2001 Course Summary (not meant to be all inclusive)- Astronomy 45

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PACS:

I. INTRODUCTION

1 Geometry: elongation, conjuction, opposition, quadrature, ecliptic, equatorial sphere,

2 Time:

$$\frac{1}{S} = |\frac{1}{P} - \frac{1}{P\oplus}| \tag{1}$$

3 Distance:

- a- Copernicus method of measuring distance
- b- Kepler method of measuring distance
- c- Parallax method for near and far objects
- d- Transit of terrestrial planets

II. THE ASTRONOMICAL CONTEXT

1 Coordinate systems:

- a- declination (δ) and right ascention (α)
- b- solid angle: an interior angle that contains all trajectories that emanate from an object (source).

2 Brightness and apparent magnitude: five magnitudes corresponds to a factor of 100 in brightness.

3 $m_1 - m_2 = 2.5 \log \frac{F_2}{F_1}$. Brightness and flux depend on frequency of radiation and are usually measured using filters in different frequency bands (U, V, B, I, etc.) The sum of fluxes in different filters is called the bolometric flux and the resulting apparent magnitude is the bolometric magnitude.

4 Absolute magnitude is the magnitude measured at a distance of 10 pc. Distance modulus is equal to $5 \log D - 5$.

4 Doppler measurements: wavelength of radiation changes with direction and magnitude of motion. Red shift is

$$z = \frac{\Delta\lambda}{\lambda} \tag{2}$$

5 proper motion is the rate of change of the angular position of star, $\mu = \frac{d\phi}{dt} = \frac{v_t}{d}$.

6 distance measurement methods

III. RADIATION

1 The specific intensity $I_\nu(r)$ at any point in space $\mathbf{r}$ is chosen such that the energy $d\epsilon$ of the photon rays that cross an area dA normal to the propagation direction in time dt in a solid angle $d\Omega$ is

$$d\epsilon = I_\nu d\nu dt dA d\Omega \tag{3}$$

2 I_ν is constant along any ray, i. e. $\frac{dI_\nu}{ds} = 0$.

3 Flux: $dF_\nu = I_\nu \cos\theta d\Omega$, where θ is the angle between the normal to area dA and the propagating rays. This means that for an isotropic $I_\nu(\theta)$, $F_\nu = 0$.

4 Energy density : $u_\nu = \frac{1}{c} J_\nu$, where $J_\nu = \int I_\nu d\Omega$. For isotropic intensities, $u_\nu = \frac{4\pi}{c} I_\nu$.

5 Radiation pressure: $P = \frac{1}{3}u$, because the photon momentum per unit time per unit area is pressure. Substituting for the energy of photon as $E = pc$ gives the above expression for radiation pressure. Substituting instead, $E = \frac{1}{2}pv$ for particles, give the matter density $P = \frac{2}{3}u$.

5 Flux from a sphere of uniform brightness: flux at a distance r from a source of brightness B and radius R is $F = \pi B (R/r)^2$. If the source luminosity is know, then the stellar radius can be obtained from $R = \sqrt{\frac{L}{4\pi^2 B}}$.

6 Blackbody: a blackbody is an ideal emitter of radiation that it absorbs. All blackbodies at the same temperature T have similar radiation spectrum,

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/KT) - 1} \tag{4}$$

There are two limits:

- a- $h\nu >> kT$
  Then, $B_\nu(T) \sim \frac{2h\nu^3}{c^2} \exp(-h\nu/kT)$. This is the Wien's tail.

b- $h\nu << kT$
Then, $B_\nu(T) \sim \frac{2\nu^2}{c^2}kT$. This is the Rayleigh-Jeans limit. This limit says that the black-body radiation increases quadratically with frequency. This is called the ultraviolet cathasrophe.

7 The BB radiation $B_\lambda(T)$ peaks at $\lambda_{max}T \sim 0.29$, where λ is in cm. This is the Wien's displacement law.

8 Stefan-Boltzmann law: the total energy density in an isotropic radiation BB field is $u = aT^4$, where $a = 7.56464 \times 10^{-15}$ erg cm^{-3} K^{-4}. The emergent flux from the surface of the BB is $F_s = \frac{c}{4}u = \sigma T^4$.

