The Acoustic Peak Primer

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1 A Physical View of the Acoustic Phenomenon

To get a simple physical understanding of what the acoustic oscillations are, it may be helpful to change the perspective. Normally, the common way of presenting the phenomenon has been in terms of standing waves where the analysis is done in Fourier space. But the baryon-photon fluid really is just carrying sound waves, and the dispersion relation is even pretty linear. So let's instead think of things in terms of traveling waves in real space.

I'm going to start with the technical description, explaining the analogy to all of the typical effects that we talk about in Fourier space. Then I'll give the layman's version of what we're seeing the large-scale structure correlation function.

The technical foundation for doing the early universe physics in real space was written down in papers by Sergei Bashinsky and Edmund Bertschinger (PRL, 87, 1301, 2001 and PRD, 65, 3008, 2002). The acoustic phenomenon itself was first predicted by Jim Peebles and J. Yu (Astrophysical Journal, 162, 815, 1970) and Rasheed Sunyaev and Yakov Zel'dovich (Astrophysics and Space Science, 7, 3, 1970).

First, what happens without baryons? At $z \gg 1000$, the dark matter and photons have small perturbations, due to the initial perturbations in the curvature of space. The spectrum of these is very blue, $P \propto k$, so the fluctuations are larger on small scales. As time goes on, the photons (and neutrinos) free stream, thereby smoothing out their perturbations on scales smaller than the light-travel time in the age of the universe. On larger scales, the dark matter perturbations continue to grow; on smaller scales, the dark matter perturbations stall (but don't decrease). [1] By the time of matter-radiation equality, the small-scale fluctuations have been softened relative to the large-scale ones, giving the usual CDM power spectrum. After matter-radiation equality, the photons/neutrinos aren't dynamically important, and the dark matter perturbations grow equally on all scales.

If one thinks of the dark matter picture in comoving coordinates (as one should throughout), one has a noise pattern at early times. Over time, the pattern stays the same, but gets a little fuzzier over time while the amplitude grows. Of course, all of these fluctuations are tiny variations!

So what happens with small amounts of baryons? The initial perturbation has created pressure imbalances in the baryon-photon fluid, and those imbalances want to smooth themselves out. This occurs by sound waves, as usual. Thinking about a single overdense region in the universe, a spherical sound wave is launched at very early times. This sound wave propagates until recombination, by which time it has reached a radius equal to the sound horizon[2], 150 Mpc in the standard cosmology.

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[1] Unfortunately, while it makes a good story and I think carries some intuition, the details on super-horizon scales depend on one's choice of GR gauge.

[2] Recall that the sound horizon is defined as the comoving distance that a sound wave can travel between the Big Bang and recombination. This includes the amount of time available, the expansion corrections for the early-time propagation, and the slowing of the speed of sound below $c/\sqrt{3}$ at late times due to the increasing inertia of the

At that point, the pressure is released, the sound speed in the baryons goes essentially to zero, and the wave is frozen in. The baryon (and photon) perturbation that was at the location of the initial dark matter perturbation has been carried out to a spherical shell. The subsequent evolution is based on the sum of the dark matter and baryon densities, so we have a large perturbation at the center with a small perturbation in a spherical shell around it.

Hence, in the correlation function, there is a boost at the distance of the sound horizon, because the center of the sphere is correlated with the shell!

Of course, this is the situation for a perturbation at a single point. In fact, the initial perturbations have excited spherical waves originating from all points, and the seed field itself has long-range correlations. This cacophony of different sound waves does tend to obscure the preferred scale in the correlations, but one still gets a signal because the perturbations are considerably large on small scales, so the shells have non-negligible weight compared to the intrinsic large-scale correlations in the dark matter.

