Astronomy 208 v. Y2K "Lecture 1" 9/19/00

Handouts

- class list form
- syllabus
- survey
- Barnard article

Introductions

Thurs 11:30 slot
Tues → Fri?

Viewgraphs "Physics of ISM"

www Web URL: cfa-www.harvard.edu/~agoodman/astro208

What's the Course About?

→ Physics of the ISM v.g. ←

What's out there? Where is it? What's it Doing? How do we find out?

FORMAT: Relies heavily on STUDENT preparation

~ • Tuesdays "lectures"
~ • Thursdays 50/50 lecture/discussion

[THIS Thursday Barnard Article – Need Volunteer "discussant"
Xtra points for volunteering for this 1st one!]

Future articles & schedule, including discussants will be on-line through web site:
Discussant provides html summary w/links by 42hrs in advance

AY208 9/19/00

More Logistical Notes

→ Auditors may attend journal discussions if they read articles.

www → Review p. 1 of Syllabus

"Texts"

really none – best refs on-line

www through web site — sug source amazon.com ($ Wolbach)

www really required readings are articles on-line through web site

Problem Sets

- every 1.5-2 weeks
- working together OK but please don't "copy"
- Some computer work / simulation
- will rely on Journal articles for some problems

Exams: Final (take-home) req. of CORE course

Grade: 40% final 35% psets 25% journal site/presentation

Office Hours: By appt AG

HJC ← prob set?

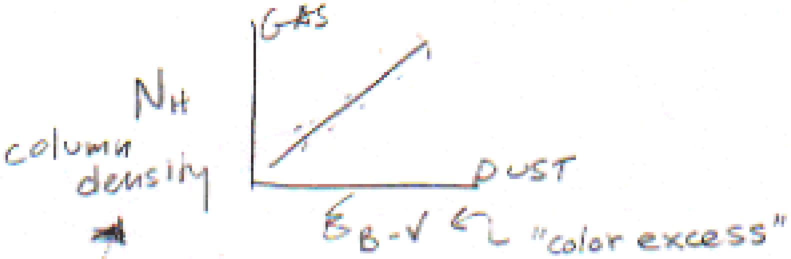
(1.1) "Earliest Observations"

How do we know there is an ISM? (Other than existence of AY 208)

in MW & other galaxies: extinction & reddening → dust (see Barnard article)

"stationary lines" from binary ☆'s
lines from ☆'s Doppler shift due to ☆'s motion
interstellar ("stationary") lines don't shift w/orbit

gas & dust correlated (see Fig 1.1 of Spitzer)



from stellar continuum

from strength of absorption lines

Even the "Dense" ISM is a Low Density Environment

$T = 0° = 273K$

$P = 1 atm$

1 mole = 22.4 ℓ for any low-density gas

$N_A = 6.02 \times 10^{23}$ (Avogadro's #) = #molecules/mole

6.02×10^{23} air molecules in 22.4 ℓ

1 ℓ = 1000 mL = 1000 cm^{-3}

~ Density in this room ~ $\boxed{6 \times 10^{20} \text{ ptcls/cc}}$

1 Torr = 1 . mm of mercury

1 atm = 760 torr

vacuum @ 10^{-8} torr is $\frac{10^{-8}}{760} = 1{,}3 \times 10^{-11}$ times less dense than air @ 0°c

$6 \times 10^{20} \times 1{,}3 \times 10^{-11} = 8 \times 10^{9} \approx \boxed{10^{10} \frac{\text{ptcls}}{\text{cc}}}$

best vacuum on Earth

208/

④

(1.2) The Modern View

a, b, c Composition, Extent, Temp Structure

[Keep in Mind : Density of ★'s in the MW ≈ $0.125\ M_\odot/pc^3$]

ⓐ Composition of (Milky Way) ISM : (constituents)

see B&O p. 346

gas: 60% H , 30% He by mass

recall $m_{He} = 4 m_H$ so if $m_{TOT} = 90$

60 H $\frac{30\,He}{4}$

~80% H , 10% He by # 60 H 7.5 He } 8 x as much H by #

other ~10% trace elements in ~☉ abundances (really < 10%)

Gas density in GALAXY ⇒ $0.025\ M_\odot/pc^3 = \frac{1.85 \times 10^{-24}\ g/cc}{1\ amu \approx 1.67\times10^{-24}} \approx \boxed{n = 1\ ptcl/cc}$

How is that density estimated?

M_{TOT} (MW) ~ $1.4 \times 10^{11}\ M_\odot$ (from rotn curve)

D ~ 40kpc (estimate from HI & CO)

scale ht ~ 140pc (from HI maps)

→ Volume ≈ 2×10^{67} cm³

⇒ $\rho_{with ★'s} \approx 1.4 \times 10^{-23}$ $\frac{M_{gas}}{M_{★'s}} \sim 0.2$ ← leftover must be gas!

⇒ $\rho_{gas} \approx 2.8 \times 10^{-24}\ g/cc$ ⇒ n = 1 or 2 ptcl/cc

Atomic? Molecular? Ionized?

At n = 10 to 50 cm⁻³ → molecular needs dust as catalyst

(will discuss ionization & recombination, association & dissociation)

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Back to Constituents

Dust

(molecules too big to name up to rocks too small to name)

$$\boxed{N(a) \propto a^{-3.5}}$$

a = size

many very small particles

e.g. 0.2 μm

Density in the Galaxy $\sim 0.002\, M_\odot/pc^3$

recall gas $\sim 0.025\, M_\odot/pc^3$

so $\boxed{\rho_{dust}/\rho_{gas} \approx 0.1}$

Cosmic Rays

(charged) (high-energy {protons, nuclei, antiprotons, e^-, e^+})

$$\boxed{0.5\ eV/cm^3}$$

mass equiv $\hat{=}\ 9 \times 10^{-34}\ g\ cm^{-3}$

Note: $1 eV = 1.6 \times 10^{-12}$ erg

$E = mc^2 \qquad m = E/c^2$

$0.5 eV = 8 \times 10^{-13}$ erg $\Rightarrow \frac{8 \times 10^{-13}}{(3 \times 10^{10})^2} = 8.9 \times 10^{-34}$ grams

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Magnetic Fields

$\sim 10^{-6}$ gauss $\sim 1\, \mu G \Rightarrow \boxed{0.2\, eV/cm^3}$ actually a bit more?

