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OBSERVATIONAL SELECTION EFFECTS AND THE M–σ RELATION

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ABSTRACT

We examine the possibility that the observed relation between black-hole mass and host-galaxy stellar velocity dispersion (the M–σ relation) is biased by an observational selection effect, the difficulty of detecting a black hole whose sphere of influence is smaller than the telescope resolution. In particular, we critically investigate recent claims that the M–σ relation only represents the upper limit to a broad distribution of black-hole masses in galaxies of a given velocity dispersion. We find that this hypothesis can be rejected at a high confidence level. We also describe a general procedure for incorporating observational selection effects in estimates of the properties of the M–σ relation. Applying this procedure we find results that are consistent with earlier estimates that did not account for selection effects, although with larger error bars. In particular, (*i*) the width of the M–σ relation is not significantly increased; (*ii*) the slope and normalization of the M–σ relation are not significantly changed; (*iii*) most or all luminous early-type galaxies contain central black holes.

Subject headings: black-hole physics — galaxies: general — galaxies: nuclei — galaxies: bulges — methods: statistical

1. INTRODUCTION

The mass of a central black hole is correlated with the properties of its host galaxy, both stellar luminosity or mass (Dressler 1989; Kormendy 1993; Magorrian et al. 1998; Häring & Rix 2004) and velocity dispersion (the M–σ relation; Gebhardt et al. 2000a; Ferrarese & Merritt 2000; Gültekin et al. 2009, hereafter G09). These correlations provide deep, but poorly understood, insights into galaxy and black-hole formation (see §4 for a brief review). The radius of the sphere of influence of a central black hole of mass M in a galaxy with velocity dispersion σ is $R_{\rm infl} \equiv GM\sigma^{-2}$, so at a distance D the angular size of the sphere of influence is $\theta_{\rm infl} = R_{\rm infl}/D$. An important determinant of the reliability of stellar dynamical detections of central black holes is the ratio of the radius of the sphere of influence to the telescope resolution. Thus we desire $\theta_{\rm infl} \gtrsim \theta_{\rm res}$ where $\theta_{\rm res}$ is some measure of the angular resolution, for example the slit width or the full width at half maximum (FWHM) of the telescope point-spread function. With few exceptions, the black holes in nearby galaxies have $\theta_{\rm infl} \lesssim 1''$ (see Table 1)—this is why most detections so far have been made with the *Hubble Space Telescope* (*HST*), which offers higher spatial resolution (FWHM$\simeq 0''\!.1$) and a more stable point-spread function than ground-based telescopes.

At a given signal-to-noise ratio, as the ratio $\theta_{\rm infl}/\theta_{\rm res}$ decreases the measurement errors in black-hole mass increase until only an upper limit can be inferred. Since $\theta_{\rm infl}$ is usually not much bigger than $\theta_{\rm res}$, an obvious concern is that resolution-dependent selection effects may bias the observed correlations between black-hole mass and galaxy properties. We focus here on possible bias in the M–σ relation, although similar considerations apply to the relations between black-hole mass and host galaxy luminosity or mass. A number of possible biases have been discussed in the literature. Several authors have argued that black-hole masses are systematically overestimated when the sphere of influence is not well-resolved (see G09 for the history of this controversy). This seems unlikely for the following reasons: (*i*) In any well-designed experimental analysis, the model parameters derived from poor data may have large error bars but should not be systematically biased. (*ii*) Gebhardt et al. (2003a) analyzed 12 galaxies twice, once using both ground-based (low-resolution) and *HST* (high-resolution) spectroscopy and once using only the ground-based spectroscopy. They found that the black-hole masses determined from these two data sets were consistent at the 1-σ level, with no evidence that the masses determined from ground-based data alone were systematically high. (*iii*) Kormendy (2004) has pointed out that the black-hole mass in M32 (NGC 0221) has remained remarkably stable—within a factor of two—over the past two decades while the spatial resolution of the spectroscopy has increased by a factor of 30, and that the mass estimates for several black holes first discovered from the ground (e.g., NGC 3115, NGC 3377, NGC 4594) did not systematically change when they were later observed with much higher resolution by *HST*. In fact, G09 pointed out a different and more important bias that is the opposite of this one: *excluding* black-hole masses from galaxies with $\theta_{\rm infl}/\theta_{\rm res} \lesssim 1$ from a mass sample systematically biases the M–σ relation derived from that sample.

Yet another bias occurs when non-detections of black holes (i.e., measured upper limits to the black-hole mass) are not included in the analysis: if the upper limits are not far from the ridgeline of the M–σ relation, then analyses that exclude them will be biased towards high mass at a given dispersion. On the other hand if upper limits are included, the analysis must account for the possibility that some galaxies do not contain black holes at all, or else a single galaxy without a black hole could drastically alter the best-fit parameters of the M–σ relation. Most studies have considered only measured black-hole masses and have ignored the numerous upper limits available in the literature; for exceptions see Valluri et al. (2005) and G09.

