SMA Technical Memo # TBA

Subject: ALGORITHMS -
Corrections for Doppler tracking errors
Date: December 30, 2021
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Abstract

We summarize a procedure that is used to correct for the error caused by wrong coordinates of the observing site used in the SMA online Doppler tracking code during the period from 2011 April 4 to 2019 April 3. The algorithm has been implemented in the SMA testing pipeline ***swarm2casa***. We also discussed the detailed algorithms for computing the astronomical parameters used in Doppler tracking model. We recommend that the information for Doppler frequency offset dopoff needs to archive in the raw data sets. We discussed the frequency offset table (FO table) in FITS-style format for wideband data, which is utilized to store the information about observing frequency change during observations. With the model described in this memo, we carried out a detailed analysis on how the coordinate errors cause errors in Doppler velocity. A precise description of the major Doppler tracking errors by smearing and bias as function of local hour angle is provided as well. In addition to the affected SWARM data, the algorithm is also being implemented in other SMA data handling software for the interferometer data produced by the old ASIC correlator.

1 Introduction

The "Doppler effect" is responsible for frequency shifts of the received signal from a source with respect to an observing station or observatory. Astronomers measure Doppler shifts to determine the velocity of an astronomical object in radial motion. However, the measurements may vary as function of time and the locations of observatories, concerning the facts: 1) the tangetial velocity varies depending on the latitude of an observatory location due to the Earth rotation around its axis; and 2) the velocity projected on the line-of-sight between an obsersatory and a celestial object varies with time owing to the orbital motion of the Earth around the barycenter of the solar system. Thus, astronomers choose a conventional reference frame when refer to their measurements in radial velocity of an celestial object. Here a few often used reference frames are introduced. These references frames are also supported by CASA, the Common Astronomy Software Applications package [1]:

- BARY - Solar System Barycenter, referenced to JPL ephemeris DE403 [2].
  Only slightly different and more accurate than heliocentric.
- LSRK - Kinematic LSR using radio definition, conventional local standard
  of rest based on average velocity of stars in the solar neighborhood [3].

- TOPO - Topocentric or a local observatory frame, fixed in observing frequency, no doppler tracking, often used for continuum observing mode.

Note: Jet Propulsion Laboratory Development Ephemeris Model (DE403) was created 1993, released in 1995, covering the time span early 1599 to mid 2199. Please check with CASA developers for updated informationi of the mathematical model.

The SMA online system supports the LSR velocity frame. With a Doppler tracking model, the online system determines the Doppler shifts of the observatory to compensate to the chnages of sky frequency. Thus the radial velocity of a celestial object with respect to the LSR can be determined using the offsets of the sky frequency from the rest frequency or the frequency in the source rest frame.

The rest of the memo is organized as follows: Section 2 describes the Doppler tracking error reported in the SMA Newsletter 2019 July. Section 3 introduces a Doppler tracking model with respect to the LSR frame. Section 4 gives a correction for the specific on-line errors occurred during 2011-2019 and a quantitative analysis of the error sources in details. Section 5 suggests a data patch for offline software in correction of the Doppler errors and a recommendation for online software to store the frequency offset (dopoff) used in Doppler tracking. Appendix A provides the detailed derivation of the formulas used in the Dopper tracking model.

2 Doppler tracking errors

A software error affecting Doppler tracking during science observations from 2011 April 4 to 2019 April 3 was reported in the SMA Newsletter 2019 July issue[4]. The report briefly described the potential effects on the data: "*This error has a negligible effect on observations of continuum and broad spectral lines. However, this error may impact observations of narrow spectral lines by a blurring of up to 0.8 km s*$^{-1}$ *(with a blue-shifted bias of 0.3 km s*$^{-1}$*).*" The error was caused by a mistake in the SMA online Doppler tracking model, that used the coordinates of the Haystack observatory location in Massachusetts instead of the SMA location in Hawaii, during the period between 2011 April 4 to 2019 April 3.

3 Doppler tracking model with respect to the LSR frame

A Doppler tracking model determines the velocity of an observatory owing to various motions of the Earth with respect to the LSR frame.

3.1 Model of motions -

In general, three velocity terms that describe an observatory motion are accounted in a Doppler tracking model.

- **Motion of the Sun** –
  The convention of Local Standard of Rest (LSR) [3] assumes the Sun to move at the rounded velocity of $\mathbf{V}_\odot = 20.0$ km s^{-1} towards RA=18$^{\rm h}$ and Dec=30$^{\rm o}$ at the epoch 1900.0†.