($\sim$enough to confine cosmic rays)

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Constituents Cont'd

Starlight $0.5\ eV/cm^3$

neglected contributors rotational energy from differential rot'n of Galaxy + turbulent energy w/in 'clouds

1.2 b, c extent & temperature structure
(see Web Pages)

+ Table 18.1 of B & D

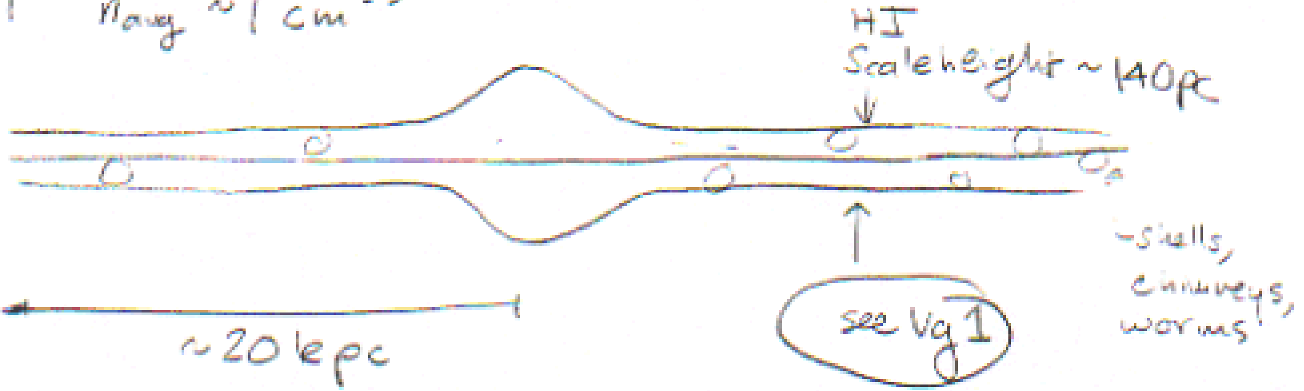
9/21/00

AY208 - Meeting #2

§1.2 Continued

Last time: Milky Way quick tour for an example

Recall $n_{avg} \sim 1\ cm^{-3}$



Neutral Material is mostly HI

$HI \rightarrow H_2$ in presence of dust in dense, cold regions $n \gtrsim 10$ to $50\ cm^{-3}$ T 10's to 100's of K

see Table 1

gas & dust correlated & $\rho_{gas}/\rho_{dust} \sim 0.1$ $\frac{\rho_{gas}}{\rho_{*'s}} \sim 0.2$

note "ρ_{dust}" means of dust per unit ISM volume, not per dust grain volume

~ Discuss Table 1 ~

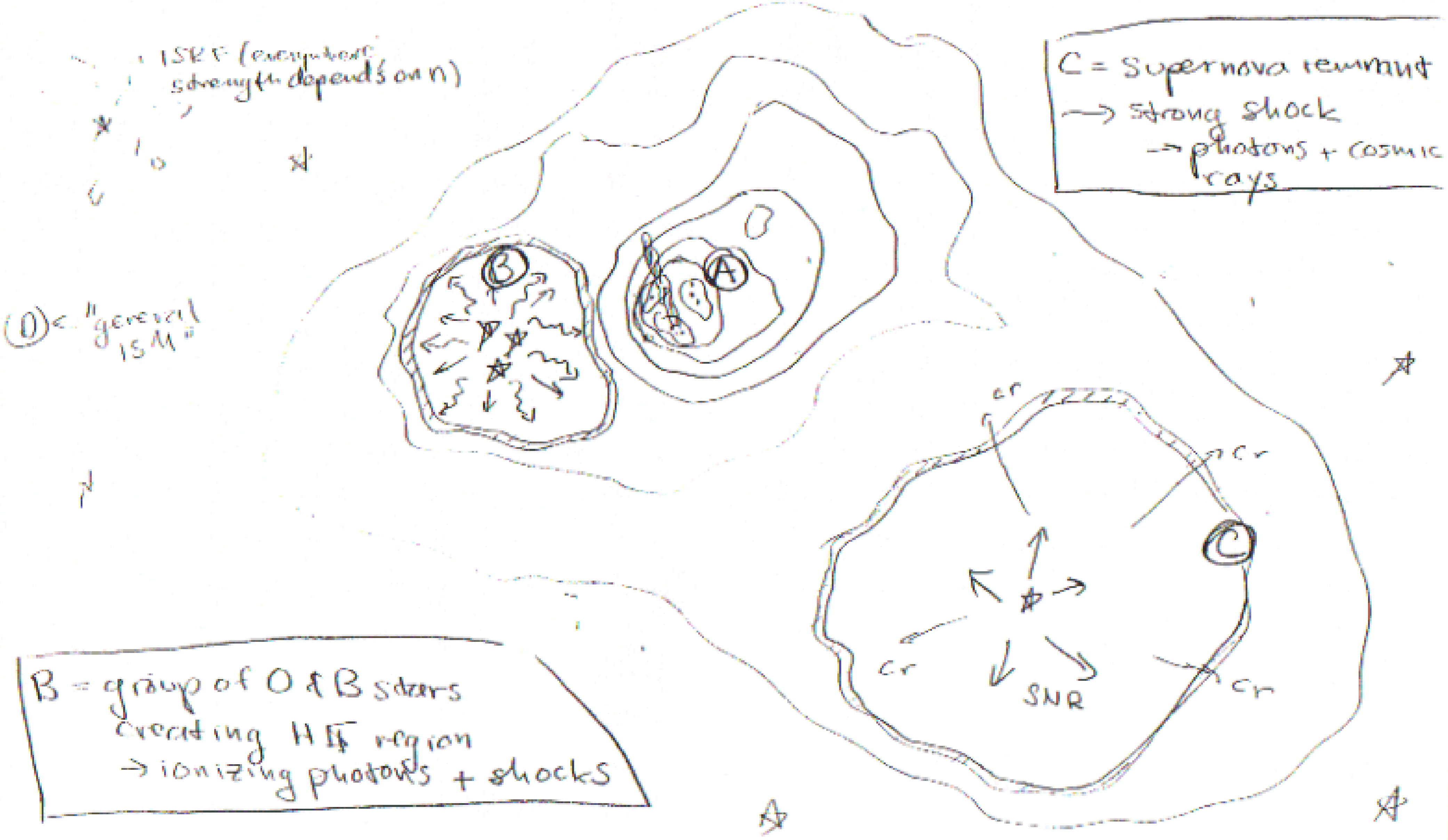
~1st 4 lines

Draw Figure on Next Page

→ Viewgraphs ←

Temperature & Ionization

A = very dense gas, possible site of ★-formation
possible outflows from young stars (shocks, heating)