9 Hertzsprung - Russell (HR) diagram: HR diagrams relate the stellar luminosity (apparent magnitude) as a function of color index (temperature).

10 OBAFGKM classification: used to categorize the stellar temperatures (color indices) ranging from the coldest (M) stars to hottest (O) stars, $2500 < T < 50,000$.

11 Refracting telescopes: make use of lenses and Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$

12 Reflecting telescopes: make use of mirrors and law of reflection, $\theta_1 = \theta_2$.

IV. CLASSICAL DYNAMICS

1 Gravitational potential of a spherical shell: If the point is outside the shell, then $V(r) = -\frac{GmM}{r}$. If the point is inside the shell, $V(r) = -\frac{GmM}{a} = constant$, where a is the radius of the shell.

2 Two-body problem: In the center of mass of two particles with masses m_1 and m_2, the relative motion of the particles is exactly similar to the motion of a fictitious particle of mass $\mu = \frac{m_1 m_2}{m_1+m_2}$ at a distance $r = |r_2 - r_1|$ from the center of mass which in the absent of external forces moves with a constant velocity.

3 Total angular momentum $L = \mu\mathbf{r}\times\dot{\mathbf{r}}$. In the center of mass, the total angular momentum is $L = \mu r^2\dot{\theta}$.

4 Total energy $E = K.E. + P.E. = \frac{1}{2}\mu\dot{\mathbf{r}}^2 + \frac{1}{2}M\mathbf{v}_{cm}^2 - \frac{GM\mu}{r}$. When angular motion is involved, the total energy minus the constant energy of the center of mass is $E = \frac{1}{2}\mu[\dot{r}^2 + r^2\dot{\theta}^2] - \frac{GM\mu}{r}$.

5 Constants of motion: Energy and angular momentum.

6 Orbits:

a- $r = r_\circ[1 + \epsilon\cos\theta]^{-1}$, where

b- $\epsilon^2 - 1 = \frac{2(E/\mu)(L/\mu)^2}{(GM)^2}$ and the semi-latus rectum $r_\circ = a(1-\epsilon^2)$, where a is the semi-major axis.

c- for $\epsilon = 0$, circular motion results.

d- for $0 < \epsilon < 1$, meaning $E < 0$, the motion is bound and elliptical.

e- for $\epsilon = 1$, $E = 0$, and the motion is bound and parabolic.

f- for bound motions, $E = -\frac{GM\mu}{2a}$.

7 Kepler's laws:

a– planets move in elliptical orbits with the center of mass (Sun) at one focus.

b– a line from the Sun (focus) sweeps equal areas in equal times, $\frac{dA}{dt} = \frac{1}{2}(L/\mu)$.

c– $\tau^2 = \frac{4\pi^2}{GM}a^3$. If a is measured in AU and τ is in years, then $\tau^2 = a^3$. Then, M is measured in $M_\odot$.

8 The escape velocity is the minimum velocity needed to remove a particle of mass μ from the gravitational attraction of the larger body of mass M. Setting the KE = PE, $v_{esc} = \sqrt{\frac{2GM}{r}}$, where r is the distance to mass M.

9 Moment of inertia of a spinning sphere is $I = \frac{2}{5}MR^2$. So, in consideration of orbital motion of the Earth and other rotating bodies about their centers of mass, one should also take account of the angular momentum due to inertia, i. e. $L = I\omega$. For instance, for the Jupiter-Sun system, the orbital period is $P = 11.86$ years (that of Jupiter). Also, the spin period of the Sun $P_\odot = 26$ days and that for Jupiter is $P_J = 10$ hours. Then, the numbers show us that $\frac{L_\odot^s}{L_odot} = \frac{6.7\times10^{45}}{1.84\times10^{47}}$ and $\frac{L_J^s}{L_J} = \frac{6.7\times10^{45}}{1.94\times10^{50}}$. The orbital angular momentum wins by several orders of magnitude.