†*SMA adopted the same convention as used at other radio observatories according to a memo posted on SMA Wiki by Taco (K. Young) [7].*

- **Annual motion** –
  This term is a motion caused by the Earth's revolution around the Sun. The Sun apparently revolves 360 degrees a year around a path on the celestial sphere called the Ecliptic. The sun moves eastward with respect to the objects on the celestial sphere. The velocity of the Earth orbiting around the Sun, $\mathbf{V_E}(t, \mathbf{x})$, is a function of Julian day (t) and heliocentric position ($\mathbf{x}$) with respect to the mean equator and equinox. Ignoring the influence of other solar system bodies, Earth's orbit is an ellipse with the Earth-Sun barycenter as one focus and a current *eccentricity* of 0.0167. The velocity slowly changes as the Earth evolving around the Sun with a *mean orbital speed* of the Earth $\mathsf{V}_{\rm E} = 29.789$ km s^{-1}. The three components of $\mathbf{V_E}$ along with the coordinates of a source direction vector $\mathbf{s}$ at transit time can be derived from the Earth orbit equation. Appendix A.1 provides an algorithm to calculate the quantities of the annual terms.

- **Diurnal motion** –
  This term is due to the Earth's rotation from west to east, which causes celestial bodies to have an apparent motion from east to west. The mean angular rate of Earth's rotation, or the Earth's *sidereal angular rate*, is $\boldsymbol{\omega}_\oplus = 7.2921150 \pm 0.0000001 \times 10^{-5}$ rad s^{-1}[5], and the Earth's *equatorial radius* is $\mathsf{r}_\oplus = 6,378.1366 \pm 0.0001$ km [5]. Multiplying the sidereal angular rate with the radius, we yeild a mean equatorial speed, or the Earth's *sidereal speed* of $\mathsf{V}_\oplus = 0.46510$ km s^{-1}. For an observatory site at higher latitute, the site velocity due to the Earth rotation becomes smaller. The speed attributed to the diurnal motion is a function of the observatory latitude. Appendix A.2 providess an algorithm to compute the diurnal term in Doppler velocity.

3.2 Doppler tracking -

In spectral line observations that are carried out while keeping the "velocity" of a particular spectral channel constant or time invariant, the sky frequency $\nu_{\rm sky}$ received at an observatory must vary with time (t) due to the motions of the observatory with respect to a reference frame. The frequency $\nu_{\rm sky}$ observed can be converted to a specific frame with a formula in a specific velocity definition depending on the requirement given a science case. Presumbly, the SMA online system supports the LSR with radio definition. The sky frequency $\nu_{\rm sky}$ can be written into a constant frequency ν_0 modified by time variable term due to the motion of the obervatory with respect to the LSR:

$$\nu_{\rm sky} = \nu_0 \left[1 - \frac{\mathsf{V}_{\rm site} + \mathsf{veldop}}{\mathsf{c}} \right], \tag{1}$$

where $V_{\rm site} = \mathbf{V}_\oplus \cdot \mathbf{s}$ is the diurnal term described in Appendix A.2; veldop is a Doppler velocity of the observatory by summing the annual term and heliocentric motion with respect to the LSR frame; and c is the light speed. For an online Doppler tracking system, the time corresponding to ν_0 must be consistent with veldop. Usually, the time at source transit is used for setting ν_0 and veldop. We denote the offset in frequency or the Doppler frequency offset that is used in a Doppler tracking system to compensate the sky frequency as

$$\mathsf{dopoff} = \nu_0 \frac{V_{\rm site}}{\mathsf{c}} \tag{2}$$

Then, Eq(1) can be rewtitten as:

$$\nu_{\rm sky} = \nu_0 \left[1 - \frac{\mathsf{veldop}}{\mathsf{c}}\right] - \mathsf{dopoff}, \tag{3}$$

In the convention of UVFITS, the observatories of performing online Doppler corrections in an observing mode for spectral line observations, a constant sky frequency ν_0, a constant veldop, and a time variable dopoff are stored in the data table for precisely tracking what is the actual the sky frequency from ν_0. In a software package for data reduction, only ν_0 and veldop are needed in corrections for the Doppler effect caused by a motion of an observatory with respect to the LSR. The time variable dopoff is only needed to track errors when something wrong occured in an online Doppler tracking program. Astronomers may use it to determine what the actual sky frequencies were used during an observing run.

For an observation that performs no online Doppler tracking, the sky frequency can be expressed as:

$$\nu_{\rm sky} = \nu_{\rm rest} \left[1 - \frac{\mathsf{veldop}}{\mathsf{c}}\right], \tag{4}$$

where $\nu_{\rm rest}$ is a rest frequency; veldop is the velocity of the observatory with respect to a target source. Then, true sky frequency $\nu_{\rm sky}$, the rest frequency $\nu_{\rm rest}$ and veldop are stored into data. Both $\nu_{\rm sky}$ and veldop are time variable. Doppler corrections caused by observatory motion for various reference frames can be done with models coded in offline pipelines during or prior to data reduction.

4 A correction for the specific errors during 2011-2019

This section provides a procedure to correct for the Doppler tracking errors occured on the SMA online system, considering the fact that no data on dopoff were recorded in the SMA data system. A model for various motions of the Earth needs to be re-built to compute the offset in veldop as a function of time caused by the online error.