An extreme possibility (Ho 1999; Batcheldor 2010) is that the M–σ relation only describes an upper limit to the black-hole mass in a host galaxy with given dispersion; that is, the black hole may have any mass at or below the mass given by the M–σ relation. In this view, the apparent narrow width of the relation is an observational selection effect that arises because black holes with much smaller masses, though common, generally cannot be detected. This hypothesis appears to be inconsistent with the experience of several of the authors, who have conducted many searches for black holes using *HST* and have found that most of these are successful.

This paper has two related goals: (*i*) to determine quantitatively whether the M–σ relation is a *ridgeline* (i.e., most black-hole masses lie close to the relation) or an *envelope* (i.e., most black-hole masses lie well below the relation); (*ii*) to investigate whether the parameters of the M–σ relation are biased by the inability to detect black holes when the angular size of the sphere of influence is too small. A by-product of the analysis will be an estimate of the fraction of luminous galaxies that contain black holes. In §2 we review the arguments by Batcheldor (2010) in favor of the envelope model. In §3 we test the predictions of the envelope model and test for bias due to selection effects in the M–σ relation. We find that we can reject envelope models at very high significance. §4 contains a discussion and conclusions.

2. REVIEW OF BATCHELDOR (2010) ARGUMENT

We have argued that detecting black holes is difficult if $\theta_{\rm infl} \lesssim \theta_{\rm res}$. This criterion is oversimplified, since the ability to measure black-hole mass depends on a number of factors in addition to angular resolution; some of these factors depend on the quality of the observations (e.g., the signal-to-noise ratio) while others depend on the properties of the galaxy (e.g., detecting black holes in galaxies with large cores is more difficult than in power-law galaxies)—see G09 for a more detailed discussion. However, it is instructive to make the simplifying assumption that a black hole can be detected if and only if $\theta_{\rm infl}$ exceeds a fixed fraction of $\theta_{\rm res}$. In this case, if galaxies are uniformly distributed in space and the number density of galaxies with given black-hole mass M and dispersion σ is $n(M,\sigma)$, the number of detected black holes with these parameters will be proportional to $M^3 n(M,\sigma)$. The additional factor of M^3 can create a bias such that galaxies of a given dispersion with low-mass black holes are strongly under-represented in the samples used to determine the M–σ relation.

Batcheldor (2010) quantifies this argument using a sample of ~ 2500 galaxies with distance $D < 100$ Mpc and measured velocity dispersion, taken from the HyperLeda catalog[1] (Paturel et al. 2003). He assigns a black hole to each galaxy, with mass chosen uniformly random in $\log M$ between a lower limit $M = 10 M_\odot$ and an upper limit given by the M–σ relation, and it assumes that the black holes can be detected if and only if $\theta_{\rm infl} > \theta_{\rm res} = 0\farcs1$. The resulting simulated dataset of black-hole masses yields an apparent M–σ relation with scatter comparable to the observed relation; Batcheldor thus argues that the envelope model is consistent with the data.

This argument implies that *HST* observations should typically yield only an upper limit to black-hole mass, not an actual detection. If $\log M$ is distributed uniformly between 10 and $10^9 M_\odot$ at $\sigma \simeq 325$ km s^{-1}, as suggested by Batcheldor, then for each of the four galaxies with measured black-hole masses between $\sim 5\times10^8$ and $5\times10^9 M_\odot$ (IC 1459, NGC 1399, NGC 4486, and NGC 4649, measured by Cappellari et al. 2002; Gebhardt et al. 2007; Gebhardt & Thomas 2009; Gebhardt et al. 2003b, respectively), there should be more than 7 with smaller black holes, for which observations would probably yield only upper limits. Instead of 28, there are only 3 upper limits in that range in the literature (NGC 315, NGC 6861, and NGC 1841; Beifiori et al. 2009). Thus published measures of black-hole masses argue against the envelope model, but the published data may not tell the whole story. Some upper limits derived from *HST* observations may not be in the literature, and the observers may have had other clues leading to an enhanced success rate (e.g., preparatory ground-based observations). Nevertheless, there is little or no positive evidence that supports the notion that very small black holes are present in galaxies with such high velocity dispersion.

3. TESTS OF THE ENVELOPE MODEL

As we have discussed, a critical test of the envelope model is whether it correctly predicts the success rate of detecting central black holes. The challenge in applying this test is that we cannot model the behavior of observers and time allocation committees, who determine which galaxies are to be observed. However, the most promising sites to prospect for black holes are the centers of those galaxies with the largest values of $\theta_{\rm infl}^{\rm pred} \equiv GM^{\rm pred}/(\sigma^2 D)$, the angular size of the sphere of influence determined using the black-hole mass $M^{\rm pred}$ predicted by the M–σ relation. Thus an objectively defined sample that provides the sharpest tests of the envelope model is the set of galaxies with the largest values of $\theta_{\rm infl}^{\rm pred}$.

We have queried the HyperLeda catalog for all galaxies with measured distance and central velocity dispersion. HyperLeda is not complete in any sense, but this method mimics the approach used by observers to identify target galaxies for black-hole searches. For each galaxy we predicted the black-hole mass using the M–σ relation in the form

$$M^{\rm pred}(\sigma) = 10^{\alpha}(\sigma/200 \text{ km s}^{-1})^{\beta} M_\odot \qquad (1)$$

with $\alpha = 8.12$ and $\beta = 4.24$ from G09. Using other values of β changes the sample, but our final results are very similar when using any β in the range 3–5 to create the initial sample. We next computed $\theta_{\rm infl}^{\rm pred}$ and sorted the galaxies by this parameter. We then found the best available distances and dispersions[2] for the galaxies near the top of the list, recomputed

[1] See http://leda.univ-lyon1.fr.

