}{2}\left[1 + (1-\epsilon^2)^{1/2}\right]. \tag{2}$$

By inverting this definition, I can solve for R in terms of R_{circ}:

$$R = 2R_{circ}[1 + (1-\epsilon)^{1/2}] \equiv R_{cric}\lambda. \tag{3}$$

The surface brightness profiles as a function of R_{circ} are now

$$I(R_{circ}) = 2^{(\beta-\gamma)/\alpha} I_b \lambda^{-\gamma} \left(\frac{R_b}{R_{circ}}\right)^{\gamma} \left[1 + \lambda^{\alpha}\left(\frac{R_{circ}}{R_b}\right)^{\alpha}\right]^{(\gamma-\beta)/\alpha}. \tag{4}$$

Note that λ is technically a function of position within the galaxy, since the measured ellipticity varies with R. Lauer *et al* 2005 only provide an "inner" and "outer" ellipticity for each galaxies. The inner ellipticity is the luminosity weighted average of the ellipticity inner to R_b, and the outer ellipticity is the luminosity weighted average of the ellipticity outer to R_b. Since I do not have the full ellipticity profile for each galaxy, but only these two ellipticity values, and to make the inverse Abel transformation easier, I make λ independent of R_{circ} by estimating $\epsilon(R)$ as $\langle\epsilon\rangle$, where the brackets denote the average of the two ellipticity values. This should not significantly affect our results since the "circularization" only affects the surface density profiles on the level of a few percent.

Figure 1: Derived mass density profiles for several galaxies (solid line). The dashed line is a power law fit to the mass density profile at the low end of r.

2 Mass densities

To obtain luminosity density profiles as a function of the 3-dimensional galactic position, r, I inverse Abel transform Eqn. **??** for each galaxy in the Lauer *et al* 2005 sample. I do this numerically, as there is no analytic solution for an inverse Abel transform of a Nuker profile. This allows me to obtain the V band luminosity density profile in units of $L_\odot/pc^3$ as a function of r, $j(r) \equiv dL/d^3r$. I check my results against Faber *et al* 1997 who provide Nuker fits to a sample of galaxies. They too do an inverse Abel transform, and list $j(r = 10pc)$ for each galaxy. When I evaluate my calculated $j(r)$ profiles at 10pc, my numbers match theirs.

By multiplying by an appropriate V band stellar mass-to-light ratio, Υ_V, we may obtain the mass density profile for each galaxy. For elliptical galaxies Υ_V is typically in the range of 1-10$\Upsilon_\odot$ (e.g., see Gebhardt & Thomas 2009, Schulze & Gebhardt 2011). For the calculations in this report, I therefore take a fiducial value of Υ_V of $5\Upsilon_\odot$. A more accurate estimate of Υ_V for each galaxy may be in order since the stellar collision rate, Γ, has a fairly strong dependence on the value of Υ_V since $\Gamma \propto n^2 \propto \Upsilon_V^2$. Lauer *et al* 2005 provide measured velocity dispersions, $\sigma_\star$ for each galaxy, so this may help us better constrain Υ_V for each galaxy. Additionally, Faber *el al* 1997 provide estimates of Υ_V for 16 of the 57 galaxies in my sample, although the derivation of these values seems a little simplistic.

I show a few examples of the mass density profiles, $\rho(r)$ that I obtain in Fig. **??**. As is evident from the figures, $\rho(r)$ is virtually a perfect power law up until $\sim R_b$. Since we are mostly interested in very small values of r, and since a power-law is very easy to work with when calculating collision rates, I fit power-laws to each profile at the lower end of r (dashed lines in Fig. **??**). In general, these power laws seem to be valid up until about 10^{-2}pc. If $\rho(r)$ is needed past this value, I can provide the full $\rho(r)$ array for each galaxy. To automate the fitting of the power laws, I consider the fractional difference between the derivative of $\rho(r)$ (in log space) as a function of r and the derivative of ρ at 10^{-6}pc:

$$frac.\ diff(r) = \frac{\left| \frac{dlog\rho}{dlogr'}\big|_r - \frac{dlog\rho}{dlogr'}\big|_{10^{-6}\mathrm{pc}} \right|}{\frac{dlog\rho}{dlogr'}\big|_{10^{-6}\mathrm{pc}}}. \quad (5)$$