4.1 Formula

If the true sky frequencies $\nu_{\rm sky}$ can be recovered with the simple formula Eq(3), one can do the offline Doppler corrections following a procedure similar to those used for the observatories that provide no on-line Doppler tracking. Unfortunately, $\nu_{\rm sky}$ can not be recomputed with Eq(3) because of missing dopoff in the SMA archived data files. Instead, the corresponding Doppler velocity correcting for the SMA Doppler tracking error can be calculated with the following formula:

$$\mathsf{veldop}' = \mathsf{veldop} + \Delta V_{\rm D}(t) + \Delta V_{\rm E}(t). \tag{5}$$

The three terms on the right side of the equation give a detailed formula to correct for SMA online Doppler tracking errors. First, veldop is the Doppler velocity of SMA recorded at the instant t_0, which is incorrect due to the error in the online model. The time t_0 corresponds to the time at which the sky frequency $\nu_{\rm sky}(t)$ is recorded as the constant sky frequency ν_0, namely,

$$\nu_{\rm sky}|_{t=t_0} = \nu_0. \tag{6}$$

For the online Doppler tracking system at SMA, t_0 was corresponding to the time at transit of the source [7]. Second, the term $\Delta V_{\rm D}(t)$ is defined as:

$$\Delta V_{\rm D}(t) = V_{\rm site}^{\rm Hay} - V_{\rm site}^{\rm SMA}, \tag{7}$$

the amount of the correction from the diurnal term, a difference of $V_{\rm site}$ at the locations (λ, ϕ) between the Haystack observatory and the SMA. For given an observatory, the coordinate (λ, ϕ) denotes its longitude and latitude. A detailed algorithm for computing $V_{\rm site}^{\rm Hay}$ and $V_{\rm site}^{\rm SMA}$ is provided in Appendix A.2. $\Delta V_{\rm D}(t)$ counts for the major error source in the SMA online Doppler tracking model used in the period between 2011 April 4 and 2019 April 3.

4.2 Analysis of the error sources

4.2.1 Errors in the diurnal term -

The error arises from two parts of the diurnal term:

- The first term varies as function of observatory latitude ϕ. This error is caused by difference in latitude $\Delta\phi$ between the Hasystack and SMA, which can be estimated from a partial derivative of the site velocity $V_{\rm site}$ as function of ϕ, see Eq(A15):

$$\Delta V_{{\rm site},\phi} = \frac{\partial V_{\rm site}}{\partial \phi}\Delta\phi = -V_{\oplus}\sin\phi\cos\delta_{\rm app}\sin(\alpha_{\rm app} - \mathsf{ST})\Delta\phi \tag{8}$$

- The second term is a function of the local apparent sidereal time ST depending on the observatory longitudes λ. The error caused by difference in longitude $\Delta\lambda$ between the two observatories can be assessed from a partial derivative of $V_{\rm site}$ as function of λ, instead:

$$\Delta V_{{\rm site},\lambda} = \frac{\partial V_{\rm site}}{\partial \lambda}\Delta\lambda = -V_{\oplus}\cos\phi\cos\delta_{\rm app}\cos(\alpha_{\rm app} - \mathsf{ST})\Delta\lambda, \tag{9}$$

  where the local sideral time $\mathsf{ST} = {\rm GMST} + \lambda$, and λ represents the east longitude of an observatory, hereafter. And GMST stands for Greenwich Mean Sidereal Time.

Giving Haystack and Mauna Kea coordinates ($\phi_{\rm Hay} \approx 42.47^{\rm o}$, $\lambda_{\rm Hay} \approx 288.51^{\rm o}$ and $\phi_{\rm SMA} \approx 19.82^{\rm o}$, $\lambda_{\rm SMA} \approx 204.53^{\rm o}$), we can assess the values of $\Delta V_{{\rm site},\phi}$ and $\Delta V_{{\rm site},\lambda}$ by choosing an astronomical object on the celestial equator, *i.e.* $\delta_{\rm app} = 0^{\rm o}$, where the calculations of the Doppler errors can be simplified,

$$\Delta V_{{\rm site},\phi} = V_{\oplus}\sin\phi_{\rm Hay}\sin({\rm HA})\Delta\phi_{\rm Hay-SMA} = 0.124 \text{ km s}^{-1}\sin({\rm HA}), \tag{10}$$

$$\Delta V_{{\rm site},\lambda} = -V_{\oplus}\cos\phi_{\rm Hay}\cos({\rm HA})\Delta\lambda_{\rm Hay-SMA} = -0.503 \text{ km s}^{-1}\cos({\rm HA}), \tag{11}$$

where the local hour angle ${\rm HA} = {\rm ST} - \alpha_{\rm app}$. Eq(10) and Eq(1) estimate the amounts of error in Doppler velocity caused by a wrong coordinate variable while other is correct. So, for a narrow maser line, the first term $\Delta V_{{\rm site},\phi}$ leads to a maximum blueshift or a minimum value in velocity $\Delta V^{\rm min}_{{\rm site},\phi} = -0.124$ km s^{-1} at the negative ${\rm HA} = -6^{\rm h}$, and a maximum redshift or a maximum value in velocity $\Delta V^{\rm max}_{{\rm site},\phi} = +0.124$ km s^{-1} at the positive ${\rm HA} = +6^{\rm h}$. Thus, the integration of line signals by tracking the source will smear the line profile. For a 12-hr track from ${\rm HA} = -6^{\rm h}$ to ${\rm HA} = +6^{\rm h}$, the accumulated smearing range in velocity is $\delta{\rm V} = \Delta V^{\rm max}_{{\rm site},\phi} - \Delta V^{\rm min}_{{\rm site},\phi} = 0.248$ km s^{-1}.