C = Supernova remnant
→ strong shock
→ photons + cosmic rays

B = group of O & B stars
creating H II region
→ ionizing photons + shocks

(AG)

Meeting 3

Velocities

← see also handout from Mtg #2

Galactic scale – spiral density wave pattern speed

e.g. Andromeda $\Omega_p = 18$ km/s/kpc (e.g. 180 km/s @ 10kpc

1km/s = 1pc in 10^6 yr

w/in ISM

"thermal" (isothermal sound) speed $= \sqrt{\frac{kT}{\mu}}$ speed of sonic disturbances

	$T =$	10	$\boxed{100}$	1000	10,000 K
for $\mu = m_H$	$c_s =$	0.3	$\boxed{1}$	3	10 km/s
			remember		
$\mu = m_{avg}$ (molecular)	$c_s =$	0.2	0.7	2	7

Alfvén speed speed of magnetic disturbances

$$V_A = \frac{B}{\sqrt{4\pi\rho}}$$

note if $B \propto \rho^{1/2}$ then V_A = constant

	atomic	molecular	molecular
	$B = 1\mu G$, $n = 1\text{ cm}^{-3}$	$30\mu G$, 10^4 cm^{-3}	1mG, 10^7cm^{-3}
$V_A =$	2 km/s	0.4 km/s	0.5 km/s

↑ at least twice as big as c_s for 10 K

(12d)

General Notes on Ionization / Dissociation ($\& T, n$ $F(\nu)$)

- generally easier (req lower $\mathcal{E}$ $h\nu$) to dissociate a molecule than to ionize something

- the lower the electronic state you're trying to ionize, the more $\mathcal{E}$ (shorter λ) you need

- $\mathcal{E}$ ionization of H from ground state $= 13.6\,eV = \frac{hc}{912\text{Å}}$
  = "Lyman Limit"

- how is (ave) ionization state (and density & temperature) measured?

(More Next week – Intro for now –)

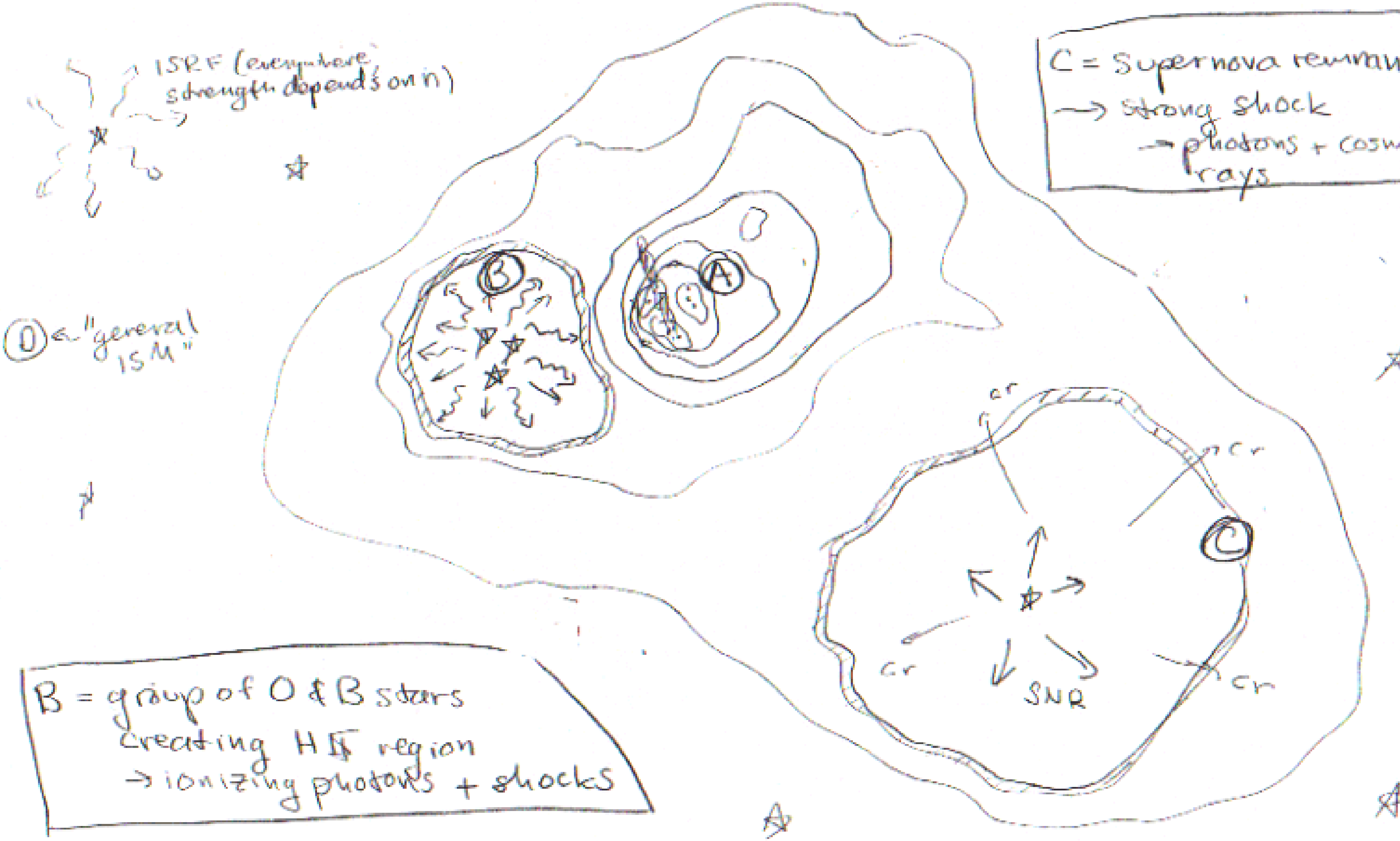
- Q. Mech tells us what ratios of certain lines should be for certain n, T, $F(\nu)$ conditions

line ratios → n, T, n_i & sometimes $F(\nu)$

- continuum "SED" (spectral energy distribution) depends on n, T

(r_c^2/d) Temperature & Ionization (from last time)

A = very dense gas, possible site of ★-formation
possible outflows from young stars (shocks, heating)



C = Supernova remnant
→ strong shock
→ photons + cosmic rays

B = group of O & B stars
creating H II region
→ ionizing photons + shocks

In Ⓐ Very Dense Region

$T \sim 10-50\,K$ {line ratios, etc}

- gas mostly molecular (T low, n high, no dissoc $h\nu$ or shocks)
- not much photoionization due to high extinction (but what if extinction "patchy")
- cosmic rays can get in and

$$\boxed{\begin{array}{l} n_i \propto n_H^{1/2} \\ \text{so } x_i = \dfrac{n_i}{n_H + n_i} \underset{\sim}{\propto} n_H^{-1/2} \end{array}}$$

but n_i very small

$x_e \sim 10^{-6}$ to 10^{-7} (presumed $= x_i$)

other considerations:

- Shocks due to impinging H II region (raise T, x_i?, n, chemistry)
- shocks due to embedded young stars w/outflows (localized changes in T, n, x_i, chemistry)
- time-evolution (what happens to chemistry?)