10 Binary systems:

a– Visual binaries: both bodies are detected orbiting about one another.

b– Astrometric binaries: only one body is observed and its motion is oscillatory due to the presence of the companion star.

c– Spectroscopic binaries: periodic oscillations of the spectrum, but visually unresolved.

d– Eclipsing binaries: two bodies eclipse one another producing periodic changes in apparent brightness.

e– Binary systems rotate with angular velocity Ω about common center of mass and have radial velocities $K_j = v_j \sin i$, where i is the angle of inclination, the angle between the line of sight and the normal to the orbital plane. For eclipsing binaries, it is safe to assume $i = 90°$, as otherwise we would not see a full eclipse.

f– $\frac{m_1}{m_2} = \frac{K_2}{K_1} = \frac{r_2}{r_1}$.

g– mass function, $f(m) = \frac{(m_1 \sin i)^3}{(m_1+m_2)^2}$.

h– $m_1 \sin^3 i = \frac{\tau}{2\pi G}(K_1 + K_2)^2 K_2$

i– $m_2 \sin^3 i = \frac{\tau}{2\pi G}(K_1 + K_2)^2 K_1$

11 Tides: when the total potential energy due to the gravitational attraction of the two bodies orbiting about one another is considered, the potential per unit mass,

$$\Phi(\mathbf{r}) = -\frac{Gm_1}{a} - \frac{Gm_2}{R}[1 + \frac{a}{R}\cos\theta + \frac{1}{2}(3\cos^2\theta - 1)\frac{a^2}{R^2}] - \frac{1}{2}\frac{G(m1_+m_2)}{R^3} [(\frac{m_2}{m_1+m_2})^2 R^2 + a^2 - 2(\frac{m_2}{m_1+m_2})Ra\cos\theta] \quad (5)$$

, where a is the radius of the larger body and R is the distance between m_1 and m_2. For Earth - moon system, the equipotential surfaces (force equal to zero) are such that $h = \frac{3}{2}(\frac{m_2}{m_1})(\frac{R_\oplus}{R})^3 R_\oplus \cos^2\theta$, where $m_1 = M_\oplus$ and $h = a - R_\oplus$ is the height above the ocean surface. The tidal height comes to $h = 0.5$ m and from the influence of the sun $h = 0.25$ m.

12 Roche Stability: The Roche limit is reached when the differential forces (gravity and centripetal) are balanced by the gravitational attraction of the small body. Any body of density ρ that comes within a critical radius, Roche radius, $R_{Roche} = (\frac{9m_2}{4\pi\rho})^{1/3}$, where m_2 is the mass of the larger body, is torn apart.

13 Roche Lobes: these are created when the equipotential surfaces intersect at the saddle points (constant potential = zero force). Roche Lobes as figure 8 intersect at L_1, the first Lagrange point, where the forces vanish. Roche lobes define the limit of a star's size. If a star becomes larger than its Roche lobe, it dumps mass through the saddle point on the companion star.

14 Virial Theorem: $<V> = -2<T>$ for a gravitational system.

V. STARS AND STELLAR STRUCTURE

1 OBAFGKMRNS classification: hottest stars — coldest stars.

2 Pop I stars are the youngest stars and hence have the highest heavy element (beyond helium) abundances, $Z \sim 0.02$.

3 Pop II stars are the oldest stars and are made mostly of light elements, as they are the survivors of the original generation of stars, $Z \sim 0.002$.

4 Nuclear Sysnthesis: ^{12}C has a mass of 12 AMU. 1 AMU = 1.66×10^{-27} kg.

a– p-p cycle: creates alpha particles (^{4}He nucleus). This occurs in low mass stars, $M < 1.5 M_\odot$.

b– CNO cycle: creates heavy elements with C as a catalyst. This occurs in high mass stars, $M > 1.5 M_\odot$.

5 Hydrostatic equilibrium: the differential in pressure is balanced with the stellar weight.

$$\frac{dP}{dr} = -\frac{G\rho(r)M(r)}{r^2} \quad (6)$$

6 for constant density, $P(r) = P_c(1 - (\frac{r}{R})^2)$, where the central pressure is $P_c = \frac{3GM^2}{8\pi R^4}$.

7 mass equation : $\frac{dM}{dr} = 4\pi r^2 \rho(r)$.

8 Equation of state: $P = P(\rho, T)$.

9 Polytropic equation of state: $P = K\rho^{1+1/n}$, where n is the polytrope index. For the main-sequence stars, such as the Sun, $n = 3$. For white dwarfs, $n = 3/2$.

10 The ideal gas law: $P = nkT$, where n is the number density of particles in a gas and k is the Boltzman constant.

11 $P \propto \rho T \propto \rho^{5/3}$ for main-sequence stars. Then, $T \propto M^{1/3}$.