The second term $\Delta V_{{\rm site},\lambda}$ involves a cosine function of the local hour angle (HA); it leads to a maximum blueshift of -0.503 km s^{-1} at the source transit or ${\rm HA} = 0^{\rm h}$ while $\Delta V_{{\rm site},\lambda} = 0$ km s^{-1} at ${\rm HA} = \pm 6^{\rm h}$. Since the symmetrics of the consine function of HA, this term leads to a bias in blueshift. The mean value[1] of the blueshifted velocity is $\overline{\Delta V}_{{\rm site},\lambda} = -0.320$ km s^{-1} with

[1] $\overline{\Delta V}_{{\rm site},\lambda} = \frac{1}{\pi}\int_{-\pi/2}^{+\pi/2}\Delta V_{{\rm site},\lambda}{\rm dHA}$;

a dispersion[2] of σ =0.404 km s^{-1} for a range of hour angle continuously from $-6^{\rm h}$ (or $-\frac{\pi}{2}$) to $6^{\rm h}$ (or $+\frac{\pi}{2}$) in a track of observing an astronomical object on the celestial equator.
A total of the errors in velocity is not a simple sum of the two estimated terms, $\Delta V_{\rm site,\phi}$ and $\Delta V_{\rm site,\lambda}$. The effect due to a combination of the errors caused by the two wrong variables (ϕ and λ) appears to be more complicated. Both the online Doppler velocity of the diurnal term $V_{\rm site}^{\rm Hay}(\phi, \lambda, \mathsf{ST})$ and the correct Doppler velocity $V_{\rm site}^{\rm SMA}(\phi, \lambda, \mathsf{ST})$ can be calculated using Eq(A15) in Appendix A.2. The difference between $V_{\rm site}^{\rm Hay}(\phi, \lambda, \mathsf{ST})$ and $V_{\rm site}^{\rm SMA}(\phi, \lambda, \mathsf{ST})$ is the correction for the Doppler velocity stored in the SMA data:

$$\Delta V_{\rm site}(\mathsf{ST}) = V_{\rm site}^{\rm Hay}(\phi, \lambda, \mathsf{ST}) - V_{\rm site}^{\rm SMA}(\phi, \lambda, \mathsf{ST}) \tag{12}$$

We computed $V_{\rm site}^{\rm Hay}$, $V_{\rm site}^{\rm SMA}$, and $\Delta V_{\rm site}$ as well as $\Delta V_{\rm site,\phi}$, $\Delta V_{\rm site,\lambda}$ for an astronomical object on the celestial equator at the local hour angle HA = $\pm$ i$^{\rm h}$ for i = 0 , 1 , 2 ... 6. These velocity values are tabulated in Table 1. Columns 1 and 2 are the hour angle (HA) and elevation (EL) at the SMA. Columns 3 and 4 are the terms of the Doppler errors caused by a wrong coordinate variable of latitude (ϕ) and longitude (λ), respectively. Columns 5 and 6 are $V_{\rm site}^{\rm Hay}$, and $V_{\rm site}^{\rm SMA}$. Column 7 is $\Delta V_{\rm site}$ that is the amount of online error in Doppler velocity of the diurnal motion, given an hour angle or a time.

Table 1: A correction for Doppler tracking errors in diurnal motion

HA (h)	EL (d)	$\Delta V_{\rm site,\phi}$ (km s^{-1})	$\Delta V_{\rm site,\lambda}$ (km s^{-1})	$V_{\rm site}^{\rm Hay}$ (km s^{-1})	$V_{\rm site}^{\rm SMA}$ (km s^{-1})	$\Delta V_{\rm site}$ (km s^{-1})
-6.0	-0.0	-0.124	0.000	0.036	0.438	-0.402
-5.0	14.1	-0.120	-0.130	-0.054	0.423	-0.476
-4.0	28.1	-0.108	-0.251	-0.139	0.379	-0.518
-3.0	41.7	-0.088	-0.356	-0.216	0.309	-0.525
-2.0	54.6	-0.062	-0.435	-0.277	0.219	-0.496
-1.0	65.3	-0.032	-0.486	-0.320	0.113	-0.433
0.0	70.2	0.000	-0.503	-0.341	-0.000	-0.341
1.0	65.3	0.032	-0.486	-0.339	-0.113	-0.226
2.0	54.6	0.062	-0.435	-0.313	-0.219	-0.095
3.0	41.7	0.088	-0.356	-0.267	-0.309	0.043
4.0	28.1	0.108	-0.251	-0.202	-0.379	0.177
5.0	14.1	0.120	-0.130	-0.123	-0.423	0.300
6.0	-0.0	0.124	0.000	-0.036	-0.438	0.402