I show this for a typical galaxy in Fig. **??**. As expected, since the slope is very constant, the fractional difference is very small across a large range of r (it reaches about 1% at about 10^{-2}pc). The noise at the lower end of r in this figure is numerical noise. I therefore fit the power law for each galaxy in the range of r_{lower} - r_{upper}, where, $r_{lower} = 10^{-6}$pc and r_{upper} equals the radius at which the fractional difference becomes 0.1% (about 10^{-3}pc for this galaxy). I visually inspected each galaxy to make sure this fitting scheme worked for the whole sample. Several of the fits were unacceptable, and I fit those by end. In general, as can be seen in the figure, the power laws are *very* good approximations to $\rho(r)$ at the lowest end of r.

To get a feel for what sort of mass profiles we have in our sample, I plot a histogram of the power law slope of the fitted power law to each galaxy in Fig. **??**. For comparison,

Figure 2: The fractional difference between the derivative of $\rho(r)$ (in log space) as a function of r and the derivative of ρ at 10^{-6}pc. Note that even though the figure says % difference, I am actually plotting the FRACTIONAL difference.

Figure 3: Distribution of the power law slope of the fitted power law for the 57 galaxies in my sample.

the power law slope of the innermost region of the Milky Way is about 1.2 The histogram shows a roughly bi-modal distribution, which is probably reflective of the fact that the Lauer sample is composed of both elliptical and lenticular galaxies.

3 Black hole masses

We would like to calculate galactic stellar collision rates as a function black hole mass, $M_\bullet$, so that we can we multiply by the black hole mass function, $\Phi(M_\bullet) \equiv dN/(dM_\bullet d^3 r_{univ})$, as calculated by Hopkins *et al* 2007. This will allow us to integrate over a suitable region of the universe to calculate the total expected stellar collision rate in this region. Fortunately, Lauer *et al* have tabulated bulge velocity dispersions from previous studies for their sample, and we can therefore use the $M_\bullet - \sigma$ relation to obtain an estimate of $M_\bullet$ for each galaxy. One of the most recent determinations of the $M_\bullet - \sigma$ relation was calculated by Schulze & Gebhardt 2011 for a sample of elliptical galaxies:

$$log\left(\frac{M_\bullet}{M_\odot}\right) = 8.18 \pm 0.06 + 4.32 \pm 0.31 log\left(\frac{\sigma_\star}{200\text{km s}^{-1}}\right). \quad (6)$$

I use this relation, and the measured velocity dispersions for each galaxy provided by Lauer *et al* 2005 to calculate the black hole mass for my 57 galaxies. I show the distribution of the black hole masses in Fig. **??**.

4 Notes about particular galaxies

Lauer *et al* 2005 measure a sample of 77 galaxies, but they only provide surface brightness profiles for 65 of these galaxies. I throw away 8 of these galaxies for the reasons listed below.

- *NGC 6876* has an incredibly flat measured surface density profile. This results in a very odd mass density profile, quite deviant from the other profiles.

- *NGC 1374, NGC 3706, NGC 4073, NGC 4406, NGC 4486B, NGC 4552,* and *NGC 5813* have derived luminosity density profiles that go negative at the lowest values of r (obviously physically impossible). This is due to the fact that the measured surface brightness profiles increase monotonically with r till about R_b, then decrease

Figure 4: Distribution of the BH masses for my 57 galaxies, M_{BH} is estimated via Eqn. **??**.

monotonically. All of the rest of the galaxies simply decrease monotonically with r. The physical picture behind the inverse Abel transform is that centers of the galaxies (low R) should have the greatest surface brightnesses (since we are seeing the projected contribution from end-to-end of the entire galaxy). The edges (high R) should have the lowest surface brightnesses since this represents the projection of a very small portion of the galaxy. In this picture, the measured surface brightness profiles should therefore decrease monotonically with R. The only way to reproduce the decrease in $I(R)$ (going from big R to low R) is to have $j(r)$ become negative at small r to deplete some of the projected light. Obviously, this simplified picture is missing something. Perhaps there is a lot of extinction in the centers of these galaxies due to dust?