Solution Set for Ay192 Problem Set 2 1999

Problem 1. This problem is also easier than you might think. First of all, the relevant limit can be written down in terms of the collecting area per speckle, the effective integration time = the correlation time, and the bandwidth. The key assumptions we need to answer are what are the timescale, the effective collecting area and the necessary signal. Other assumptions needed are the wavelength and the bandpass.

For this exercise, we'll work in the V with a 100Å bandpass. We'll assume that we need to detect at least one photon to guide (optimistic, I know). And we'll assume that the effective timescale and collecting area are set by the atmosphere — the seeing cell size and the wind speed. We have

$$1 \text{ photon/sec} \;=\; N_p(\lambda)\left(10^{-m_L/2.5}\right)(\Delta\lambda)\left(\frac{\pi}{4}r_0^2\right)t$$

where $N_p(\lambda)$ is our calibration of photons/sec/Å/cm^2 for a 0th magnitude object, m_L is the limiting magnitude,r_0 is the scale length (assumed to represent the approximate diameter of an atmospheric cell), t is the correlation time, and $\Delta\lambda$ is the bandpass. Since the number of speckles is proportional to the square of the diameter of the telescope — the number is simply related to the number of seeing "cells" over the primary, $N_S = D^2/(2r_0)^2$, the limit does **not** depend on the diameter of the primary telescope! It does, however, go as the square of the seeing scale, and goes linearly with the correlation time.

The correlation time can be estimated via the size of the seeing cell and the average wind speed. Lets say the windspeed is $\sim$ 20 km/hr (about 13 miles per hour), so the correlation time for 40 cm cells is

$$t \;=\; (40\ cm/20\ km/h) \times (\frac{3600\ s/h}{10^5\ cm/km}) \;=\; 0.07 \text{ seconds}$$

The estimate of the magnitude limit requires you to pick a bandpass and a wavelength — the second just to get the calibration of photon flux. For a 100Å bandpass at V (with $N_p(5500\AA) = 950/cm^2/sec/\AA$), we have

$$\left(10^{-m_L/2.5}\right) \;=\; N_p^{-1}(\lambda)(\Delta\lambda)^{-1}\left(\frac{\pi}{4}r_0^2\right)^{-1}t^{-1}$$

or

$$m \;=\; 2.5\ log\Big[(950)(100)\left(\frac{\pi}{4}40^2\right)(0.07)\Big]$$

which is **only** 17.3 magnitude! Note again that this is independent of the telescope diameter as long as the telescope is larger than r$_0$. By the way, 17th mag is pretty good for this problem; Mt. Wilson is a very good site.

In actuality, we usually can do a bit better by using slightly broader bandpasses (though they can't be too broad, $\Delta\lambda/\lambda \;\leq\; 0.1$, otherwise there would be no co herence) and by going to sites where the correlation times are long ($\sim$ 0.1 sec), and the scale legnths are long ($\sim$ 20 cm). Remember, though that one photon per speckle is a relatively optimistic limit, especially considering losses in the telescope and imperfect detectors.

Problem 2.

Start with the basics. Refraction is given by

$$z_t - z_a \;=\; R_0 tan(z_a) \;-\; 0.067" tan^3(z_a)$$

and

$$R_{P,T} \;=\; R_0(\frac{P}{(760 \text{ Torr} + 2.9T)})$$

so for the summit of Mt. Hopkins, which is at 8600 feet, and assuming T=15°C,

$$R_0 \;=\; 60.38"(0.7) \;\sim\; 42"$$

since the pressure scale height of the atmosphere is about 25,000 feet.

We can then, using the result from problem 4 (!) compute the refraction as a function of hour angle at the top, middle and bottom of the field-of-view. The galactic center is at -29° so the top and bottom of the field are at -24° and -34° respectively. Mt. Hopkins is at +31°41.'

The second part of the effect is a change in the relative angles between stars at the top, middle and bottom because refraction is not along declination, except exactly on the meridian. The angle of refraction is defined as the parallactic angle. Atmospheric refraction is along a line from the zenith through the object to the horizon, which is generally only along ya line of celestial coordinates (declinatiion) when the object is on the meridian and the refraction is aligned north-south. Otherwise, the line of refraction makes and angle with the celestial coordinates known as the parallactic angle, η,

$$\sin\eta \;=\; \frac{\sin h \, \cos\phi}{[1 - (\sin\phi \, \sin\delta + \cos\phi \, \cos\delta \, \cos h)^2]^{1/2}}$$

where h and δ are the object's hour angle and declination and ϕ is the observers latitude (as above).