[2] Some of these dispersions seem implausible to us, e.g., 500 km s^{-1} for

$\theta_{\rm infl}^{\rm pred}$ and resorted. The galaxies with the top 50 resulting values of $\theta_{\rm infl}^{\rm pred}$ are listed in Table 1.

Our results will be based on a sub-sample of galaxies from this Table with the $N \leq 50$ largest predicted angular spheres of influence, and we must choose N. If N is too small, the statistical uncertainties will be unnecessarily large. If N is too large the power of the test will be diluted by galaxies that have not been examined for black holes. We normally work with $N = 30$, but we have experimented with other values of N and find, as described below, that our results are quite insensitive to N so long as $N \gtrsim 20$. Of the top 30 galaxies in Table 1, 15 have published black-hole mass determinations and 5 have published upper limits.

We present two tests with these data. The first tests for the probability of obtaining these data given the envelope hypothesis as presented by Batcheldor (2010). The second examines more generally how limited resolution affects our inferences about the M–σ relation and its properties.

3.1. *Test A*

We make the simplifying assumption that a black hole can be detected if and only if its angular sphere of influence exceeds $\theta_{\rm min} = 0\farcs01$, a factor of two smaller than the smallest angular sphere of influence for a published black-hole mass (NGC 2778; Gebhardt et al. 2003b). This assumption is conservative, in that a larger value of $\theta_{\rm min}$ would yield results that are even harder to reconcile with the envelope model (see Figure 1b). For a given galaxy, with known distance and velocity dispersion, there is then a minimum black-hole mass that can be detected, $M_{\rm limit}$. In the envelope model as presented by Batcheldor (2010), logarithmic black-hole masses $\mu = \log_{10}(M/M_\odot)$ are distributed uniformly between some upper and lower limits $\mu_{\rm max}$ and $\mu_{\rm min}$, so the probability of detecting a black hole in a given galaxy is

$$P_{\rm detect} = \frac{\mu_{\rm max} - \mu_{\rm limit}}{\mu_{\rm max} - \mu_{\rm min}}. \qquad (2)$$

Following Batcheldor (2010) we take $\mu_{\rm min} = 1$ (minimum black-hole mass of $10 M_\odot$) and $\mu_{\rm max}$ given by equation (1) with $\alpha = 8.7$ and $\beta = 5.0$. (The assumed values of α and β do not strongly influence our results, and our conclusions below are made stronger if values that more closely match the best-fit ridgeline relation are used.) We use this model to calculate the probability of making as many black-hole detections as found in the top 30 galaxies of Table 1. Of these, 15 have black-hole mass detections; the rest have either an upper limit or no published results from a black-hole search. We make the conservative assumption that these galaxies have no published detection because their black holes are too small ($\theta_{\rm infl} < \theta_{\rm min}$). This is unlikely to be the case; for example, some of these are unpromising galaxies to examine for black holes on account of their peculiar or irregular classifications.

To quantify the probability of finding 15 black-hole masses (p), we ran 10^6 Monte Carlo realizations of simulated observations of the top 30 galaxies in Table 1 using equation (2) to calculate detection probabilities of each galaxy. Only 7166 realizations resulted in 15 or more simulated black-hole mass detections ($p = 7.2 \times 10^{-3}$), allowing us to reject the null hypothesis (Batcheldor's version of the envelope model) at the 99.3% confidence level.

NGC 4055, and we are engaged in a program to remeasure high-dispersion galaxies including some from this list.

