Collisional Effect of Excited Dark Matter on Halo Profiles

Below are some follow-up thoughts about the effect of cold dark matter (DM) with internal excitation energy that is released in DM-DM collisions. For the characteristic densities and velocities in galaxy cores, we need the collision cross-section to be $\sim 10^{-25}$-10^{-24} cm^2 in order to get one collision in a Hubble time. Interestingly, this is comparable to the geometric cross section of ordinary nucleons. Dwarf galaxies have an escape speed of $\sim 10^{-4}c$ and will lose their dark matter if the excitation energy is $\gtrsim 10^{-8}$ of the rest mass of the DM particle. Let us denote the excess velocity acquired by a DM particle after a collision by v_0.

A halo of mass M collapsing at redshift $z \gg 1$ has a virial radius

$$r_{\rm vir} = 1.5 \left(\frac{M}{10^8 M_\odot}\right)^{1/3} \left(\frac{1+z}{10}\right)^{-1} \ {\rm kpc}\ , \tag{1}$$

and a corresponding circular velocity,

$$V_{\rm vir} = \left(\frac{GM}{r_{\rm vir}}\right)^{1/2} = 17.0 \left(\frac{M}{10^8 M_\odot}\right)^{1/3} \left(\frac{1+z}{10}\right)^{1/2} \ {\rm km\ s^{-1}}\ . \tag{2}$$

We may also define a virial temperature

$$T_{\rm vir} = \frac{\mu m_p V_c^2}{2k} = 1.04 \times 10^4 \ \left(\frac{\mu}{0.6}\right) \left(\frac{M}{10^8 M_\odot}\right)^{2/3} \left(\frac{1+z}{10}\right) \ {\rm K}\ , \tag{3}$$

where μ is the mean molecular weight and m_p is the proton mass. Note that the value of μ depends on the ionization fraction of the gas; for a fully ionized primordial gas $\mu = 0.59$, while a gas with ionized hydrogen but only singly-ionized helium has $\mu = 0.61$. The binding energy of the halo is approximately,

$$E_b = \frac{1}{2}\frac{GM^2}{r_{\rm vir}} = 2.9 \times 10^{53} \left(\frac{M}{10^8 M_\odot}\right)^{5/3} \left(\frac{1+z}{10}\right) \ {\rm erg}\ . \tag{4}$$

Numerical simulations of hierarchical halo formation indicate a roughly universal spherically-averaged density profile for the resulting halos, though with considerable scatter among different halos. This profile has the NFW form [after Navarro, J. F., Frenk, C. S. & White, S. D. M. *Astrophys. J.* **490**, 493 (1997)],

$$\rho(r) = \frac{3H_0^2}{8\pi G}(1+z)^3 \Omega_m \frac{\delta_c}{c_{\rm N} x (1 + c_{\rm N} x)^2}\ , \tag{5}$$

where $x = r/r_{\rm vir}$, and the characteristic density δ_c is related to the concentration parameter $c_{\rm N}$ by

$$\delta_c = \frac{\Delta_c}{3} \frac{c_{\rm N}^3}{\ln(1 + c_{\rm N}) - c_{\rm N}/(1 + c_{\rm N})}\ , \tag{6}$$

with $\Delta_c = 18\pi^2 \simeq 178$. The concentration parameter itself depends on the halo mass M at a given redshift z, with a value of order ~ 4 for newly collapsed halos.

As a result of collisions, DM particles will oscillate with a velocity v_0 around the center of a galaxy and their distribution will acquire a core out to a radius where the gravitation potential equals twice their kinetic energy (from the virial theorem). We can find the resulting core radius by comparing the circular velocity profile of the NFW profile to the excess velocity acquired by particle collisions,

$$v_{\rm vir}^2 \left\{ \frac{1}{x} \frac{[\ln(1+cx) - cx/(1+cx)]}{[\ln(1+c) - c/(1+c)]} \right\} = \frac{2}{3} v_0^2, \tag{7}$$

where $x = r/r_{\rm vir}$ is the radius as a fraction of the virial radius of the halo, and the factor of $\frac{2}{3}$ on the right-hand-side is because v_0 is the velocity dispersion in 3D, and the left-hand-side is the circular velocity $v_{\rm c} = GM/r$ in 2D. If $v_{\rm vir}^2 < \frac{2}{3} v_0^2$ then the halo could lose most of its dark matter through DM collisions. The value of v_0 therefore sets a minimum halo mass, $M_{\rm min}$. By tuning v_0 we can resolve the dwarf satellite problem of LCDM. This possible solution was not discussed in the literature.

Massive halos with $M \gg M_{\rm min}$ retain most of their dark matter and only develop a core in their DM profile based on equation (7). In the regime where the core radius is smaller than the break radius $r < r_{\rm vir}/c$ (or equivalently $x < 1$), we can derive a simple expression for the surface mass density of the core.

The NFW density profile simplifies for $x \ll 1$ to the scaling,

$$\rho \propto \frac{\rho_{\rm vir} r_{\rm vir}}{r} \tag{8}$$

implying a **spatially uniform** inner profile for the DM surface density,

$$\Sigma \sim \rho r \sim 10 \left(\frac{c}{10}\right)^2 \frac{M}{r_{\rm vir}^2} \sim 0.1 \frac{\rm g}{\rm cm^2} \left(\frac{M}{10^8 M_\odot}\right)^{1/3} \left[\frac{(1+z)}{10}\right]^2. \tag{9}$$

Interestingly, in LCDM halos with $M \sim 10^8 M_\odot$ form at $z \sim 10$ whereas halos with $M \sim 10^{12} M_\odot$ form at $z \sim 2$. This implies that the central DM surface density is nearly the same for all galaxy halos and when multiplied by G obtains a value close but somewhat lower than the threshold acceleration in MOND: $G\Sigma \sim 10^{-8}$ cm s^{-2}. At the radius where the baryons contribute a comparable mass within the core of the DM halo, the acceleration will roughly double (inducing adiabatic contraction of the DM halo) and potentially agree with the favored value of $a_0 \sim 2 \times 10^{-8}$ cm s^{-2}. This would be the acceleration of the DM core irrespective of the core radius, as long as that radius is smaller than the break radius $r_{\rm vir}/c$. The latter condition is naturally satisfied in halos with $M \gg M_{\rm min}$. We can calculate the characteristic halo mass as a function of redshift in LCDM and plot the surface density of the core for different choices of v_0.