In summary, the two wrong coordinates (ϕ and λ) together could cause a velocity smearing in a spectral line profile and bias in blueshift of a line center. The maximum smearing in the resultant velocity $\delta V = \Delta V_{\rm site}^{\rm max} - \Delta V_{\rm site}^{\rm min} \approx 0.825$ km s^{-1} in a 10 h track from HA = -5 h to HA = 5 h. The resultant velocity bias at local hour angle HA = $0^{\rm h}$ leads to a line center blueshifted up to -0.341 km s^{-1} that is mainly due to the longitude error. We note that the blueshift of a line center could exceed -0.341 km s^{-1} and is up to -0.525 km s^{-1}, *e.g.*, for a snapshot at HA ~ -3 h of an astronomical object on the celestial equator.

[2] $\sigma^2 = \frac{1}{\pi}\int_{-\pi/2}^{+\pi/2} \left[\Delta V_{\rm site,\lambda} - \overline{\Delta V}_{\rm site,\lambda}\right]^2 {\rm dHA}$..

4.2.2 Possible errors in the annual term -

Finally, $\Delta V_{\mathrm{E}}(t = t_0)$ accounts for the error in calculation of the Earth orbiting velocity at a source transit. Appendix A.1 provides detailed algorithms for computing $V_{\mathrm{E}}(t)$ given an epoch time t. If the online Doppler tracking model also mistakely used the local sideral time (ST) at Haystack observatory in computing $V_{\mathrm{E}}(t_0)$ to determine the constant sky frequency ν_0 (Eq(6)), then the longitude error would lead to a shift in ν_0 although the error appears to be relatively minor, varying annually between -0.121 and $+0.121$ km s^{-1}. For observations of narrow spectral line sources, this term appears to be significant if the problematic online Doppler tracking model contained such an error.

5 A data patch and recommendations -

Two specific recommendations for online and archival software are summarized concerning the issues occured in the SMA online Doppler model and data storage systems.

5.1 A patch for the Doppler tracking error

A correction for Doppler velocity veldop′ given in Eq(5) needs to be computed to replace the veldop in the original data that are subject to the Doppler error. One may follow a simple procedure along with the formulas discussed above to implement a data patch for corrections of the Doppler tracking errors occured in the period between 2011 April 4 and 2019 April 3. There appear at least two ways for the data remendy:

- (1) Create a routine to re-produce the integration-structure table in_read[‡] with veldop′ corrected for the Doppler tracking error to replace the wrong value of veldop for each of the archived data sets produced during the period between 2011 and 2019 when the problematic online model was used.

[‡] *According to the SMA Wiki document "Current SMA data file format" [8], beginning at the 37th byte, the* float vc *appears to be defined as the observatory Doppler velocity* (veldop) *in the integration-structure table* in_read.

- (2) Create a routine of computing veldop′ and integrate this routine into a pipeline program to automatically replace the online-recorded value of veldop while loading the raw data that involved the Doppler tracking error into a software platform for data reduction. This algorithm has been implemented in the SMA testing pipeline (swarm2casa) for corrections of the Doppler errors in the SWARM correlator data produced prior to April 3, 2019. The procedure is also being used in SMA offline software for handling archived data produced from old ASIC correlator.

5.2 A recommendation to archive the Doppler frequency offset

According to the discussion in section 3.2, three parameters ν_0, veldop and dopoff are key to track down what actually did on the online Doppler tracking. With the three Doppler tracking parameters, one can restore the actual sky frequency ν_{sky} from Eq(1) and Eq(2). Fixing possible errors occured in the online Doppler traching would be straight forward.

In handling wideband data with multiple spectral chunks or windows, an ancillary table for fre-

Table 2: AIPS FO table

XTENSION	= 'BINTABLE'	/ Extension type
BITPIX	= 8	/ Binary data
NAXIS	= 2	/ Table is a matrix
NAXIS1	= 36	/ Width of table in bytes
NAXIS2	= 1450	/ Number of entries in table
PCOUNT	= 0	/ Random parameter count
GCOUNT	= 1	/ Group count
TFIELDS	= 7	/ Number of fields in each row
EXTNAME	= 'AIPS FO '	/ AIPS table file†
EXTVER	= 1	/ Version number of table
TFORM1	= '1D	/ FORTRAN format of field 1
TTYPE1	= 'TIME	/ Type (heading) of field 1
TUNIT1	= 'DAYS	/ Physical units of field 1
TFORM2	= '1E	/ FORTRAN format of field 2
TTYPE2	= 'TIME INTERVAL	/ Type (heading) of field 2
TUNIT2	= 'DAYS	/ Physical units of field 2
TFORM3	= '1J	/ FORTRAN format of field 3
TTYPE3	= 'SOURCE ID	/ Type (heading) of field 3
TUNIT3	= ' '	/ Physical units of field 3
TFORM4	= '1J '	/ FORTRAN format of field 4
TTYPE4	= 'ANTENNA NO. '	/ Type (heading) of field 4
TUNIT4	= ' '	/ Physical units of field 4
TFORM5	= '1J '	/ FORTRAN format of field 5
TTYPE5	= 'SUBARRAY '	/ Type (heading) of field 5
TUNIT5	= ' '	/ Physical units of field 5
TFORM6	= '1J '	/ FORTRAN format of field 6
TTYPE6	= 'FREQ ID '	/ Type (heading) of field 6
TUNIT6	= ' '	/ Physical units of field 6
TFORM7	= '2E '	/ FORTRAN format of field 7
TTYPE7	= 'DOPPOFF '	/ Type (heading) of field 7
TUNIT7	= 'HZ '	/ Physical units of field 7
NO_ANT	= 29	
NO_IF	= 2	
REVISION	= 0	
END		