Table 1
Top 50 galaxies by predicted angular sphere of influence

Galaxy	Type	σ	D (Mpc)	$M^{\rm pred}$ ($10^8 M_\odot$)	$\theta_{\rm infl}^{\rm pred}$ ($''$)	M ($10^8 M_\odot$)	Ref.
N0224	Sb	160	0.8	0.52	2.27	1.5	1
N4649	E2	385	16.5	22	0.79	21	2
N6861	S0	414	28.1	29	0.54	<15	3
N4486	E1	324	17.0	10	0.52	62.7	4
N3998	S0	305	14.9	8.1	0.52	2.4	5
N1399	E1	337	21.1	12	0.46	5.1	6
N4751	S0	349	23.5	14	0.44	…	…
N4594	Sa	240	10.3	2.9	0.44	5.7	7
N4472	E2	294	17.0	6.9	0.42	…	…
N4374	E1	296	17.0	7.1	0.42	15	8
N3115	S0	230	10.2	2.4	0.40	9.6	9
N0221	E2	75	0.9	0.02	0.39	0.03	10
N1332	S0	321	22.9	10	0.38	14.5	11
N4143	SB0	271	16.0	4.9	0.37	<1.4	12
N5128	S0/E	150	4.4	0.4	0.36	3.0	13
N1161	S0	336	27.5	12	0.35	…	…
N3031	Sb	143	4.1	0.33	0.34	0.8	14
N4945	Sc	134	3.7	0.25	0.33	…	…
N4552	E1	254	15.4	3.7	0.33	<19	3
N4526	SAB0	264	16.9	4.4	0.33	<3.2	3
N2787	SB0	189	7.9	1.1	0.33	0.43	15
N2293	SAB0pec	261	17.1	4.2	0.32	…	…
IC1459	E4	340	30.9	13	0.32	28	16
E137−044	SAB0	489	69.3	60	0.32	…	…
N3034	Irr	130	4.0	0.22	0.29	…	…
E138−005	SB0pec	349	36.1	14	0.29	…	…
N4055	E?	500	87.5	66	0.27	…	…
N3379	E0	206	11.7	1.5	0.27	1.2	17
N4365	E3	256	20.4	3.8	0.26	…	…
N4278	E1	237	16.7	2.8	0.26	<1.8	3
N1023	SB0	205	12.1	1.5	0.26	0.5	18
N5087	S0?	283	26.2	5.9	0.25	…	…
N4406	E3	235	17.0	2.7	0.25	…	…
N4261	E2	315	33.4	9.3	0.25	5.5	19
N2663	E	291	27.5	6.6	0.25	…	…
N4621	E5	225	17.0	2.2	0.23	…	…
N3923	E4–5	257	22.9	3.9	0.23	…	…
N5062	S0pec	389	60.0	23	0.22	…	…
N4342	S0	225	18.0	2.2	0.22	3.6	20
N4105	E3	262	26.6	4.2	0.21	…	…
N1407	E0	272	28.8	4.9	0.21	…	…
N1270	E?	427	76.7	34	0.21	…	…
N0253	SABc	103	3.2	0.08	0.21	…	…
N5838	S0	266	28.5	4.5	0.20	…	…
N6587	SAB0?	333	48.9	12	0.19	…	…
N1395	E2	245	24.6	3.2	0.19	<0.14	3
IC2586	E4	346	53.1	14	0.19	…	…
N4291	E2	242	25.0	3	0.18	3.2	2
N2841	Sb	206	17.8	1.5	0.18	…	…
N4442	SB0	187	15.3	1	0.17	…	…

References. — (1) Bender et al. (2005), (2) Gebhardt et al. (2003b), (3) Beifiori et al. (2009), (4) Gebhardt & Thomas (2009) and Gebhardt et al. (2011), (5) de Francesco et al. (2006), (6) Gebhardt et al. (2007), (7) Kormendy (1988), (8) Bower et al. (1998), (9) Emsellem et al. (1999), (10) Verolme et al. (2002), (11) Rusli et al. (2010), (12) Sarzi et al. (2002), (13) Silge et al. (2005), (14) Devereux et al. (2003), (15) Sarzi et al. (2001), (16) Cappellari et al. (2002), (17) Gebhardt et al. (2000b), (18) Bower et al. (2001), (19) Ferrarese et al. (1996), (20) Cretton & van den Bosch (1999).

Note. — The 50 galaxies with the largest angular sphere of influence, as predicted by the M–σ relation. Hubble types are mostly from NED. The velocity dispersions come from G09 or HyperLeda, and the distances are our best estimates from G09, Tonry et al. (2001), NED redshift-independent distances, or HyperLeda. Predicted black-hole masses are from equation (1), and $\theta_{\rm infl}^{\rm pred} = GM^{\rm pred}\sigma^{-2}D^{-1}$. We also list the best black-hole mass measurements for these galaxies. The final column gives a reference code for the mass measurement or upper limit.