12 For stability, KE (thermal energy) = PE, i. e. $MkT \sim \frac{GM^2}{R}$, or $R \propto M^{2/3}$.

13 mean molecular weight for a neutral gas: $\mu_n = \frac{\sum_j N_j A_j}{\sum_j}$, where N_j are the number of particles j with mass ratio (relative to H) of A_j.

14 mean molecular weight for a fully ionized gas: $\mu_i = \frac{\sum_j N_j A_j}{\sum_j N_j(1+Z_j)}$, where Z_j the nuclear charge.

15 Adiabatic index: $\gamma = 1 + 1/n$, where n is the polytrope index.

16 for a thermal gas, $P \propto \rho$, meaning that $n = \infty$, referring to an infinite number of degrees of freedom for particle motion.

17 transport of heat:

a– radiation: heat is carried by photon absorption or emission b– conduction: heat is generated due to collision between gas particles.

c– conduction: heat is transferred by the actual motion of gas particles.

18 Degenerate matter: fermions exert pressure due to Pauli Exclusion Principle:

a– Fermi pressure: for non-relativistic electrons $P_F = \frac{1}{20}(\frac{3}{\pi})^{2/3}\frac{h^2}{m_e}n_e^{5/3}$.

b– for relativistic electrons: $P_F = \frac{1}{8}(\frac{3}{\pi})^{1/3}hcn_e^{4/3}$.

18 White Dwarfs: the electron degenerate pressure supports the gravitational (hydrostatic) pressure

a– non-relativistic WD: $P_F = \frac{1}{20}(\frac{3}{\pi})^{2/3}\frac{h^2}{m_3 m_H^{5/3}\mu_e^{5/3}}\rho^{5/3}$

b– relativistic WD: $P_F = \frac{1}{8}(\frac{3}{\pi})^{1/3}\frac{hc}{m_H^{4/3}\mu_e^{4/3}}\rho^{4/3}$

c– since the hydrostatic pressure is $P \propto \rho^2 R^2$, then for non-relatistic WD, we find that

d– $R \sim \mu_e^{-5/6}$, and

e– $M \sim \mu_e^{-5/2}$. Then,

f– $MR^3 \sim \mu_e^{-5}$ = constant. Also, $\rho \sim M^2$.

19 Chandrasekhar mass: $M_{ch} = 1.457 M_\odot$.

20 Virial Theorem for stellar structure: $E_{tot} = \frac{3\gamma-4}{3(\gamma-1)}U$, where U is the total gravitational potential energy. For $\gamma > 4/3$, the total energy < 0 and the star is bound. For $\gamma < 4/3$, the total energy > 0 and the star is unbound.

VI. COSMOLOGY

1 Hubble relation: $v = HD$.

2 Cosmic background radiation: isotropic blackbody radiation at a temperature of $T = 2.73$K. The red shift $z = 1 - \frac{\nu}{\nu_0} \sim 1500$ relates the temperature then to now

$$T_{then} = T_{now}(z+1). \tag{7}$$

3 Newtonian Dynamics ($\frac{v}{c} << 1$):

4 Lemaitre equation: A sphere of radius R expanding with velocity $\dot{R} = H_0 R$, where H_0 is the Hubble constant now. Let $R(t) = R_\circ a(t)$ subject to $a(t_\circ) = 1$, then

$$\dot{a}^2 = \frac{8\pi}{3}G\rho a^2 - kc^2 \tag{8}$$

where k is a constant of dimension (length)$^{-2}$. The density $\rho = \rho_\circ/a^3$. The starting point is the Newton's equation.

5 Define a critical density $\rho_c(t) = \frac{3H^2}{8\pi G}$ such that $\rho = \rho_c + \frac{3kc^2}{8\pi G a^2}$. The starting point is the Lemaitre equation and the realization that $H = \frac{\dot{R}}{R} = \frac{\dot{a}}{a}$.

6 The density parameter $\Omega = \frac{\rho}{\rho_c}$ and the deceleration parameter is $q = -\frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{2}\Omega$.