(See "Molecular Clouds" chart)

Note Idea of Photoionization-Regulated Star-Formation (McKee)

How long does a molecular cloud last?

(see Ch)

“P^hms^roton D^issociation^ominated Region”

In Ⓑ H II Region $\bar{7{,}000} \lesssim T_k \lesssim 10{,}000\,K$ } line ratios, etc.

- gas primarily ionized, due to photons (← but some neutral left) shortward of Lyman Limit $13.6\,eV = h\left(\frac{c}{912 Å}\right)$ produced by O ★'s (& some B ★'s)

notes ① elements other than H have different ionization energy so will ionize more or less easily, (depending also on how ionized they already were... these elements have $>1\,e^-$!!)

② evidence that H II regions are clumpy (in many cases)
rms value of n_e ⊛note from continuum radiation ← (avg'd over vol) is only ~ 1/6 of what's derived from line ratios ⟶ radiation is not produced in a filled volume... note ≠ what is "filling factor" of ___ ← in this case, ionized gas

VERY IMPORTANT CONCEPT → $$\boxed{\text{filling factor} = \frac{\text{filled volume}}{\text{total volume}}}$$

③ dust present in H II regions (evidence scattered ★ light)
Smaller grains may be destroyed... study thermal emission SED

④ much free-free radio emission, synchrotron, & recomb-line (e.g. H76α)
$\bullet e^- \rightarrow \bullet$ ion bremsstrahlung

⑤ chemistry very dependent on time, n, T, flux

*For ref
n_e $1.6 \times 10^4\,cm^{-3}$ near Trapezium 2.6×10^2 3pc away

In Ⓒ SNR

- gas can be ionized in shocks by collisions (high v required, to produce high-energy collisions, T)

~~e.g.~~ if $v > 1000 \text{ km s}^{-1}$ $T > 10^6 K$

atom—electron collisions will

- ionize H & He,
- produce X-rays, ← observed ✓
- produce highly ionized atoms of elements heavier than H, He ← observed in abs lines.

$$\sqrt{\frac{kT}{\mu}} \quad \sqrt{\frac{1.38\times10^{-10}}{1.67\times10^{-24}}} \sim \sqrt{10^{8}} \sim 10^{4}$$

→ 100 km/s thermally

- gas is also excited (e.g. "shocked H_2" (vibrational) emission) and dissociated by shocks

In Ⓓ

UV photons from ISRF produce the "mean ionization"

best measure is n_e from Pulsar DM

Role of B-fields depends critically on $\underline{x_i}$, n (B has no effect on neutrals, they need to collide w/ ions ∴ need to know |B|, n, x_i)

Meeting #4 of Astronomy 208

On syllabus, we've now covered 1.1, and 1.2 a-g

let me just mention (h) "Time Scales & Stability"
e.g. "dynamical time" "crossing time" "free-fall time" "cooling time" (explain)

Always an important consideration:
what happens to some entity as a function of time

→SEE also B & D Chapter 22←

good 1st guess: Virial Analysis for Volume w/in "S"

$$(\emptyset =)\ \frac{1}{2}\frac{D^2 I}{Dt^2} = \overset{\text{kinetic}}{2T} + \overset{\text{internal}}{3\Pi} + \mathcal{M} + W + \underbrace{\frac{1}{4\pi}\int_S (\vec{r}\cdot\vec{B})\vec{B}\cdot d\vec{S}}_{\text{magnetic stresses}} - \underbrace{\int_S \left(p + \frac{B^2}{8\pi}\right)\vec{r}\cdot d\vec{S}}_{\text{surface pressure term}}$$

equilibrium

Spitzer 217-218

$I = \int \rho r^2 dV$ = moment of inertia

✓ $T = \frac{1}{2}\int \rho v^2 dV$ = kinetic energy of the fluid (macroscopic) mass motion

$\Pi = \int p\,dV = \frac{2}{3}$ of random K.E. of thermal ptcls + (microscopic) molecular motion
$\frac{1}{3}$ " " " relativistic ptcls

✓ $\mathcal{M} = \frac{1}{8\pi}\int B^2 dV$ = magnetic energy w/in S

✓ $W = -\int \rho\vec{r}\cdot\vec{\nabla}\phi\, dV$ = total grav energy of system, if masses outside S don't contribute to pot'l

✓ = terms used most

often, many terms ignored, e.g. "$2T + W = 0$"
this kind of simple analysis often used to determine how "bound" a system is & predict its future (e.g. collapse, expansion, evaporation)
(specific examples later in course, including instability analyses)

$$\frac{d^2 I}{dt^2} < 0 \quad \text{etc.}$$

? timescales for this

Chandrasekhar & Fermi's 1953 virial theorem: very useful in ISM

$$\begin{array}{ccccccccc} 2[T_m & + & T_K] & + & \Omega & + & \mathfrak{M} & = & 0 \\ \nearrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ T & & \frac{3}{2}\Pi & & W & & \mathfrak{M} & & \frac{D^2 I}{Dt^2} \end{array}$$

(much different notation used in the literature, but its all the same idea)

Note: Virial thm always holds — "inapplicability" is only a problem when important terms are omitted!

2. Kinetic Equilibrium & Radiative Processes

2.1 Thermodynamic Equilibrium

often "Local Thermodynamic Equilibrium" or "LTE" is used as "local" assumption

ISM is not usually really in equilibrium, unlike stars
q in ★'s emission, abs., scatt, coll., (almost) all take place on very short t-scales, in comparison w/any dynamical or evolutionary time scales — in ISM, n is low things are SLOW!