†SMA may need to coordinate with CASA-NRAO for support if the AIPS FO table adopted and EXTNAME = 'SMA FO' to be implemented.

quency offset is needed for an offline software to unambiguously record the online Doppler tracking information as well as to build a standard format to communicate the frequency information between offline software packages for data reduction. For example, in recent, AIPS has invented a FO table to convey information about frequency changes occuring during the observations [6]. The detailed Doppler tracking information is kept so that one can find out exactly what the observing sky frequency was. AIPS used to hide this information, obtained from the on-line data, in a column in calibration table CL table [6]. The AIPS FO table replaces the usage of the "hidden" column of

Table 3: Mandatory and optional keywords for AIPS FO table headers

Keyword	Value type	Value
EXTNAME	A	'AIPS FO'
NO_ANT	I	Maximum antenna number
NO_IF	I	Number IFs (n_{IF})
REVISION	I	File format revision code

Number of antennas - the value of the NO_ANT keyword shall specifies the maximum antenna number to occur in the AIPS FO Table.

Number of spectral windows - The value of the NO_IF keyword specifies the number of spectral windows (IFs) in the data set. In the frequency offset table, this c ontrols the dimension of the Doppler offset column.

Table 4: Mandatory columns for AIPS FO table headers

Title	Type	Units	Description
TIME	1D	days	time of center of interval
TIME INTERVAL	1E	days	length of time interval
SOURCE ID	1J		Source ID number
ANTENNA NO.	1J		Antenna number
SUBARRAY	1J		Subarray number
FREQ ID	1J		Frequency setup number
DOPOFF	E($n_{\rm IF}$)	Hz	Doppler offset

Time - The time in days since the reference date for the center of the interval represented by the table row is given in the TIME column. The length of that interval is given in the TIME INTERVAL column.

Source identification number - If the file contains observations of more than one source, then the identification number of the source being observed will be given as the value of the SOURCE ID column. A value ≤ 0 is taken to apply to all sources.

Antenna number - The ANTENNA NO. column contains a positive integer value that uniquely defines the antenna within the array. This is the antenna identification number that is used in other tables, including the visibility data. If the same antenna appears in more than one array, it needs not have the same station number in each array.

Subarray number - The SUBARRAY column that is one array for the SMA.

Frequency setup number - The FREQ ID column shall contain a positive integer value that uniquely identifies the array number to which the other data in the table row apply.

Doppler offset - The DOPOFF column conveys $n_{\rm IF}$ values giving the actual observed frequency minus the time-independent frequency for each IF. The time-independent frequencies are described for each IF and frequency setup number in the FQ table, and the frequency offsets described for each source in the SU table, as well as the reference frequency given in the uv-data header.

the CL table that is not supported in other software packages.

Tables 2 to 4 describe an example of the AIPS FO tables [6]. The FO table is maintained with single- as well as multi-source data sets for the data reduction software packages, CASA and AIPS for example, at NRAO. It would be convenient for offline software reduction packages and/or a pipeline software at SMA to adopt the FO table. However, it was alerted that the SMA online software did not store the Doppler frequency offset dopoff in the SMA archival data with MIR-format although both the frequency ν_0 and Doppler velocity veldop are recorded.

Therefore, it is recommended for the SMA online storage system to store the value of dopoff into the frequency structure table sp_read at the position of, *e.g.* double sparedbl1, one of the six spare doubles designed for future use, in the current MIR format for SMA data [8]. The structure of MIR binary file sp_read is flexible to store and retrieve dopoff as a spectral-chunk-based variable.

Appendix -

A Algorithms for computing the Doppler corrections

In this Appendix, we provide detailted algorithms to computing the annual term (A.1) and the diurnal term (A.2) in Doppler velocity.

A.1 Annual term in Doppler velocity

The quantities of the annual term in Dopper velocity are related to the orbital motion of the Earth around the Sun. Figure 1 illustrates a sketch for an orbital plane of a celestial object (the Earth) and a reference plane (the ecliptic plane) via three orbital elements (the three orientation angles), ω, Ω, i.