In the Table below, TM is the difference in refraction between the top of the field (at -24 degrees) and the middle, MB is the middle to the bottom, and η is the parallactic angle.

HA	Top	Middle	Bottom	TM	MB	η_{top}	η_{bottom}
±0.5	62.38	75.67	93.83	0.20	0.35	7.7	7.0
±0.4	61.99	75.21	93.24	0.13	0.22	6.2	5.6
±0.3	61.69	74.86	92.79	0.08	0.12	4.6	4.2
±0.2	61.48	74.60	92.47	0.03	0.06	3.1	2.8
±0.1	61.36	74.45	92.27	0.00	0.10	1.5	1.4
0.00	61.31	74.40	92.21	0.00	0.00	0.0	0.0

Our images then essentially form little arcs at the top and bottom of the field-of-view, with the arcs at the southern edge about 0.35" in diameter, and those in the north about half that excursion. Note that these little arcs actually represent a rotation around the field center, so one way of partially correcting for this is to rotate the wide field camera when you're observing at large zenith angles.

Problem 3. The throughput of a telescope is modified by losses suffered in the telescope due to optical surfaces and obstructions (*e.g.* vignetting). In typical reflecting optical telescopes the losses are usually due to the obstruction of the secondary and its spiders, to reflective losses from the mirror coatings and, when not freshly cleaned or aluminized, to scattering losses by dust on the mirror. This last is most likely to affect the primary mirror surface (or any other uplooking surfaces) since gravity helps keep the secondary clean. For our purposes, we will neglect the scattering losses, I will note here (and you should remember for further work!) that the actual optical (5500Å) surface reflectivity of telescope primaries rarely exceeds 90% and is usually nearer 85% because of dust coverage, e.g. the 5% loss due to dust scattering mentioned for the HST primary.

If we take the numbers from our class handount for front surface AL, the normal incidence reflectivity of Al is 0.84 at 3600Å and 0.88 at 7000Å. Those aren't bad for real usage.

So, let's count mirrors and measure the obscuration of the secondary, etc. To the Cassegrain foci, the MMT 6.5-m has 2 mirrors and the WFPC2 7! mirrors. We'll cheat a little and assume that each camera has a filter plus a window plus a lens accounting for 6 glass surfaces. Optimistically these are 1each. The secondary to primary ratios are 1.8m/6.5m for the 6.5-m MMT, and 0.82m/2.4m for HST (the size of the sheild and not the size of the secondary. The "secondary" obscurations are then 3%, 8% (with the 1.8-m diameter f/5) and 12%, respectively. The 6.5-m is better in the IR as the f/15 secondary is smaller (approx 30 inches in diameter), so the obscuration is of order 1%, but lets not consider that case just yet.

We'll also benchmark the detectors with our Loral CCD for the MMT where the CCD response at 3600Å and 7000Å is 53% and 70%, respectively, and with a thick coated CCD for WFPC2, which would have a response of 14% and 34% respectively.

Thus the effective throughputs, including the detector, on the MMT and HST are:

$$T = (0.84)^2 \times (0.99)^6 \times (6.5^2 - 1.8^2)/6.5^2 \times 0.53 = 0.33 \quad \text{MMT 3600Å}$$

$$T = (0.88)^2 \times (0.99)^6 \times (6.5^2 - 1.8^2)/6.5^2 \times 0.70 = 0.47 \quad \text{MMT 7000Å}$$

$$T = (0.84)^6 \times (0.99)^6 \times (2,4^2 - 0.82^2)/2.4^2 \times 0.14 = 0.04 \quad \text{HST 3600Å}$$

$$T = (0.88)^6 \times (0.99)^6 \times (2.4^2 - 0.82^2)/2.4^2 \times 0.34 = 0.13 \quad \text{HST 7000Å}$$

Note that our numbers for HST are remarkably close to those given in the WFPC2 handbook — physics works! (Or they did the same calculation we did....)

The area of each telescope with the secondary obscuration already accounted for in the throughput is just $\pi d^2/4$, or 3.31×10^5 cm^2 for the MMT and 45,239 cm^2 for HST.

Last but not least we need the sky brightness and its calibration. From problem definition and the the ESD notes the sky on the ground and space is 20.7 and 21.0 mags/sq arcsec and 24.0 and 23.7 mags per sq arcsec respectively at 3600Å and 7000Å.

