Solution Set for Ay192 Problem Set 1 1999

Problem 1. (a) This is pretty straightforwardly the integration of the Airy function over a circular aperture. You want to do the integral

$$\text{Airy Fraction(r)} = \frac{\int_0^r 2\pi r A(r/\pi) dr}{\int_0^\infty 2\pi r A(r/\pi) dr} \quad ,$$

where

$$A(r/\pi) = \frac{4[J_1(r) - tJ_1(tr)]^2}{[r(1-t^2)]^2}$$

and $J_1(r)$ is the 1st order Bessel function of the first kind, and t is the ratio of the central obscuration (the secondary) to the aperture (the primary). — See the attached plots. Note that the Airy function for the real 6.5-m is the dark line, and that for a 6.5-m w/o central obstruction is the lighter line.

For a 6.5-m telescope with a 1.2-m secondary (18.5% central obstruction), At 5500Å, the 50% *diameter* is 0.019," and the 95% diameter is 0.143." The diameter of the aperture is what is relevant and what is usually quoted. At 20μ, the 50% diameter is 0.7," and the 95% diameter is $\sim$ 6.0" Aren't diffraction limited 6.5-meter telescopes wonderful!

(b) This is a bit trickier. If you calculate the two relevant functions, the Airy function and the Gaussian, you find that at small radii, and for the seeing diameters given, the Gaussian *always* dominates the profile so the simple answer to the question of what is the 50% or 95% encircled energy diameter is that for the Gaussian. The Airy function looks like a delta function to the Gaussian so the convolution looks essentially like the Gaussian. This is *not* true at wavelengths like 10μ, where the Airy function is as broad as the Gaussian, but I didn't ask you that. In this case the 50% and 95% diameters are 1.0" and 2.0," which you can also just get from the encircled energy function for a Gaussian,

$$C(r) = 1 - e^{-(r/r_c)^2} \quad .$$

The atmosphere **!!%$%!!!##.

(c) The ratio of the Airy functions for a clear 6.5-m aperture and for a 6.5-m with a large secondary can be seen in the attached Log Airy and encircled energy plots. Basically, the presence of a central obstruction widens and softens the diffraction pattern, throwing light into outer odd diffraction rings.

Problem 2. To compute the DQE, first compute the input S/N, which is the same for both detectors operated in these modes.

$$S/N_{in} = \frac{nt}{(nt + 2st)^{1/2}}$$

where n is the count rate from the object, s is the count rate from the sky, and t is the integration time. The factor of two comes from the fact that the noise contribution from the sky comes from both apertures (think about that!).

The count rates are easily computed by finding that 0th magnitude corresponds to 950 photons/second/cm^2/Å. The 48-inch telescope, minus central obstruction = 10940 cm^2, and the bandpass is either 1500Å, or 10Å
for the second part. One should add in thr throughput losses in the telescope (0.89^2 for the two aluminum surfaces) and the atmospheric extinction, which I'll take as 0.14 magnitude (or about a 15% loss). Doing the appropriate algebra,

$$n = 6.61\text{photons/sec} \quad \text{or } 0.044\text{photons/sec for spectroscopy}$$

$$s_p = 769\text{photons/sec} \quad \text{or } 5.13\text{photons/sec} \quad \text{"} \quad \text{"}$$

$$s_{ccd} = 1092\text{photons/sec} \quad \text{or } 7.28\text{photons/sec} \quad \text{"} \quad \text{"}$$

input to the detector (photmeter has a 5" diameter aperture, but the CCD ends up with a 8x8 pixel square in the first example to cover the 5" round aperture). These numbers neglect the transmission losses in the filters, lenses, etc., but that's ok for now.

The output signal to noise, as explained in class, depends on the detector noise sources and the relative quantum efficiency η.

$$S/N_{out,PMT} = \frac{\eta nt}{(\eta nt + 2\eta st + 2dt)^{1/2}}$$

where $d = 0.05$ /s is the dark count rate of the PMT's.

$$S/N_{out,CCD} = \frac{\eta nt}{(\eta nt + 2\eta st + 2r^2)^{1/2}}$$

where $r^2 = 100$ /pixel times the number of pixels required (64). To beat this, the usual trick is to bin or use point spread function fitting with array detectors rather than just circular apertures.

The DQE is the square of the ratio of output to input S/N. You should/could write a little computer program to evaluate these functions for different input parameters. The output of my program is attached.

The DQE for the PMT is constant with time at either 39.99% or 39.05% for the spectroscopic and photometric cases, respectively. The DQE of the PMT will depend on the ratio of n+s to d; the DQE of the CCD increases with integration time asymptotically approaching its RQE. For this example, it does not "beat" out the PMT until the integration time is greater than 1000 seconds in the spectroscopic case with the large pixels. For the photometry, as described, the PMT reaches S/N of 100 in a little under 10^6 seconds (!) whereas the CCD does it in about 6×10^5 seconds. Clearly you do not want to photometer a 23rd magnitude object on a 1.2-m if you require S/N = 100. S/N = 10 is actually reached by the CCD in about 12000 seconds, which is more reasonable.

Note also, and this is *very important*, for spectroscopy, even the CCD does not give a S/N observation of 100 until the integration time goes well over a year of integration! You will understand exactly how bad this is later when we take all of the limiting factors into account.

The attached plots are my solution.

In the real world, we would both bin the CCD and use point spread function routines, which would decrease the effective read noise substantially. Also, for spectroscopy, we would use a slit, which effectively limits ones ability to do spectrophotometry. but which improves the S/N immeasurably compared to use of a large focal plane aperture.

Problem 3. This problem is just the application of the lensmaker's equation. You need to assume something about the index of refraction of the glass to do the problem. And the easiest way to test your assumption is to measure the focal length of the lens!

If we assume that you have one of the standard crown glasses with an index of refraction of about 1.5, then our estimate of the lens focal length is just

$$\frac{1}{f} \;=\; (\eta-1)(\frac{1}{R_1}-\frac{1}{R_2})$$

$$= (1.5-1.0)\times(0.1-0.05) \;=\; 0.025$$

$$\Rightarrow \quad f = 40 \text{ cm}$$