7 Three different scenarios:

a– $k > 0$: (closed universe), where $\rho > \rho_c$, $\Omega > 1$ and $q < 0.5$.

b– $K < 0$: (open universe), where $\rho < \rho_c$, $\Omega < 1$, and $q > 0.5$.

c– $k = 0$: (flat universe), $\rho = \rho_c$, $\Omega = 1$, and $q = 0.5$.

8 For a flat universe ($k = 0$ and $\Omega = 1$):

a– matter dominated Einstein - deSitter Universe:

$$\rho a^3 = \rho_\circ a_\circ^3 \tag{9}$$

and

$$a(t) = (6\pi G\rho_\circ)^{1/3}t^{2/3} \tag{10}$$

b– radiation dominated Einstein - deSitter Universe:

$$\rho a^4 = \rho_\circ a_\circ^4 \tag{11}$$

and

$$a(t) = (32\pi G\rho_\circ/3)^{1/4}t^{1/2} \tag{12}$$

9 Cosmological constant: Einstein added at term to the Lemaitre equation to counteract gravity.

$$\dot{a}^2 = \frac{8\pi}{3}G\rho a^2 - kc^2 + \frac{\Lambda}{3}a^3 \tag{13}$$

By choosing $\Lambda = 4\pi G\rho$, one can make $H = 0$ and halt the expansion.

10 Recombination: when the temperatures were low enough to about $T = 2.7(z = 1500 + 1) \sim 4000$ K, the Universe ran out of photons to ionize and recombination era began.

VII. INTERSTELLAR MEDIUM (ISM)

1 Emission nebulae: diffuse patches of emission surrounding hot O and B-type stars. HII regions are emission nebulae with electron densities $n_e \sim 5 \times 10^5$ cm^{-3} and masses $M > 30 M_\odot$.

2 Planetary nebulae: produced from the ejection of outer stellar atmospheres and material is ionized. Called planetary because they look like planets.

3 Radiation balance: total number of ionization events = total number of recombination events

4 Total number of ionization events = intensity (luminosity) of photons divided by the photon energy in each wavelength.

5 Total of recombination events = probability for recombining two particles (rate coefficient $\alpha(T)$) times the densities of two recombining particles, say electrons and protons, over a volume of radius r_s, called the Stromgren radius. The larger the densities of the recombining particles, the larger the number of recombination events.

$$Q = \frac{4}{3}\pi r_s^3 n_1 n_2 \alpha(T) \tag{14}$$

where n_1 and n_2 are the densities. For hydrogen and helium, the first ionization energies are, respectively, 13.6 and 24.6 eV, or 3.3×10^{15} and 6.0×10^{15} Hz.

6 Interstellar dust and gas reduce the transmission of light and hence the magnitudes and fluxes:

$$m_\lambda - m_{\lambda_\circ} = A_\lambda \tag{15}$$

where A_λ is called the extinction. Recall that in Chapter III, we learned that intensity remains constant along a defined path, i. e. $\frac{dI_\lambda}{ds} = 0$. If there is absorption or scattering of light by dust particles, the intensity must diminish. If we define this diminishing to be exponential,

$$\frac{dI_\lambda}{ds} = -\kappa_\lambda I_\lambda \tag{16}$$

Then,

$$I_\lambda = I_\lambda(0) \exp(-\tau_\lambda) \tag{17}$$

where $\tau_\lambda = \int_0^s n_H \sigma_H(\lambda) ds$ is the optical depth and $\sigma(\lambda)$ is the cross section in cm^2 for scattering and/or absorption. Then,

$$\begin{aligned} m_\lambda - m_{\lambda_\circ} &= -2.5 \log \frac{I_\lambda}{I_\lambda(0)} \\ &= -2.5 \log \exp(-\tau_\lambda) = 1.086\tau_\lambda \end{aligned} \tag{18}$$

Therefore, $A_\lambda = 1.086\tau_\lambda$. By measuring m_λ for a star of the same spectral type near us, such that for this star, $A_\lambda = 0$, we can derive A_λ by knowing the distance to the star and the measured flux ratio.

7 Phases: hot phase (10^6 K), warm phase (10^5 K), and cold phase (100 K). The molecular clouds exist in the coldest reaches of the cold phase of ISM. The typical temperatures for molecular cloud is about 20 K. The best example of a hot gas is the solar corona (the diffuse ionized gas stretching from the Sun).