- also, in ISM hot ★'s, cosmic rays, X-ray bkgd, shocks mixed in → non LTE

⟹ level populations in atoms & molecules are not always in their "equilibrium distribution" (for a Maxwellian v-distr.)
(closest in regions where em & abs of $h\nu$ not too important & collisional ex & de-ex rule)

I'll use Spitzer's notation:

$$n_j(X^{(r)}) \quad ①$$

ptcl dens
\# of times ionized
state
element

$HI = H^{(0)}$

Spitzer's definition:
"ETE" equivalent thermodyn. equil
$T = T_{kinetic}$ ← actual, but may not tell whole story
n_e = actual e^- density

for ETE above written n_j^* | absvd value written n_j

and $n_j^* \neq n_j$ since in real ISM, dist. of atoms in dif stages of ionization will be different

Define: $$\boxed{b_j = \frac{n_j}{n_j^*}} \quad ② \quad = \text{ratio of particle density to ptc dens in ETE}$$

{note: b_j close to 1 when collisions dominate ionization & recombination}

$$\boxed{\text{"LTE"} \equiv b_j = 1 \text{ for bound levels}}$$

Relative popln of levels for the same atom or ion given by the Boltzmann eqn:

$$\boxed{\frac{n_j^*(X^{(r)})}{n_k^*(X^{(r)})} = \frac{g_{rj}}{g_{rk}} e^{-(E_{rj}-E_{rk})/kT}} \quad ③$$

E_{rj} = energy & g_{rj} = stat. wt. of level j, ionization r

So, using ② & dropping the "r" designation:

$$\boxed{\frac{n_k}{n_j} = \cancel{\frac{b_k}{b_j}} \frac{g_k}{g_j} e^{-h\nu_{jk}/kT}} \quad ④$$

ν_{jk} = freq of radiative trans from k → j
convention $E_k > E_j$
$E_{jk} > 0$

To find the fraction of $X^{(r)}$ atoms which are excited to level j:

Define

$$\boxed{\sum_k n_k^*(X^{(r)}) = n^*(X^{(r)})} \quad ⑤$$

= particle density of r-times ionized X in all states

↓ using Boltzmann ↓

$$\text{Fraction} = \boxed{\frac{n_j^*(X^{(r)})}{n^*(X^{(r)})} = \left(\frac{g_{rj}\, e^{-E_{rj}/kT}}{\sum_k g_{rk}\, e^{-E_{rk}/kT}}\right)} \quad ⑥$$

= fraction of $X^{(r)}$ in level j

Define

$$\boxed{f_r = \text{partition function for ion } X^{(r)} = \sum_k g_{rk}\, e^{-E_{rk}/kT}} \quad ⑦$$

So then ⑥ becomes:

$$\boxed{\frac{n_j^*(X^{(r)})}{n^*(X^{(r)})} = \frac{g_{rj}}{f_r} e^{-E_{rj}/kT}} \quad ⑧$$

gives distribution of atoms over different levels

Astronomy 208 Introduction to Meeting #5

Goal for today (& next time) ...

What distribution of observable photons are produced / absorbed etc. by "reactions" in the ISM, under specific conditions?

Points to Remember. (Very Important)

① → Except in H II regions, we're NOT usually talking about electronic transitions (e.g. Bohr Hydrogen)

② → "Upper level" and "lower level" just refer to two different QM states of an atom or molec.
$E_{upper} > E_{lower}$

③ → "transitions" can be induced by
- photons
- "interactions" w/ cosmic rays
- "collisions" w/ atoms, molec, ions
- " interactions" w/ electrons

④ → "levels" can refer to electronic, rotational, vibrational, spin, or "magnetic" states

⑤ → Need to know:
- Chemical Composition (Atoms, Molec, Ions, Elec, c rays, e^-)
- Photon Bath (Radiation Field)
- Velocity Distribution (from μ & T)

We'll discuss Collisions & Transition Probabilities Today
n-n, n-i A_{ul}, B_{lu}, B_{ul} etc

(2.1b non-equilibrium ~~states~~ distributions e.g. masers – later)

2.2. EXCITATION PROCESSES

note nice calc of t to estab Maxwellian dist

{Much is determined by collisions among various species...
{ion-ion or ion-e⁻ are essentially Coulomb forces (see Spitzer 2.1)

Short-Range Forces amongst neut-ion or neut-neut; inherently $\boxed{QM'l}$

Neutral-Neutral: Very weak interaction until electron clouds overlap ~> behave like "hard spheres", so given $r_{atom} \sim 1$ Å

$$\boxed{\sigma_{nn} \approx \pi(\underset{\text{atom 1}}{r_1^2} + \underset{\text{atom 2}}{r_2^2}) \sim 10^{-15}\ \text{cm}^2} \quad ①$$

What collision rate does that imply?

$$\boxed{\text{m.f.p.} = \ell_c \approx (n_n \sigma_{nn})^{-1} = \frac{10^{15}}{n_H}\ \text{cm}} \quad ②$$

That's about 3×10^{-4} pc

length = [(# density) × (area)]$^{-1}$

∴ In gas at a temperature T, mean atomic velocity given by

$$\boxed{\frac{3}{2} m_n V^2 = kT} \quad ③$$

m_n = mass of a neutral

So

$$\boxed{\frac{1}{\tau_{nn}} \approx \frac{V}{\ell_c} \approx \left(\frac{2kT}{3m_n}\right)^{1/2} n_n \sigma_{nn} = 7 \times 10^{-12}\, n_n\, T^{1/2}\ \text{s}^{-1}} \quad ④$$

$$[\tau = 4.5 \times 10^{3}\, n^{-1}\, T^{-1/2}\ \text{yrs}]$$

So, if $n_n = 1$ and $T = 80$ K $\Rightarrow \boxed{\tau_{nn} = 500 \text{ years}}$!!

(core) $n_n = 10^4$ and $T = 10$ K $\Rightarrow$ 1.7 months

(hot neut ISM) $n_n = 1$ $T = 10^4$ K $\Rightarrow$ 45 years

Remember
3.2×10^7 s/yr

$$\boxed{\text{Density Matters More than T} \quad \frac{1}{\tau} \propto n T^{1/2}}$$

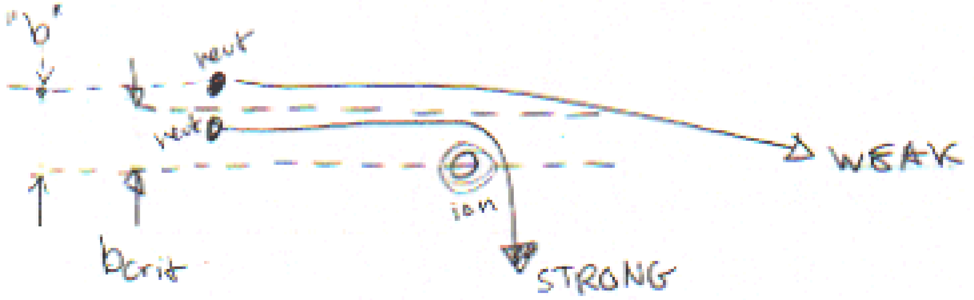
What about $\boxed{\text{Ion - Neutral}}$ Collision Rates?

