Answer Key - Spectroscopic Eclipsing Binary System

Part A. Use the first equation and first row of H-line emission data to determine the min/max radial velocities of each star.

StarA

Min: $\frac{656.202nm - 656.273nm}{656.273nm} = \frac{v_r}{c} \qquad \Rightarrow v_r = -32.5\,{}^{km}\!/_{s}$

Max: $\frac{656.532nm - 656.273nm}{656.273nm} = \frac{v_r}{c} \qquad \Rightarrow v_r = 118.4\,{}^{km}\!/_{s}$

StarB

Min: $\frac{656.332nm - 656.273nm}{656.273nm} = \frac{v_r}{c} \qquad \Rightarrow v_r = 27.0\,{}^{km}\!/_{s}$

Max: $\frac{656.402nm - 656.273nm}{656.273nm} = \frac{v_r}{c} \qquad \Rightarrow v_r = 59.0\,{}^{km}\!/_{s}$

Since each star traverses a circular orbit, Kepler's Second Law requires that the velocity of each star around the center of mass is constant.

The average velocity of StarA = The average velocity of StarB
Since the stars maintain a constant distance relative to each other (remain in their circular orbits), the average velocity = The velocity of the system.

$$v_{system} = \tfrac{1}{2}\left(v_{r_{\min}} + v_{r_{\max}}\right) = \tfrac{1}{2}\left(-32.5\,{}^{km}\!/_{s} + 118.4\,{}^{km}\!/_{s}\right) = \tfrac{1}{2}\left(27.0\,{}^{km}\!/_{s} + 59.0\,{}^{km}\!/_{s}\right)$$

$v_{system} = 43.0\,{}^{km}\!/_{s}$ away from the Earth.

Part B. Use the second equation to determine the radius of orbit.

Orbital velocity:

$v_{orbital} = v_{radial} + v_{system}$

StarA: $v_A = 75.4\,{}^{km}\!/_{s}$

StarB: $v_B = 16.0\,{}^{km}\!/_{s}$

Convert the units of Period (P) from days to seconds:

$$\mathrm{P} = (88.2days)\left(\frac{24hrs}{1day}\right)\left(\frac{3600\,\mathrm{sec}}{1hr}\right) = 7.6\times10^{6}\,\mathrm{sec}$$

$$\frac{2\pi r_{StarA}}{\mathrm{P}} = v_A$$

$$\frac{2\pi r_{StarB}}{\mathrm{P}} = v_B$$

$$\frac{2\pi a}{\mathrm{P}} = \frac{2\pi}{\mathrm{P}}\left(r_{StarA} + r_{StarB}\right) = \left(v_A + v_B\right)$$

$$2\pi a = \left(91.4\,{}^{km}\!/_{s}\right)\left(7.6\times10^{6}\,s\right)$$

$$\Rightarrow a = 1.1\times10^{8}\,km$$

Convert the units of Orbital Radius (a) from kilometers to AU.

$$a = \frac{\left(1.1\times10^{8}\,km\right)}{\left(1.5\times10^{8}\,km\right)} = 0.73AU$$

Part C. Use the third equation to determine the combined mass of the stars.

Convert the units of Period (P) from days to years:

$$\mathrm{P} = \left(88.2days\right)\left(\frac{1year}{365days}\right) = 0.242years$$

$$M_{StarA} + m_{StarB} = \frac{(0.73AU)^3}{\left(0.242yr\right)^2} = 6.64M_{\odot}$$

$$\frac{M_{StarB}}{M_{StarA}} = \frac{v_{StarA}}{v_{StarB}} = \frac{118.4\,{}^{km}\!/_{s}}{59.0\,{}^{km}\!/_{s}} = 2.0$$

$$\Rightarrow 2M_{StarA} = M_{StarB}$$

$$M_{StarA} = 2.21M_{\odot}$$

$$M_{StarB} = 4.42M_{\odot}$$