The standard calibration for photon rates for a 0th magnitude star are 788.5 and 620.3 photons/s/cm^2/Å at U and R, respectively. The sky is not extincted, so atmospheric extinction does not have to be taken into account.

Since we've included the obscuration in the above throughput calculation, the collecting areas of the MMT and HST are $\pi 325^2$ and $\pi 120^2$, respectively. A 1" diameter round aperture has an area of $\pi/4" = 0.785$ square arcseconds.

Plugging through we get, for example

$$U_{MMT} = (5.25 \times 10^{-9})(789)(1000)(0.33)(0.785)(3.31 \times 10^5) = 355$$

My solutions:

λ	Ground Sky	6.5-m	Space Sky	HST
3600Å	20.7	355	24.0	0.28
7000Å	21.0	301	23.7	0.95

Counts per second in 1" aperture.

Problem 4. Arcturus is at $\alpha = 14^h\ 15^m\ 08.2^s$ and $\delta = +19°\ 14'\ 31."$ The North Galactic Pole is at $\alpha = 12^h 58^m$ and $\delta = +28°\ 55.'$ The North Ecliptic Pole is at $18^h 00$ and +66°30' CTIO is at latitude -30° 10.'

Using the formulae

$$sec\ z = (sin(\phi)\ sin(\delta) + cos(\phi)\ cos(\delta)\ cos(h))^{-1}$$

and the correction formulae for the airmass x, we have for Arcturus

x_A at 14^h 00 = 1.54

x_A at 18^h 30 = 4.99

x_A at 22^h 20 = its below the horizon, sorry.

At 1.54 airmasses, to correct to the zenith required correcting for 0.54 airmasses but the full 1.54 airmasses to space. The extinction at 3500Å is ~ 0.65 magnitudes per airmass (from Palomar handout - CTIO is a little higher but its a good approximation) so its about 0.35 magnitudes or 40% fainter than at the zenith, and 1.0 magnitudes or 2.5 times fainter than outside the atmosphere. Ugh.

At 5 airmasses (!) its 11 times fainter than at the zenith and 20 times fainter than outside the atmosphere. Isn't HST fun! Or, fortunately, Arcturus is pretty darn bright.

For the NGP at transit, the zenith angle, Z, is 59.08° *plus* a small refraction correction of $\sim$ 60"$\times$ tan Z, yielding an observed zenith angle of 59.04°. The minimum airmass is then 1.94.

The NEP is never above the horizon at CTIO, sorry again! Trick question.

Also note that airmass is airmass — its defined as unity at the zenith at whatever observatory you are at, and the extinction and refaction are scaled accordingly to that. Essentially, the extinction terms vary with observatory altitude plus other factors depending on wavelength, etc.

Problem 5. The hardest part of this problem is finding the van de Hulst Curve. Its an excercise in astro-library-science. van de Hulst's curve was published in Johnson's article on dust in *Nebulae and Interstellar Matter*, Vol VII in Stars and stellar Systems edited by Middlehurst and Aller. The original version was published in 1949 in Rech. Astr. Obs. Utrecht 11, 1, 1. My plot of the curve (the ugly one on log-log paper) is exactly from Johnson updated with numbers for the H band. Cardelli, Clayton and Mathis is relatively easy to program up given the equations in their paper. If you tabulate the output at the standard wavelengths given for vdH, we have, normalizing both curves to $A_V = 1.0$:

Filter	Wavelength	vdH	CCM
U	3600Å	1.56	1.57
B	4400Å	1.33	1.32
V	5600Å	1.00	1.00
R	6900Å	0.74	0.76
I	9000Å	0.47	0.48
J	1.25μ	0.25	0.28
H	1.60μ	0.155	0.190
K	2.20μ	0.085	0.114
L	3.50μ	0.045	0.06
M	5.00μ	0.033	—
N	10.0μ	0.013	—

Note that the vdH curve #15 used by Johnson (which is the same as #14) has $R=A_V/E(B-V) = 3.05$. The CCM curve I've computed here is with their standard value of 3.1. The two curves are nearly identical in the optical but diverge in the IR, with CCM giving slightly higher values of the extinction relative to V. To answer the question I posed, the biggest difference is at K, and is of order 0.039 mag per A_V=1.0. Not very much, but it could be important, especially when observing near the Galactic Center. With $A_V \sim 30$ magnitudes to the GC, the predicted CCM reddening at K is more than a magnitude larger than the vdH.