AY 145: HW 8

Due Wednesday, April 20th

1. **Minkowski Metric**

   Consider the Minkowski metric for flat spacetime,

   $$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2. \tag{1}$$

   (a) The time between two events that occur at the same location is the proper time τ, where

   $$\mathrm{d}\tau \equiv \frac{\mathrm{d}s}{c}. \tag{2}$$

   Using the Minkowski metric, show that the time measured between two events by an observer at rest, Δt_{rest}, is related to the time measured by an observer moving with speed v, $\Delta t_{\mathrm{moving}}$, by the relation

   $$\Delta t_{\mathrm{moving}} = \frac{\Delta t_{\mathrm{rest}}}{\sqrt{1 - v^2/c^2}}. \tag{3}$$

   The factor of $\gamma \equiv 1/\sqrt{1 - v^2/c^2}$ is called the Lorentz factor.

   (b) Show that the Minkowski metric written is spherical coordinates is

   $$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2. \tag{4}$$

   Is spacetime still flat when the metric is written in this way?

2. **Curvature of Space**

   The properties of geometric objects are different on curved surfaces than in flat space. Here we consider the properties of triangles and circles in a curved space.

   (a) In a positively curved space (e.g. a sphere) the interior angles in a triangle sum to $> 180°$, while in a negatively curved space, the angles sum to $< 180°$. Draw a sphere. On the sphere's surface, draw a triangle whose interior angles sum to 270° (yes, that can be a triangle with three right angles). Can you draw a triangle whose interior angles have a larger sum? Draw a triangle whose angles have the smallest possible sum.

   (b) In flat space the relationship between the radius, r and the circumference, C, of a circle is $C = 2\pi r$. This is not true in curved space. On the surface of a sphere of radius R, find the expression for the relationship between a circle's radius and circumference. Draw a circle on a sphere such that the ratio of the circumference to the radius is exactly 4 ($C = 4r$).

   (c) Space does not have to be a plane to be flat. Can you think of non-planar surfaces that are flat? Is the surface of a cone a flat space? What about that of a cylinder? Now, connect the ends of the cylinder to form a donut. Is the surface of the donut a flat space?