If the acoustic effect in large-scale structure is simply seeing these shells, what is happening in the CMB? The differences in this case are: 1) that we are not seeing the three-dimensional picture, but instead only a two-dimensional cross-section of it, and 2) that we are seeing the photon perturbations directly, not the dark matter (save by gravitational redshifts, aka Sachs-Wolfe, on large scales). Instead of a central perturbation with a faint spherical echo, all we see is the spherical shell. But we only see slices through it. The slices themselves are moderately thick (tens of Mpc), because the optical depth doesn't switch from infinity to zero instantly. When those thick slices intersect the limb of a sphere, we see a patch on the sky with a size related to the size of the sphere. This is less rare than you'd think — imagine slicing the sphere into slabs a fair fraction of the radius — and so the CMB maps are dominated by this characteristic scale.

What has become of all of the other physical effects discussed in the CMB reviews? First, Silk damping. This is caused by the diffusion of the photons relative to the baryons. This corresponds to a slight broadening of the sound wave as it propagates outwards. By recombination, even a point seed has created a shell tens of Mpc thick (compared to 150 Mpc in radius). That means that structure in the initial perturbation field smaller than this eventual thickness don't imprint on the baryon distribution in the acoustic shells.

Next, the driving and inertial terms. The small-wavelength Fourier modes are increased in amplitude by the decay of the gravitational potential caused by the free-streaming of the photons and the stall of the dark matter growth. The timing of this decay of the force is such that it boosts the oscillations. In the Fourier picture, the result is an increasing envelope for the oscillations in the power spectrum as one goes to small scales. Meanwhile, the non-zero mass of the baryons causes an overshoot in the oscillations, causing the odd-even asymmetry of the acoustic peaks in the CMB.

My intuition, which I haven't proven in detail but is surely treated in Bashinsky and Bertschinger, is that both of these effects simply control the exact waveform of the spherical sound wave that is being launched. Both effects are imprinted at early times, before the wave has had a chance to propagate away from its seed perturbation. The late-time wave propagation is simply a sound wave with diffusion.

Importantly, the waveform matters for the CMB because one is primarily looking at the shells

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baryons relative to the photon pressure. In formula, $s = \int_0^{\rm rec} dt\; c_s(t)(1+z)$, where c_s is the sound speed and $z(t)$ and $t_{\rm rec}$ depend on the matter and radiation densities.

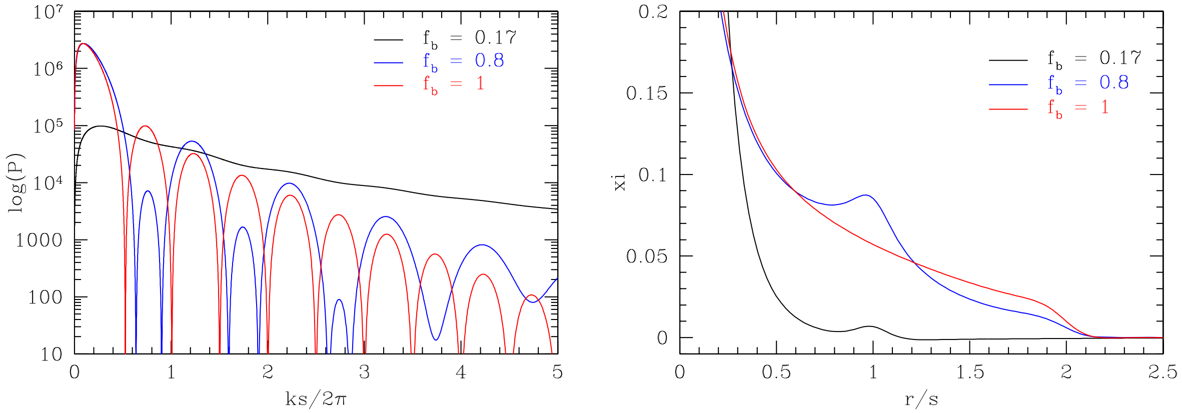

Figure 1: The linear-regime power spectrum and correlation functions for models with 3 different baryon fractions, shown with the physical scale tied to the sound horizon in each model. For low baryon fraction, the acoustic effect is a simple peak at $r = s$; this is due to the correlation of the dark matter correlation with the spherical sound wave it drives into the baryon. For a pure baryon model, no perturbation remains at the center of the shell, and so the correlations are driven by all pairs along the sphere, which means that there is a drop at the diameter of the sphere ($r = 2s$). In Fourier space what is happening is that in the low baryon fraction model, the baryon rarefactions downweight the CDM compressions so as to give a trough in the power, but in the high baryon fraction, both the baryon compressions and rarefactions both give peaks in power. This doubles the frequency of the oscillations, but also moves the model much further from a pure sinusoid.