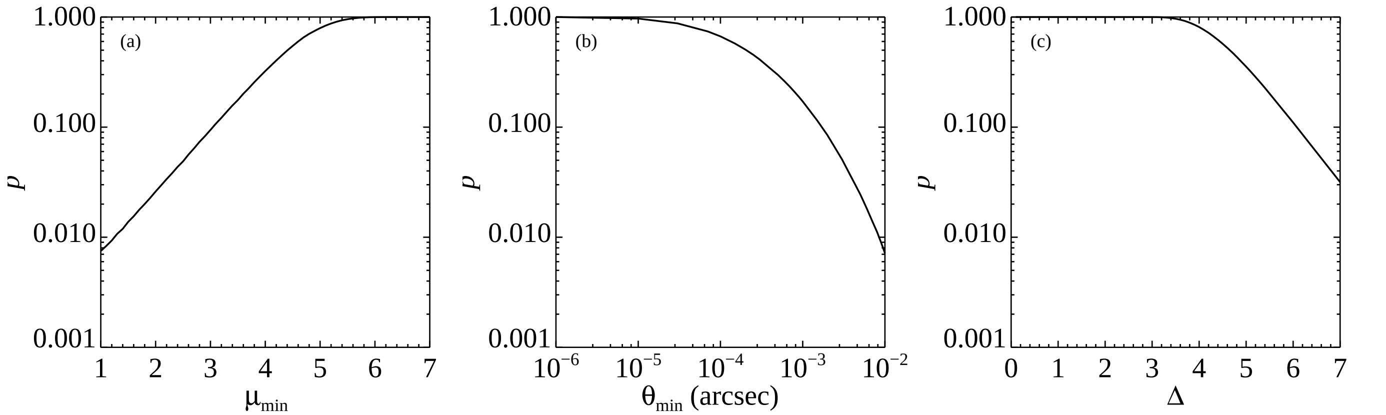

Figure 1. Plot of the probability p of detecting 15 or more black holes in the top 30 galaxies as a function of the following envelope model parameters: (*a*) $\mu_{\rm min} = \log_{10}(M_{\rm min}/M_\odot)$ where $M_{\rm min}$ is the minimum black-hole mass in the envelope model. Values of $\mu_{\rm min} < 3$ can be rejected at the 90% confidence level. (*b*) The minimum detectable angular sphere of influence $\theta_{\rm min}$. The p-values are very low for all plausible values of $\theta_{\rm min}$, showing that our test is insensitive to the exact value assumed. (*c*) Δ, the range of log mass in the envelope model.

The above calculation assumes that the minimum logarithmic black-hole mass is $\mu_{\rm min} = 1$, which was chosen for consistency with Batcheldor (2010). We plot p as a function of $\mu_{\rm min}$ in Figure 1a. Even for $\mu_{\rm min} = 3$ (minimum mass of $1000\,M_\odot$) we can reject the envelope model at about the 90% confidence level.

We plot p as a function of the minimum detectable angular sphere of influence $\theta_{\rm min}$ in Figure 1b. Only at $\theta_{\rm min} < 0\farcs0015$ is $p > 0.1$; thus for all plausible values of $\theta_{\rm min}$ the envelope hypothesis can be rejected at high confidence.

Our results are insensitive to the number N of galaxies in our sample ($N = 30$). For $N = 20$, $N = 30$, and $N = 50$ and our standard parameters ($\mu_{\rm min} = 1$ and $\theta_{\rm min} = 0\farcs01$) the envelope model is ruled out at the 99.7%, 99.3%, and 96.5% confidence levels respectively.

We stress again the conservative nature of the assumption that all galaxies without published black hole mass measurements have black holes with masses too small to be measured. If, at the other extreme, we only considered the 20 galaxies with published mass estimates or upper limits, then we would rule out the Batcheldor envelope model at the 99.998% confidence level or at the 90% level for $\mu_{\rm min} = 5.1$.

An alternative to the assumption that $\mu_{\rm min}$ is constant is to assume constant width of the M–σ relation so that $\mu_{\rm min} = \mu_{\rm max} - \Delta$ with a constant value for Δ. In Figure 1c we plot p as a function of Δ. The probability of finding 15 black holes in our sample is $p < 0.1$ for $\Delta > 6$. This result is sensitive to our assumed value for the normalization parameter α of the M–σ relation, taken to be $\alpha = 8.7$ following Batcheldor. For example, if instead we assume $\alpha = 8.12$ from G09, we find the much more stringent constraint $p < 0.1$ for $\Delta > 2.8$. In order to better constrain this alternative model as well as to determine the extent that selection effects alter our inferences, we present a more sophisticated test.

3.2. *Test B*

We construct a parametrized model for the distribution of black-hole masses and the observational constraints on their detection. The model has seven free parameters $X \equiv \{\alpha, \beta, b, s, \Delta, x_r, s_r\}$, and is based on the following assumptions:

(*i*) The probability that a given galaxy has a central black hole is b_1. The parameter b_1 is assumed to be independent of galaxy properties. This is almost certainly an oversimplification but the galaxies in Table 1 are mostly luminous ellipticals, lenticulars, and early-type spirals so are likely to have similar properties.

(*ii*) The probability that a galaxy in Table 1 has been examined carefully for evidence of a black hole is some constant b_2. Only the product $b \equiv b_1 b_2$—the combined probability that a galaxy has a black hole *and* has been examined for one—can be determined from the data.

(*iii*) If a galaxy has a black hole, the probability distribution of its logarithmic mass $\mu = \log_{10}(M/M_\odot)$ is determined by the M–σ relation (1) and takes the form $dp = p_1(\mu|\sigma, X)d\mu$ where

$$p_1(\mu|\sigma,X) = g\left[\mu - \alpha - \beta\log_{10}(\sigma/200\ \mathrm{km\ s^{-1}})\right]. \quad (3)$$

The parameters α and β are to be fit from the data, and the function g is assumed to have the form

$$g(x) = k\begin{cases} \exp(-\frac{1}{2}x^2/s^2), & x > 0 \\ 1, & -\Delta \le x \le 0 \\ \exp(-\frac{1}{2}(x+\Delta)^2/s^2), & x < -\Delta. \end{cases} \quad (4)$$

The case $\Delta = 0$ corresponds to the usual assumption of a ridgeline M–σ relation with a Gaussian distribution of the residuals in logarithmic mass. In the case of a large value of Δ, the M–σ relation only defines an upper envelope to the range of black-hole masses. Since g is a probability density, the constant k must be chosen so that $\int g(x)dx = 1$, that is, $k^{-1} = \Delta + \sqrt{2\pi}s$. With this parametrization the variance in log mass is

$$\epsilon^2 = \frac{\Delta^3/12 + \sqrt{\pi/8}\Delta^2 s + 2\Delta s^2 + \sqrt{2\pi}s^3}{\Delta + \sqrt{2\pi}s}. \quad (5)$$