Neutral is "polarized" by $\vec{E}$ field of Ion so that interaction energy

$$\boxed{U(r) \cong \underset{\substack{\uparrow\\ \vec{E}\text{-fld due}\\ \text{to polarization}}}{\vec{E}} \cdot \vec{P} = \overset{\vec{E}\text{ field of ion}}{\frac{Ze}{r^2}} \Big(\alpha \overset{\vec{E}\text{ field created by}}{\frac{Ze}{r^2}} \Big) = \frac{\alpha Z^2 e^2}{r^4}} \quad ⑤$$

(2U)

α = polarizability of neutral
$\approx a_0^3$ (a_0 = Bohr radius = 0.529Å)



Two types of interactions depending on masses & rel velocity

μ = reduced mass = $\dfrac{m_1 m_2}{m_1 + m_2}$

Impact parameter = b

⑥a

WEAK $\dfrac{\alpha Z^2 e^2}{r^4} \ll \dfrac{\mu v^2}{2}$	STRONG: $\dfrac{\alpha Z^2 e^2}{r^4} \gg \dfrac{\mu v^2}{2}$
interaction < kinetic	interaction energy > kinetic

Boundary corresponds to "critical impact parameter" = b_{crit} given by

$$\boxed{\frac{\alpha Z^2 e^2}{b_{crit}^4} = \frac{\mu v^2}{2}} \quad ⑥b$$

∴ Effective Collision X-sectn, σ_{ni} given by re-arranging 6b

$$\boxed{\sigma_{ni} \cong \pi b_{crit}^2 = \pi Z e \left(\frac{2\alpha}{\mu}\right)^{1/2} \frac{1}{v}} \quad ⑦$$

Since $n_i \neq n_n$ necessarily, we can't just say (eg. 2 ¢ 4)

$$\frac{1}{\tau_{ni}} \approx \frac{v}{\ell_c} = v\, n_{?}\, \sigma_{ni}$$

Instead, we'll leave "n" out ¢ calculate a $\overbrace{\text{rate coefficient}}^{k}$ in $\boxed{cm^3 s^{-1}}$

(8a) $$\boxed{k = \langle \sigma_{in} v \rangle}$$ but ⑦ $\Rightarrow$ $\sigma_{ni} \propto \frac{1}{v}$ so k indep. of v

(8b) $$\boxed{k = \pi z e \left(\frac{2\alpha}{\mu}\right)^{1/2}} \quad \boxed{\approx 2 \times 10^{-9}\ cm^3 s^{-1}}$$ e.g. $C^+ - H$

ion-neutral scattering rate coefficient

for most exotherm ion-neut reaxns

Take "AY209" to find out more! (e.g. calculate 'α' from Q.M.)

⑨ So $$\boxed{\frac{\text{Rate}}{\text{Volume}} = n_i n_n \langle \sigma_{ni} v \rangle \ cm^{-3} s^{-1}}$$

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example say $n_i = n_n = 1$, then $\frac{\text{Rate}}{\text{Vol}} \cong \boxed{2 \times 10^{-9}\ cm^{-3} s^{-1}}$ ion-neut

(15 yrs btwn transitions!)

For n-n, we found $\tau_{nn} = 500yr$ for $n_n = 1$ ¢ $T = 80K$ that corresponds to $6.3 \times 10^{-11}\ s^{-1}$, or $\boxed{6.3 \times 10^{-11} cm^{-3} s^{-1}}$ in a box $1 cm^3$ w/ 1 ptcl.

In this case, ion-neut interactions are more common by a factor of ~30.

FYI Exothermic (chemical) ion-neutral reactions $\rightarrow$ $k \sim 2 \times 10^{-9} cm^3 s^{-1}$ as above

e.g. $CH^+ + H_2 \rightarrow CH_2^+ + H$

Also Exothermic charge-exchange reaxns often have $k \approx 2 \times 10^{-9}\ cm^3\ s^{-1}$

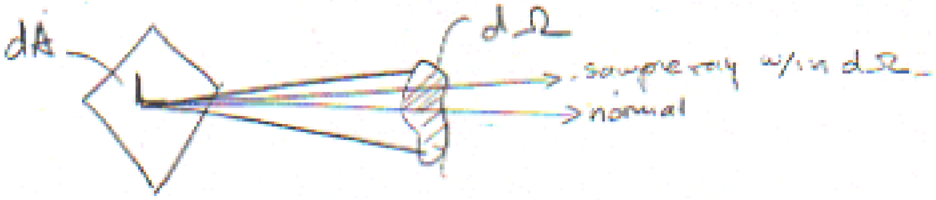
e.g. $O^+ + H \rightarrow O + H^+$

See Rybicki & Lightman
Ch. 1

Ion-Ion Rates given by Coulomb Interactions, same for Ion-Electron (see Prob Set 3?)

Radiative Transfer Definitions

(more depth next time — this is just so I can use J_ν in discussing Einstein A, B)



dE = energy crossing dA in time dt & freq range $d\nu$
$= I_\nu \, dA \, dt \, d\Omega \, d\nu$ ⓇⒹ R1

Ⓡ R2 I_ν = specific intensity (distance independent) $\text{erg s}^{-1}\text{ster}^{-1}\text{cm}^{-2}\text{Hz}^{-1}$

R3 $F_\nu = \int_{4\pi} I_\nu \overset{\cos\theta}{\wedge} d\Omega$ = Flux $\text{erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}$
(Actually $= \int I_\nu \cos\theta \, d\Omega$) assuming normally incident rays ($\cos\theta = 1$)

R4 $u_\nu = \frac{1}{c}\int_{4\pi} I_\nu \, d\Omega$ = Energy Density $\text{erg cm}^{-3}\text{Hz}^{-1}$

R5 $J_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu \, d\Omega$ = Mean Intensity $\text{erg s}^{-1}\text{cm}^{-2}\text{Hz}^{-1}$
(averaged over 4π steradians)

$\left(u_\nu = \frac{4\pi}{c} J_\nu\right)$

next time

Astronomy 208 Meeting #6 (+)

① – The Very Basics of Radiative Transfer (Part I: Definitions) (2.3a)

