Ay 145 — Problem Set 3

Due Wednesday, March 2, 2005

Problem 1. The Rayleigh-Jean's Limit and Opacity

a.) What is the condition under which the Rayleigh-Jeans approximation is valid?

b.) At a radio telescope a researcher observes at $\nu = 23$ GHz ($\lambda = 1.3cm$). For what temperature, T, does $h\nu = kT$? The telescope will measure a temperature. That temperature will include contributions from the source, the atmosphere and the noise of his telescope. How large must the measured temperature be in order for the Rayleigh-Jeans approximation to $B_\nu(T)$ to be valid?

c.) Start with the Planck Law and the condition in part (a.) to explicitly derive the Rayleigh-Jeans limit of $B_\nu(T)$, showing that $B_\nu(T) \propto T$.

d.) Recall the equation of radiative transfer. If we make two simplifying assumptions, first that there is no background source and second that the source function is homogeneous (S_ν is not a function of τ_ν) and thermal ($S_\nu \rightarrow B_\nu$) then the sky emission temperature (T_{sky}) that the radio telescope sees will be the emission and self absorption of the atmosphere alone. Show that the Rayleigh-Jeans approximation can be used to rewrite the simplified equation of radiative transport as:

$$T_{sky} = T_{atm}\left(1 - e^{-\tau_0}\right) \tag{1}$$

where T_{atm} is the temperature of the atmosphere (which we assume is constant) and τ_0 is the optical depth through the entire atmosphere.

e.) The above expression for T_{sky} is valid only when looking directly overhead (i.e. $z =$ zenith angle $= 0^o$). Show that when looking at some other z, the correct expression for T_{sky} is:

$$T_{sky} = T_{atm}\left(1 - e^{-\tau_0 sec(z)}\right) \tag{2}$$

f.) Simplify the expression for T_{sky} for an optically thin ($\tau_0 << 1$) atmosphere.

g.) The radio telescope will measure $T_{system} = T_{sky} + T_{noise}$ where T_{noise} is the additive noise introduced by the telescope itself. If we assume that $T_{atm} = 273$ K, make a fit to the data in Table 1 to determine τ_0.

Hint: Plot T_{sys} vs. $\sec(z)$.

z [deg]	T_{sys} [K]
9	107.5
29	111.3
30	110.0
45	111.3
56	113.8
58	116.3
63	120.0
67	125.0
72	132.5
75	138.8
80	152.5

Table 1: The measured system temperature as a function of zenith angle, z. Note that there is no contribution to the sky temperature other than the atmosphere itself (i.e. the telescope is not pointing at a bright radio object