

DETECTING THE MEMORY EFFECT FROM A MASSIVE BLACK HOLE
MERGER AT THE GALACTIC CENTER THROUGH LUNAR RANGING

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ABSTRACT

A gravitational wave pulse from a major merger of massive black holes at the Galactic center induces a permanent increase in the Earth-Moon separation. For black holes of mass $\sim 10^6 M_{\odot}$, the shift in the local gravitational potential is comparable to the Earth-Moon potential, leading to the Moon being perturbed relative to the Earth during the passage of the pulse. The permanent increase in the Earth-Moon separation is a fraction of a millimeter, measurable by lunar ranging for future merger events.

1. INTRODUCTION

The black hole at the Galactic Center, SgrA*, grows in part through mergers of black holes of mass $M_{\text{BH}} \sim 10^6 M_{\odot}$ (Micic et al. 2011; Greene et al. 2020). Here we calculate the imprint of such mergers on the Earth-Moon separation.

2. RESULTS

A merger between black holes of the above mass results in a gravitational wave pulse of a characteristic duration,

$$(\Delta t)_{\text{GW}} \sim \left(\frac{GM_{\text{BH}}}{c^3} \right) \sim 5 \text{ s} . \quad (1)$$

The mass equivalent of the radiated energy, $(\Delta M)_{\text{GW}}$, changes the near-Earth gravitational potential by an amount,

$$(\Delta \phi)_{\text{GW}} \sim \frac{G(\Delta M)_{\text{GW}}}{d_{\text{GC}}} = 0.6 \times 10^{-11} c^2 \left[\frac{(\Delta M)_{\text{GW}}}{M_{\text{BH}}} \right] , \quad (2)$$

where c is the speed-of-light, $d_{\text{GC}} \approx 8 \text{ kpc}$ is the distance of the Galactic center from the Sun (Reid et al. 2019), and typically $(\Delta M)_{\text{GW}} \lesssim 0.1 M_{\text{BH}}$ (Healy et al. 2014).

Coincidentally, this shift in gravitational potential as a result of the energy carried by the pulse happens to be comparable to the gravitational potential that binds the Moon to Earth,

$$\phi_{\oplus} = \frac{GM_{\oplus}}{d_{\text{Moon}}} = 10^{-11} c^2 , \quad (3)$$

where $M_{\oplus} = 6 \times 10^{27} \text{ g}$ is the mass of the Earth and $d_{\text{Moon}} \approx 4 \times 10^{10} \text{ cm}$ is the Earth-Moon distance.

The gravitational radiation pulse traverses the Earth-Moon system over a light-crossing time $(\Delta t)_{\text{cross}} \sim (d_{\text{Moon}}/c) \sim 1.3 \text{ s}$, during which the gravitational-potential change affects one of the objects before the other. For $(\Delta \phi)_{\text{GW}} \lesssim \phi_{\oplus}$, the temporary weakening of the gravitational binding between the Earth and the Moon during the passage period $(\Delta t)_{\text{cross}}$ leads to an increase in the Earth-Moon separation by an amount,

$$\left(\frac{\Delta d_{\text{Moon}}}{d_{\text{Moon}}} \right) \sim \left(\frac{(\Delta \phi)_{\text{GW}}}{\phi_{\oplus}} \right) \times \frac{1}{2} \left(\frac{v_{\text{Moon}} t_{\text{cross}}}{d_{\text{Moon}}} \right)^2 \sim 0.3 \times 10^{-11} \left[\frac{(\Delta M)_{\text{GW}}}{M_{\text{BH}}} \right] , \quad (4)$$

where $v_{\text{Moon}} \approx 1 \text{ km s}^{-1}$ is the Moon’s orbital speed, and the geometric calculation ignored the small eccentricity in the Moon’s orbit.

The above increase in distance as a result of the motion of the Moon relative to Earth is of the same magnitude as the known “memory effect” (Zel’dovich & Polnarev 1974; Braginskii & Grishchuk 1985; Christodoulou 1991; Bieri et al. 2012), for which the permanent change in separation between free-floating objects of negligible mass initially at rest relative to each other, is also of order, $(\Delta d_{\text{Moon}}/d_{\text{Moon}}) \sim [(\Delta \phi)_{\text{GW}}/c^2]$.

3. IMPLICATIONS

The resulting permanent change $\Delta d_{\text{Moon}} \sim 1 \text{ mm}[(\Delta M)_{\text{GW}}/M_{\text{BH}}]$ is above the ultimate sensitivity threshold of lunar ranging (Murphy et al. 2012), $(\Delta d_{\text{Moon}}/d_{\text{Moon}}) \sim 10^{-14}$, and could be measured for future merger events.

A tight binary of black holes with individual masses $\sim 2 \times 10^6 M_{\odot}$ and a separation a would merge on a timescale of $\sim 40 \text{ yr} (a/10^{14} \text{ cm})^4$ (Peters 1964). The existence of such a binary is not ruled out by the orbits of the S-stars which are observed at much larger distances, $\gtrsim 10^{15} \text{ cm}$ (Gualandris et al. 2010).

The permanent displacement from the memory effect would increase slightly the eccentricity of stellar binaries at wide separations $\gtrsim 10^{16} \text{ cm}$, but this imprint is not detectable at the precision enabled by astronomical surveys such as Gaia (Hwang et al. 2022), even when considering the increase in its amplitude with decreasing Galactocentric distance.

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