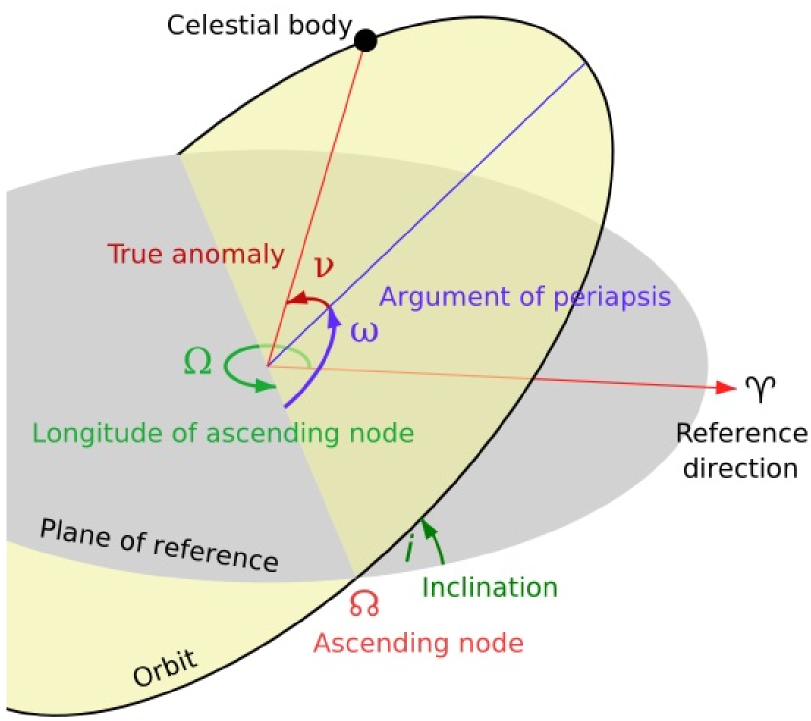

Figure 1: An illustration of transformation between orbital plane and equatorial plane (plane of reference) for an celestial object in an elliptical orbit. This plot is adopted from https://en.wikipedia.org/wiki/Orbital_plane_(astronomy).

A.1.1 Orbital elements

Here are the three orbital elements along with the true anomaly (ν) defined:

ω – is the *argument of periapsis* (or perihelion), an angle between the ascending node to the periapsis, defining the orientation of the ellipse in the orbital plane.

Ω – is the *longitude of the ascending node*, an angle from a specified reference direction (original longitude) to the direction of the ascending node.

i – is the *inclination angle*, describing a vertical tilt of the ellipse with respect to the reference plane (the ecliptic plane).

ν – is the *true anomaly* at epoch (t), defining the position of the orbiting body along the ellipse at a specific time.

A.1.2 Orbital trajectory and velocity

The orbital motion of the Earth around the Sun follows Kepler's three laws by ignoring the perturbations from a third body. The motion of objects concerned in classical celestial mechanics is governed by Newton's laws. The solutions of trajectory and velocity for an orbiting object (the Earth) around a massive object (the Sun) in the orbital plane with a polar coordinate system are straightforward:

Trajectoy or orbit equation –

$$r = \frac{a(1-e^2)}{1+e\cos\nu}. \tag{A1}$$

Angular velocity –

$$r\dot{\nu} = \frac{a(1-e^2)V_{\mathrm{E}}}{r}. \tag{A2}$$

Radial velocity –

$$\dot{r} = -eV_{\mathrm{E}}\sin\nu. \tag{A3}$$

The additional parameters for the Earth's elliptical orbit motion are summarized here:

e – is the eccentricity of the Earth's orbit.
a – is the mean distance from the Earth to the Sun.
V_{E} – is the mean speed of the Earth's orbiting around the Sun.

The orbit equation of Eq(A1) and velocity components of Eq(A2) and Eq(A3) can be written in an heliocentric Cartesian coordinates on the orbital plane. If we choose the direction of the perihelion as x'-axis and the direction of angular momentum as z'-axis, in a right-handed Cartesian cooridnate system, the orbit equation of polar coordinate syetm Eq(A1) is writen as:

$$\mathbf{x}' = \begin{bmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{bmatrix}. \tag{A4}$$

Translating the solutions Eq(A2) and Eq(A3) from polar coordinate system to a Cartesian coordinates on the obital plane, velocity components can be written as:

$$\mathbf{v'} = \begin{bmatrix} r\dot{\nu}\sin\nu - \dot{r}\cos\nu \\ r\dot{\nu}\cos\nu - \dot{r}\sin\nu \\ 0 \end{bmatrix} . \tag{A5}$$

A.1.3 The matrix of transformation between orbital and equatorial planes

The matrix of a transformation from the Earth orbital plane to the equatorial plane can be expressed as:

$$\mathfrak{T} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} . \tag{A6}$$

The elements in the matrix for a rotational transformation can be derived as a function of three independent angular variables. The elements of the matrix to transform orbital plane (x', y', z') to equatorial plane (x, y, z) are a function of the three angular variables (ω, Ω, i) described in A.1.1. They are derived and listed as follows:

$$\begin{aligned}
T_{11} &= +\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i \\
T_{12} &= -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i \\
T_{13} &= +\sin\Omega\sin i \\
\\
T_{21} &= +\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i \\
T_{22} &= -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i \\
T_{23} &= -\cos\Omega\sin i \\
\\
T_{31} &= +\sin\omega\sin i \\
T_{32} &= +\cos\omega\sin i \\
T_{33} &= +\sin i \ .
\end{aligned}$$