② – Einstein Coefficients (2.3b)

③ – Introductory Radiative Transfer (Part II)

4 – Absorption & Emission in Terms of Einstein Coefficients

5 Radiative Transfer in Terms of Einstein Coefficients

("1") Radiative Transfer Definitions

(enough to use J_ν, the mean intensity, in discussing Einstein coefficients)

For this example:
differentially small $d\Omega$
"normally incident rays"
w/in $d\Omega$ of normal
(more general expressions later)
(Projected area ≡ real area)

$$\boxed{\begin{array}{l} dE = \text{energy crossing } dA \text{ in a time } dt\text{, w/in freq range } d\nu \\ \quad = I_\nu\, dA\, dt\, d\Omega\, d\nu \end{array}}$$ (R1)

(R2) I_ν = specific intensity along a ray (distance independent $dI_\nu/ds = 0$ in free space) erg s^{-1} ster^{-1} cm^{-2} Hz^{-1}

(R3) $F_\nu = \int_{4\pi} I_\nu \, d\Omega$ = Flux erg s^{-1} cm^{-2} Hz^{-1} ← note – not per $d\Omega$

(R4) $u_\nu = \frac{1}{c}\int_{4\pi} I_\nu \, d\Omega$ = Energy Density erg cm^{-3} Hz^{-1} $(= \frac{4\pi}{c} J_\nu)$

(R5) $J_\nu = \frac{1}{4\pi}\int_{4\pi} I_\nu \, d\Omega$ = Mean Intensity erg s^{-1} cm^{-2} Hz^{-1}
same units as F_ν

Re-write Xfer Eqn (10) as:

$$\boxed{\frac{dI_\nu}{d\tau} = -I_\nu + S_\nu} \quad ⑫ \qquad \text{where} \quad \boxed{S_\nu \equiv \frac{j_\nu}{\alpha_\nu}} \quad ⑬ \quad = \text{"Source function"}$$

emission

absorption (incl. scatter.)

note $d\tau$ is a more natural "line-of-sight" unit than ds

So then, integrating ⑫ gives soln to Xfer eqn:

$$\boxed{I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')} S_\nu(\tau_\nu')\,d\tau_\nu'} \quad ⑭$$

initial intensity diminished by absorption (see picture next page)

integrated source function along depth τ_ν from obsv'r, diminished by absp'n

τ_ν' is dummy variable so that we can allow S_ν to depend on τ_ν'

Note

$\tau_\nu > 1$	optically thick (opaque)
$\tau_\nu < 1$	optically thin (transparent)

If S_ν is independent of τ_ν' (e.g. a blob of uniform comp, T, n) then

"constant source function"

$$⑭ \Rightarrow \boxed{\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \\ &= S_\nu + e^{-\tau_\nu}(I_\nu(0) - S_\nu) \end{aligned}} \quad ⑮$$

Interesting limits

$\tau_\nu \to \infty \qquad I_\nu \to S_\nu \qquad$ (e.g. a rock)

$$\boxed{\tau_\nu \to 0 \qquad I_\nu(\tau_\nu) \to I_\nu(0) + S_\nu \tau_\nu} \quad (15a)$$

recall $e^{-x} \to 1 - x$ for small x & $\tau_\nu I_\nu(0) \ll I_\nu(0)$

Some Important Definitions/Equations to Remember from Rad Xfer

j_ν = emission coeff $= \frac{\epsilon_\nu \rho}{4\pi}$ $\qquad \epsilon_\nu$ = emissivity

α_ν = absorption coeff $= \rho \kappa_\nu$ $\qquad \kappa_\nu$ = opacity

$\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')ds' = \alpha_\nu s$ = optical depth

↑for α_ν indep of s

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau_\nu')} S_\nu(\tau_\nu')d\tau_\nu'$$

$S_\nu \equiv j_\nu/\alpha_\nu$

If S_ν indep of τ_ν' $\qquad \left[\begin{array}{ll} \tau_\nu \to \infty & I_\nu \to S_\nu \\ \tau_\nu \to 0 & I_\nu(\tau_\nu) \to I_\nu(0) + S_\nu \tau_\nu \end{array}\right]$

For Thermal Radiation:

Rayleigh-Jeans Limit ($h\nu \ll kT$): $I_\nu^{RJ} = \frac{2\nu^2}{c^2} kT$ $\qquad (I_\nu \propto \nu^2 T)$

Wien Limit ($h\nu \gg kT$): $I_\nu^{W} = \frac{2h\nu^3}{c^2} \exp\left(-\frac{h\nu}{kT}\right)$

For R-J case $T_B = T_B(0)e^{-\tau_\nu} + T(1-e^{-\tau_\nu})$

as $\tau \to \infty$ $\quad T_B \to$ true temp of material

as $\tau \to 0$ $\quad T_B \to T_B(0) + T\tau_\nu$

A note on statistical weights

Full specification of an atomic energy level involves four quantum #'s : n, ℓ, s, m

- n: electronic or "principal"
- ℓ: orbital quantum #
- s: spin dirn vector
- m: projection of $j = \ell + s$

Note: mg mom of e⁻ – orbital mg mom of atom — spin-orbit — fine struct

mg mom of e⁻ – spin of proton — spin-spin — hfs

---

e.g. For H with $n=2$

$\ell=1$: $j = 3/2$, $m = -3/2, -1/2, 1/2, 3/2$; $j = 1/2$, $m = -1/2, 1/2$

$\ell=0$: $j = 1/2$, $m = -1/2, 1/2$

8-fold degeneracy

Number of sublevels of principal quantum number n usually $= \boxed{2n^2} = \text{statistical weight} = \boxed{g_n}$

= "number of distinct quantum states contained in level n"

What about molecules, ions? "g" is still ~ # of available sub-levels (ie degeneracy) but a bit more complex to calculate

– Einstein Coefficients –

M2 Einstein's Motivation in 1917

In 1901 Planck found expression for BB

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1}$$

recall this is for photons in thermodynamic equilibrium

Previously (1894) there was just Wien's Law

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{kT}\right)$$

which applies when $h\nu >> kT$ and is derived semi-classically $\left(\frac{h\nu}{kT} = \frac{E}{kT}\right.$, a Boltzmann factor$\left.\right)$

GOAL: Quantum processes, at least in equilibrium, must give rise to the same distribution of 'atoms' over their quantum states as predicted by general laws of thermodynamics.

Einstein's approach: match Planck law by assigning transition probabilites to absorptive & emissive processes.

Note: If only "ordinary" (abs) & (spontaneous emission) are included one can only get Wien Law.