(*iv*) The probability that a black hole will be detected depends only on the ratio of the angular radius of the sphere of influence to the resolution limit of the telescope. Thus the detection probability is

$$p_2(\mu|\sigma, D, \theta_{\rm res}, X) = f\left[\log_{10}(\theta_{\rm infl}/\theta_{\rm res})\right]. \quad (6)$$

All of the detections in Table 1 are based on observations with *HST* or telescopes with inferior resolution (and thus these could easily have been detected at *HST* resolution); moreover all of the upper limits are from *HST*. Thus we can assume *HST* resolution for all of the measurements in this list, $\theta_{\rm res} = 0\farcs1$,

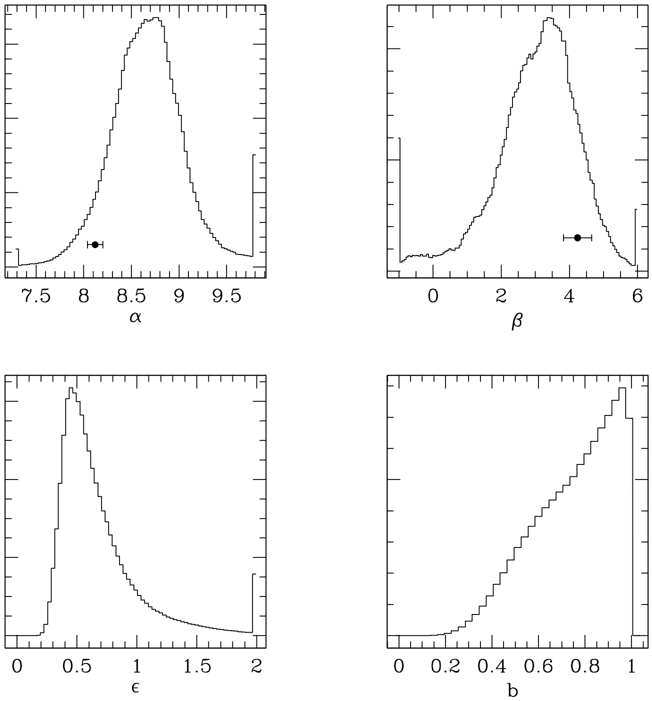

Figure 2. The marginalized probabilities of the parameters of the M–σ relation, as determined by a Markov chain Monte Carlo solution of equation (8) for the data in Table 1. The points with error bars denote the estimates of normalization α and slope β from G09.

and henceforth we suppress this argument. We parametrize the detection probability as

$$f(x) = \begin{cases} 1, & x > x_r \\ \exp[-\frac{1}{2}(x - x_r)^2/s_r^2], & x \le x_r. \end{cases} \tag{7}$$

This equation involves two free parameters: x_r is the value of $\log_{10}(\theta_{\rm infl}/\theta_{\rm res})$ at which detection of the black hole becomes certain, and s_r is a measure of the range of logarithmic black-hole mass over which detection is possible but not certain. We restrict the ranges of these parameters to $-1 < x_r < 1$ and $0 < s_r < 1$. For example, the restriction $x_r = 1$ reflects the conservative assumption that detection of a black hole should be certain if the sphere of influence is more than ten times the resolution of *HST*. Our results are insensitive to the values chosen for the range of x_r and s_r. The particular functional form in equation (7) is chosen so that the integral in equation (8) below is analytic, which greatly speeds up the time-consuming Markov chain Monte Carlo calculations.

Now suppose that we have a sample of N galaxies with dispersions σ_i and distances D_i. In K of these galaxies a black hole has been detected with logarithmic mass μ_i, and in the remaining $N-K$ galaxies no black hole has been detected (we ignore the extra information available from the upper limits to the black-hole mass in a galaxy). Then the posterior probability distribution of the parameter set X is

$$\begin{aligned} p(X) =& c\,\Pi(X) b^K \prod_{i=1}^{K} p_1(\mu_i|\sigma_i, X) p_2(\mu_i|\sigma_i, D_i, X) \\ & \times \prod_{j=1}^{N-K} \left[1 - b \int d\mu\, p_1(\mu|\sigma_j, X) p_2(\mu|\sigma_j, D_j, X) \right], \end{aligned} \tag{8}$$

where $\Pi(X)$ is the prior probability distribution and the constant c is chosen so that $\int p(X) dX = 1$. We assume that the prior distribution is flat in all of the parameters X, with the range restrictions on x_r and s_r mentioned above. We then evaluate the probability distribution (8) using a Markov chain Monte Carlo simulation and the top $N = 30$ galaxies in Table 1. To account for fluctuations in the best-fit parameters because of the limited sample size, we resample the top 30 galaxies (with replacement) 100 times, run the Markov chain Monte Carlo each time, and average the results. The marginalized probability distributions over the parameters α and β (the normalization and slope of the M–σ relation), b (the combined probability that a galaxy has a black hole and has been observed), and ε (the standard deviation in logarithmic mass of the M–σ relation) are shown in Figure 2.