We note that the coordinate systems with little case letters are heliocentric. It would be more convenient for the Earth-based observers to translate the coordinate system into geocentric. If we choose the X-axis pointing toward the vernal equinox and Z-axis toward the north pole, in a right-handed equatorial coordinate system, $\mathsf{R} = \sqrt{\mathsf{X}^2 + \mathsf{Y}^2 + \mathsf{Z}^2}$ denotes a fourth distance coordinate between the Earth and the Sun, forming an equatorial coordinate system. The equatorial coordinate system is geocentric. The orbit equation Eq(A4) on the orbital plane in a heliocentric system needs to translate into a geocentric Cartesian coordinates system $(\mathsf{X}', \mathsf{Y}', \mathsf{Z}')$ on the orbital plane:

$$\mathbf{X'} = \begin{bmatrix} -r\cos\nu \\ -r\sin\nu \\ 0 \end{bmatrix} . \tag{A7}$$

And the velocity in the geocentric coordinate system on the orbital plane can be derived as:

$$\mathbf{V'} = \begin{bmatrix} V_{\mathrm{E}} \sin \nu \\ -V_{\mathrm{E}}(e + \cos \nu) \\ 0 \end{bmatrix}, \tag{A8}$$

where V_{E} is the mean speed of the Earth evolving around the Sun.

A.1.4 Computing the positional components

The orbit equation from the (X', Y', Z')-system on the orbital plane can be translated into the geocentrical equatorial system with the transforming matrix Eq(A6):

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix}. \tag{A9}$$

Thus, the solution for the orbit equation in the Equatorial coordinate system becomes:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -r \cos(\nu + \omega) \\ -r \sin(\nu + \omega) \cos i \\ -r \sin(\nu + \omega) \sin i \end{bmatrix}, \tag{A10}$$

where the distance r between the Earth and the Sun is given by Eq(A1).

A.1.5 Computing the velocity components

The velocity components in the Equatorial systems can be obtained by transforming the solutions on the orbital plane:

$$\begin{bmatrix} V_{\mathrm{X}} \\ V_{\mathrm{Y}} \\ V_{\mathrm{Z}} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} V'_{\mathrm{X}} \\ V'_{\mathrm{Y}} \\ V'_{\mathrm{Z}} \end{bmatrix}. \tag{A11}$$

With the Eq(A8) and the elements of the transformation matrix listed in A.1.3, the components of the earth velocity are obtained:

$$\begin{bmatrix} V_{\mathrm{X}} \\ V_{\mathrm{Y}} \\ V_{\mathrm{Z}} \end{bmatrix} = \begin{bmatrix} +V_{\mathrm{E}}(\sin(\nu + \omega) + e \sin \omega) \\ -V_{\mathrm{E}}(\cos(\nu + \omega) + e \cos \omega) \cos i \\ -V_{\mathrm{E}}(\cos(\nu + \omega) + e \cos \omega) \sin i \end{bmatrix}. \tag{A12}$$

A.2 Diurnal term in Doppler velocity

If $[\alpha_{\mathrm{app}}, \delta_{\mathrm{app}}]$ are the apparent right ascension and declination of the Doppler tracking source, the source vector $\mathbf{s}$ is a function of α_{app} and δ_{app}. In the equatorial coordinate system, the vector $\mathbf{s}$ can be expressed as the direction cosines:

$$\mathbf{s} = \begin{bmatrix} \cos \alpha_{\mathrm{app}} \cos \delta_{\mathrm{app}} \\ \sin \alpha_{\mathrm{app}} \cos \delta_{\mathrm{app}} \\ \sin \delta_{\mathrm{app}} \end{bmatrix}. \tag{A13}$$

Given an observatory's geocentrical latitude (ϕ) and a local apparent sidereal time (ST), the observatory site's velocity ($\boldsymbol{V}_{\mathbf{site}}$) due to the Earth rotation can be derived, in a simple algorithm, from the sidereal speed of the Earth equator. The velocity vector can be written as in an equatorial coordinate system:

$$\boldsymbol{V}_{\mathbf{site}} = \begin{bmatrix} -V_{\oplus} \cos\phi \sin \mathsf{ST} \\ +V_{\oplus} \cos\phi \cos \mathsf{ST} \\ 0 \end{bmatrix}. \tag{A14}$$

Thus, the diurnal term in Doppler velocity or the site speed (V_{site}) can be determined by projecting the velocity vector ($\boldsymbol{V}_{\mathbf{site}}$) on to the direction vector ($\boldsymbol{s}$) of the Doppler tracking source ($\boldsymbol{V}_{\mathbf{site}} \cdot \boldsymbol{s}$):

$$V_{\mathrm{site}} = V_{\oplus} \cos\phi \cos\delta_{\mathrm{app}} \sin(\alpha_{\mathrm{app}} - \mathsf{ST}). \tag{A15}$$

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