being cut by planes at their edges, so the variations in the waveform get imprinted into the exact angular extent of the hot and cold spots. However, for large-scale structure, this is a second-order effect, namely the exact shape of the peak in the correlation function. The fact that there *is* a peak is simply the propagation of this large spherical shell away from the dark matter overdensities. Of course, at late times, non-linear structure formation on small scales blurs out the peak and obscures our view of this waveform at $z = 0.3$.

Finally, the dark matter perturbation that seeded the wave initially itself grows a factor of a few while the wave is speeding outwards, so the correlations in the matter are weaker than one might have guessed from a 17% baryon fraction.

1.1 High baryon fractions

It is interesting to consider what happens in a baryonic model without dark matter. One still has the initial perturbations launching a spherical wave (albeit with a slightly different waveform). But after recombination, all of the matter perturbation is in this shell; there is no perturbation left at the center! So one does *not* get a sharp peak in the correlation function — that was the consequence of correlating the center and the sphere, thereby putting all of the weight into one separation. There is still a correlation, namely the auto-correlation of two points on a sphere, which does have a preferred scale but not a sharp peak. Indeed, the correlations have a sharp edge at *twice* the sound horizon, corresponding to diametrical opposing points on the sphere.

In the usual Fourier space picture, what has happened is that the baryon transfer function oscillates between positive and negative peaks (that is, the fluid goes through compressions and rarefactions). When added to the purely positive CDM transfer function, this gives a series of peaks and troughs, which remain so when squared to give the power spectrum. However, for a pure baryon model, both the rarefactions and compressions give peaks in the power spectrum. In other words, instead of having a sinusoidal power spectrum, one has a *squared-sinusoidal* power spectrum. The former has a nice peaked Fourier transform; the latter has broader support and twice as high a characteristic frequency.

Figure 1 illustrates this point. Eisenstein & Hu (1998, ApJ, 496, 605) has much more discussion of the pure baryonic case and the mixture with CDM.

2 The Essential Version

Viewed in real space, the essential point of what we've detected is very simple. Fluctuations in the dark matter dominate the formation of large-scale structure, but in the plasma era of the early universe, the dark matter fluctuations are accompanied by initial overdensities in the plasma, which then propagate outwards as spherical sound waves. The plasma age ends abruptly, freezing in those sound waves and leaving a shell of material at a radius of what today is 500 million light-years. The excess of gas now found in those shells make those regions slightly more dense and hence more likely to form galaxies. There is hence a correlation at late times between galaxies in the central regions that caused the shell and those in the shell. Of course, the universe is composed of many shells and centers, so that one can't literally see the shells, but we detect the effect by finding that galaxies are slightly more likely to be separated by 500 million light-years than by 400 or 600.

The complications like driving, inertia, and damping are irrelevant for this basic picture. And indeed, I think the basics are a lot easier to visualize in 3-d with the galaxy survey than in 2-d in the CMB!

The launching of the sound waves is very similar to dropping a rock in a pond and seeing the circular wave come off (obviously that a gravity wave, not a compressional wave, but I'm focusing on the geometry). The difference here is that the area where the "rock" entered is still the most likely region to form galaxies; the spherical shell that it produced is only carrying 5% of the mass.

Hopefully, this demystifies the effect: we're seeing the imprint of spherical sound waves launched from the sites of dark matter overdensities in the early universe. But also I hope it makes it more clear as to why this effect is so robust: the propagation of sound in the baryon-photon plasma is very simple, and all we're doing is measuring how far it got.