The best-fit values for the parameters of the M–σ relation (1) are $\alpha = 8.7 \pm 0.4$, $\beta = 3.1^{+1.4}_{-1.5}$ (median and 68% or 1-σ confidence interval). These are consistent with the estimates $\alpha = 8.12 \pm 0.08$, $\beta = 4.24 \pm 0.41$ derived by G09 but the error bars are much larger and the medians are only consistent at about the 10% level. Part of this difference arises because of the different samples. The present sample contains only 15 black-hole masses—less than a third of the 49 masses used and has a larger median dispersion (271 km s^{-1} compared to 175 km s^{-1}), both necessary byproducts of choosing a sample based on $\theta_{\rm infl}^{\rm pred}$ (see figure 9b in G09). Fitting the masses and upper limits in the present sample using the methods of G09 yields $\alpha = 8.29 \pm 0.13$, $\beta = 3.61 \pm 0.62$—thus about 30% of the difference in the normalization α and about half of the difference in the slope β can be attributed to changes in the sample. A second reason for the differences is that we are fitting a more general model—the fit in G09 assumes $\Delta = 0$ and ignores observational selection effects. Our results suggest that accounting for observational selection may lower the slope and increase the normalization of the M–σ relation from its standard value, but this is not a secure conclusion since the changes are less than the 1-σ confidence interval.

The standard deviation in log mass from the M–σ distribution is $\varepsilon = 0.6^{+0.4}_{-0.2}$. This is consistent with the estimate by G09 that $\varepsilon = 0.44 \pm 0.06$ for their entire sample and 0.31 ± 0.06 for the ellipticals in their sample (more precisely, these are estimates of the intrinsic scatter after removing measurement error, a correction that we do not apply in this paper). Thus there is no evidence that the width of the M–σ relation derived in prior analyses has been artificially narrowed by observational selection effects.

Batcheldor (2010) considers models in which the distribution of logarithmic black-hole mass is uniform over a range of about 6–9. These are similar to models in which the parameter Δ in equation (4) is between 6 and 9. Models in which $\Delta > 6$ are excluded at about the 99% confidence level.

The probability that a galaxy has a black hole and that it has been examined for one is $b = 0.8^{+0.15}_{-0.2}$. We note that 20 of the top 30 galaxies in Table 1 have measured masses or upper limits, suggesting that the probability that a galaxy in this list has been examined is $b_2 = 20/30 \approx 0.67$. Thus our results are consistent with $b_1 = b/b_2 \simeq 1$, i.e., all galaxies in the list contain central black holes.

A possible concern with this analysis is that our model contains too many variables to be constrained by the data. One symptom of this problem would be strong covariances between the model parameters. We find a significant correlation between α and Δ (correlation coefficient 0.5–0.6), which presumably arises because the mid-point of the ridgeline of the M–σ relation at a given dispersion is determined by the combination $\alpha - \frac{1}{2}\Delta$ (eqs. 3 and 4). There is a strong anticorrelation between α and s (correlation coefficient -0.6) which presumably arises because the upper envelope of the M–σ relation at a given dispersion is determined by $\alpha + xs$ where x is of order unity. All other correlation coefficients are typically $\lesssim 0.3$ in absolute value. Thus there is no strong covariance between most of the fitted variables.

The model parameters remain stable as we vary the number of galaxies in the sample between $N = 20$ and $N = 50$, The normalization parameter α declines by only 3% over this range; the width s increases by about 15–20%; and the standard deviation in log mass ε increases by 25–30%.

Finally, we ask: given the error bars on the parameters of the M–σ relation found here, should we believe the smaller error bars from the analysis of G09? There are good reasons why the error bars in G09 should be smaller: (*i*) the G09 sample contains more than three times as many black-hole masses, including some (NGC 4258 and the Milky Way) with very small error bars (this argument assumes, as did G09, that the M–σ relation is the same for these Sbc spirals as it is for early-type galaxies); (*ii*) the G09 analysis accounts for measurement errors in the mass determinations and for upper limits; (*iii*) the G09 analysis fits only three parameters (slope, normalization, and scatter of the M–σ relation), while the present analysis fits seven. Given that the present analysis finds no evidence for bias due to selection effects in the three parameters that G09 *do* measure, it is plausible—though not proven—that such bias is small enough to be negligible.

4. DISCUSSION AND CONCLUSIONS

The M–σ relation was predicted by simple theoretical models of self-regulated black-hole growth, in which the wind from an accreting black hole ejects the gas from a galaxy and thereby quenches further accretion. For an energy-driven wind the predicted relation is (Silk & Rees 1998)

$$M = \frac{1}{2\pi}\frac{\sigma_T}{G^2 m_p c}\frac{f_{\rm gas}}{f_w}\sigma^5, \tag{9}$$

where σ_T is the Thomson cross section, m_p is the proton mass, G is the gravitational constant, c is the speed of light, $f_{\rm gas}$ is the gas fraction of the galaxy's total mass, and f_w is the mechanical power of a wind coming from accretion onto the black hole, expressed as a fraction of the Eddington luminosity[3]. For a momentum-driven wind (Fabian 1999),

$$M = \frac{1}{2\pi}\frac{\sigma_T}{G^2 m_p}\frac{v_w}{c}\frac{f_{\rm gas}}{f_w}\sigma^4, \tag{10}$$

where v_w is the wind velocity. Again, above this mass, all gas is expelled so that growth by accretion cannot continue unless an additional source of gas is provided, e.g., by a merger. Momentum-driven winds are favored because energy-driven winds appear to be too weak once cooling is accounted for (Silk & Nusser 2010).

