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Collisional Effect of Excited Dark Matter on Halo Profiles

Below are some follow-up thoughts about the effect of cold dark matter (DM) with internal excitation energy that is released in DM-DM collisions. For the characteristic densities and velocities in galaxy cores, we need the collision cross-section to be $\sim 10^{-25}$-10^{-24} cm^2 in order to get one collision in a Hubble time. Interestingly, this is comparable to the geometric cross section of ordinary nucleons. Dwarf galaxies have an escape speed of $\sim 10^{-4}c$ and will lose their dark matter if the excitation energy is $\gtrsim 10^{-8}$ of the rest mass of the DM particle. Let us denote the excess velocity acquired by a DM particle after a collision by v_0.

A halo of mass M collapsing at redshift $z \gg 1$ thus has a virial radius

$$r_{\rm vir} = 1.5 \left(\frac{M}{10^8 M_\odot}\right)^{1/3} \left(\frac{1+z}{10}\right)^{-1} \ {\rm kpc}\ , \tag{1}$$

and a corresponding circular velocity,

$$V_{\rm vir} = \left(\frac{GM}{r_{\rm vir}}\right)^{1/2} = 17.0 \left(\frac{M}{10^8 M_\odot}\right)^{1/3} \left(\frac{1+z}{10}\right)^{1/2} \ {\rm km\ s^{-1}}\ . \tag{2}$$

We may also define a virial temperature

$$T_{\rm vir} = \frac{\mu m_p V_c^2}{2k} = 1.04 \times 10^4 \ \left(\frac{\mu}{0.6}\right) \left(\frac{M}{10^8 M_\odot}\right)^{2/3} \left(\frac{1+z}{10}\right) \ {\rm K}\ , \tag{3}$$

where μ is the mean molecular weight and m_p is the proton mass. Note that the value of μ depends on the ionization fraction of the gas; for a fully ionized primordial gas $\mu = 0.59$, while a gas with ionized hydrogen but only singly-ionized helium has $\mu = 0.61$. The binding energy of the halo is approximately,

$$E_b = \frac{1}{2}\frac{GM^2}{r_{\rm vir}} = 2.9 \times 10^{53} \left(\frac{M}{10^8 M_\odot}\right)^{5/3} \left(\frac{1+z}{10}\right) \ {\rm erg}\ . \tag{4}$$

As a result of collisions, DM particles will oscillate with a velocity v_0 around the center of a galaxy and their distribution will acquire a core out to a radius where the potential energy equals twice their kinetic energy (from the virial theorem). Assuming an initial NFW profile, we can find the resulting core radius from the relation,

$$v_{\rm vir}^2 \left\{ \frac{1}{x} \frac{[\ln(1+cx) - cx/(1+cx)]}{[\ln(1+c) - c/(1+c)]} \right\} = \frac{2}{3} v_0^2 \tag{5}$$

where $x = r/r_{\rm vir}$ is the radius as a fraction of the virial radius of the halo, c is the concentration parameter (typically, $c \sim 4$ for a newly formed halo), and the factor of $\frac{2}{3}$ on the right-hand-side is because v_0 is the velocity dispersion in 3D, and the left-hand-side is the circular velocity $v_c = GM/r$ in 2D. If $v_{\rm vir}^2 < \frac{2}{3}v_0^2$ then the halo could lose most of its dark matter. The value of v_0 therefore sets a minimum halo mass, $M_{\rm min}$. By tuning v_0 we can resolve the dwarf satellite problem of LCDM.

Massive halos with $M \gg M_{\rm min}$ retain most of their dark matter and only develop a core in their DM profile based on equation (5). In the regime where the core radius is smaller than the break radius $r < r_{\rm vir}/c$ (or equivalently $x < 1$), we can derive a simple expression for the surface mass density of the core.

Numerical simulations of hierarchical halo formation indicate a roughly universal spherically-averaged density profile for the resulting halos, though with considerable scatter among different halos. This profile has the NFW form [after Navarro, J. F., Frenk, C. S. & White, S. D. M. *Astrophys. J.* **490**, 493 (1997)].

$$\rho(r) = \frac{3H_0^2}{8\pi G}(1+z)^3\Omega_m \frac{\delta_c}{c_{\rm N}x(1+c_{\rm N}x)^2} \ , \tag{6}$$

where $x = r/r_{\rm vir}$, and the characteristic density δ_c is related to the concentration parameter $c_{\rm N}$ by

$$\delta_c = \frac{\Delta_c}{3}\frac{c_{\rm N}^3}{\ln(1+c_{\rm N}) - c_{\rm N}/(1+c_{\rm N})} \ , \tag{7}$$

where $\Delta_c = 18\pi^2 \simeq 178$. The concentration parameter itself depends on the halo mass M, at a given redshift z, with a value of order ~ 4 for newly collapsed halos.

The NFW profile,

$$\rho \propto \frac{1}{x(1+cx)^2}, \tag{8}$$

simplifies for $x \ll 1$ to the scaling,

$$\rho \propto \frac{\rho_{\rm vir} r_{\rm vir}}{r} \tag{9}$$

implying a spatially **constant surface density**,

$$\Sigma \sim \rho r \sim 10\left(\frac{c}{10}\right)^2 \frac{M}{r_{\rm vir}^2} \sim 0.1\frac{\rm g}{\rm cm^2}\left(\frac{M}{10^8 M_\odot}\right)^{1/3}\left[\frac{(1+z)}{10}\right]^2 . \tag{10}$$

Interestingly in LCDM, halos with $M \sim 10^8 M_\odot$ form at $z \sim 10$ whereas halos with $M \sim 10^{12} M_\odot$ form at $z \sim 2$. This implies that the central DM surface density is nearly constant and when multiplied by G obtains a value close to the threshold acceleration in MOND $G\Sigma \sim 10^{-8}$ cm s^2. If the baryons contribute a comparable mass in the core of the DM halo, then the acceleration will roughly double and agree with the favored value of $a_0 \sim 2 \times 10^{-8}$ cm s^{-2}. This would be the acceleration of the DM core irrespective of the core radius, as long as that radius is smaller than the break radius $r_{\rm vir}/c$, a condition which is naturally satisfied for halos with $M \gg M_{\rm min}$. We can calculate the characteristic halo mass as a function of redshift in LCDM and plot the core surface density.