These theories do not, however, predict whether the growth of black holes should inevitably continue until these limits are reached, or whether instead the black-hole growth stalls in many galaxies at smaller masses, i.e., they do not predict whether M–σ is a ridgeline or an envelope relation.

[3] There appears to be an error of a factor of $(4\pi)^2$ in this formula as given in Silk & Rees (1998); of course, this is only an approximate result in any case.

The tests described in this paper provide strong evidence that M–σ is a ridgeline relation. In particular, Test A shows that the envelope relation advocated by Batcheldor (2010) is ruled out because it predicts far fewer black-hole detections than are found in the literature. Quantitatively, our standard envelope model (flat distribution in log mass down to $10 M_\odot$, minimum detectable sphere of influence $0\farcs01$, $N = 30$ galaxies) is inconsistent with the data at the 99% level, and the envelope model is ruled out at $> 90\%$ confidence for a wide range of other assumptions. Test B shows that after accounting for observational selection effects the rms scatter in log mass at given dispersion σ is only $\varepsilon = 0.6^{+0.4}_{-0.2}$, consistent with the estimate of G09 and inconsistent with the envelope model.

The analysis in Test B also provides a framework for estimating the bias introduced into the M–σ relation by observational selection effects. Our principal findings are that (*i*) the scatter in the M–σ relation remains small after accounting for observational selection; (*ii*) the normalization and slope of the relation are consistent with those derived in analyses that neglect selection effects, such as G09, but with much larger uncertainties. These uncertainties could be reduced by (*i*) searching carefully for black holes in all of the high-ranked galaxies in Table 1 using *HST* or ground-based adaptive optics; (*ii*) generalizing the analysis to include the black holes in the Milky Way and those in maser galaxies (Greene et al. 2010a)—of course, a danger in the second step is that the M–σ relation may depend on galaxy morphology.

Finally, our analysis suggests that most galaxies in the list in Table 1 do contain a central black hole; in particular, a lower limit to the probability that a black hole is present—assuming *all* of the galaxies in the list have been searched—is $b = 0.8^{+0.15}_{-0.2}$.

The distribution of black-hole masses as a function of host-galaxy properties is relevant to the demographics of active galactic nuclei (AGN), since black holes are believed to be the engines that power AGN. In particular, the famous Sołtan (1982) argument estimates the local mass density in black holes from the density of AGN photons determined from quasar surveys at optical and X-ray wavelengths. The estimate is based on an assumed radiative efficiency ε, the ratio of bolometric radiative energy emitted by an AGN to the rest-mass energy of fuel consumed. The Sołtan density can be compared to the local density of black holes determined from the density of galaxies as a function of velocity dispersion and the M–σ relation. For plausible estimates of the radiative efficiency (typically $\varepsilon = 0.1$–0.3 for thin-disk accretion onto a black hole) these two independent estimates for the black-hole mass density agree within a factor of two or so (Marconi et al. 2004; Yu & Lu 2008). These results assume a ridgeline M–σ relation so the agreement suggests that the ridgeline model is not far from correct. This is not a strong argument because of several uncertain factors in the Sołtan argument such as the radiative efficiency, the bolometric corrections, and the population of black holes ejected from galaxy centers by gravitational-wave recoil. It is also possible to account for the agreement by invoking frequent super-Eddington accretion from relatively underweight black holes (King 2010), although this hypothesis requires an active fraction of nuclei much higher than is observed. Nevertheless, it would be a surprising coincidence if a combination of errors accidentally canceled in such a way that the simple estimates we have described for the local black-hole mass density agreed so well.

Our estimate that the fraction of galaxies in our data set containing massive black holes is consistent with unity ($b_1 \approx 1$)

sheds light on the process of gravitational-wave recoil in black-hole mergers. As two black holes inspiral and coalesce, asymmetric emission of gravitational waves imparts a kick to the merged black hole (e.g., Fitchett 1983; Baker et al. 2008; van Meter et al. 2010). If this kick is larger than the escape velocity at the galaxy center, as can happen for high black-hole spins and particular orientations, then the black hole will be ejected. If the merging galaxies are typical, gas-poor ellipticals at late times, there will not be enough cold gas at their centers to fuel the growth of another black hole and reëstablish the M–σ relation. Our results therefore imply that ejection of black holes is rare in galaxies of this kind, a result consistent with theoretical calculations (Schnittman 2007; Volonteri et al. 2008).

We note that because we select galaxies based on $\theta_{\rm infl}^{\rm pred}$, our sample tends to have high velocity dispersions—the median dispersion of the top 30 galaxies in Table 1 is $\sigma = 271$ km s^{-1} compared to a median $\sigma = 175$ km s^{-1} in the sample of G09. Thus the conclusions that we draw may not apply to the population of black holes in the smallest galaxies, which are still poorly understood (e.g., G09; Greene et al. 2010b; Kormendy et al. 2010; Volonteri et al. 2010; van Wassenhove et al. 2010; Kormendy et al. 2011).

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