Einstein needed to include stimulated emission (Prob Set)

Note: $\boxed{\Delta E_{ul} = h\nu_{ul} \mp h\nu_0}$

Absorption

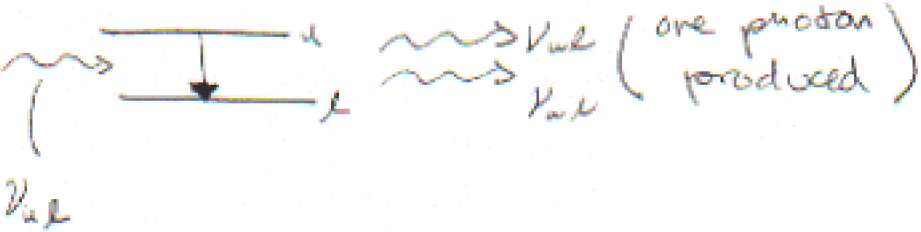
B_{lu}

Spontaneous Emission

A_{ul}

Stimulated Emission

B_{ul}



Note: Equ #'s in what follows continue from Meeting 5, as they form a "unit"

Einstein Coefficients $A_{u\ell}$, $B_{\ell u}$

(see Figure on prev page)

(i)

$A_{u\ell} \equiv$ transition probability per unit time for spontaneous emission (sec^{-1})

(ii) $\boxed{B_{\ell u}\bar{J} =}$ transition prob. per unit time for absorption (sec^{-1})

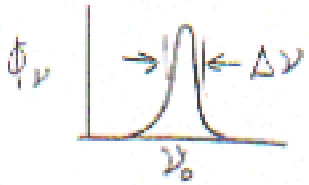
where

⑩ $$\boxed{\bar{J} \equiv \int_0^\infty \phi_\nu J_\nu \, d\nu = \int_0^\infty \phi_\nu \frac{1}{4\pi}\int I_\nu \, d\Omega \, d\nu}$$ (from E5)

= line-profile-integrated mean intensity

and

ϕ_ν = line-profile shape, normalized so that ⑪ $\boxed{\int_0^\infty \phi_\nu \, d\nu = 1}$



(We'll return to line-profile shapes later...)

(iii) $B_{u\ell}\bar{J}$ = transition prob per unit time for stimulated emission

Einstein fnd necessary to get Planck funcn back from all terms (see below)

note: If J_ν changes slowly over $\Delta\nu$, then one can approximate $\phi(\nu)$ as a δ funcn. so that

(ii) → $B_{\ell u} J_{\nu_0}$ and (iii) → $B_{u\ell} J_{\nu_0}$

(Einstein did this implicitly)

Relations between Einstein Coefficients (in Thermodynamic Eq)

"detailed balance"   # trans into state i = # trans out of state i

$n_u =$ }
$n_\ell =$ } number density in u, ℓ state

$$\boxed{n_\ell B_{\ell u} \bar{J} = n_u A_{u\ell} + n_u B_{u\ell} \bar{J}}$$ ⑫

(absorption) = (spont em + stim em)

Solving for J from ⑫

skip

$$\boxed{\bar{J} = \left(\frac{n_u A_{u\ell}}{n_\ell B_{\ell u} - n_u B_{u\ell}} \right) = \frac{A_{u\ell}/B_{u\ell}}{(n_\ell/n_u)\left(\frac{B_{\ell u}}{B_{u\ell}}\right) - 1}}$$ ⑬

Recall from Meeting #4 eq ④, in Th.Eq. $\frac{n_\ell}{n_u}$ given by

$$\boxed{\frac{n_\ell}{n_u} = \frac{g_\ell}{g_u} \exp\left(h\nu_0/kT\right)}$$ ⑭

(Boltzmann eqn.)

↖ see handout on "g"s

so then ⑬ becomes

$$\boxed{\bar{J} = \frac{A_{u\ell}/B_{u\ell}}{(g_\ell B_{\ell u}/g_u B_{u\ell}) \exp(h\nu_0/kT) - 1}}$$ ⑮

↑ ↑ ↑

Fundamental Equation giving Mean Intensity in terms of Q.M. transition probabilities & stat. wts & T!

But we know that in Th. Eq. $\overset{\text{Mean Intensity}}{J_\nu} = \overset{\text{Planck Function}}{B_\nu} = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right)-1\right]^{-1}$

and B_ν varies slowly on the scale of virtually any $\Delta\nu$

so $\left[\bar{J} = \underset{⑩}{\int_0^\infty \phi(\nu) J_\nu \, d\nu} = B_\nu\right]$ ⑯ since $\underset{⑪}{\int_0^\infty \phi(\nu)\, d\nu = 1}$

So, then for eq ⑮ to give $\bar{J} = B_\nu$, the Einstein coeff are related by:

"detailed balance"

⑰ $$\boxed{g_\ell B_{\ell u} = g_u B_{u\ell}}$$ {"absorp & stim em related by ratio of stat. wts"} $\frac{g_\ell}{g_u} = \frac{B_{u\ell}}{B_{\ell u}}$

⑱ $$\boxed{A_{u\ell} = \frac{2h\nu^3}{c^2} B_{u\ell}}$$ "amount of spontaneous emission related to stim. emission by $\frac{2h\nu^3}{c^2}$"

note: this is independent of T { must hold even out of Th Eq — Q.M. property!

(Prob Set will show why Einstein had to include Stim. Em. to get Planck Law rather than just Wien law ($I_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT}$)
$h\nu >> kT$
upper level underpop.)

Note: If we know any 1 of $A_{u\ell}, B_{u\ell}, B_{\ell u}$ we can get other two : very USEFUL

Next task: We'll relate this "microscopic, atomic" discussion of atomic/molec states & rate coeff to "macroscopic" absorption & emission coeff (More Radiative Xfer, etc)

Highlights of Rybicki & Lightman, Ch. 1 ...

"3" Introductory Radiative Transfer (Part II) (+ see handout)

Recall $\boxed{\overset{\text{specific intensity}}{I_\nu} = \text{constant along rays}}$ (see R&L p. 7)

erg s^{-1} ster^{-1} cm^{-2} Hz^{-1}

$$\therefore \boxed{\frac{dI_\nu}{ds} = 0} \quad ①$$

when there are no extra sources of emission and no absorption & no scattering in/out of beam

s = path length

Ⓠ What changes this and makes $\boxed{\frac{dI_\nu}{ds} \neq 0}$?

Ⓐ (i) Emission (ii) Absorption (iii) Scattering

(i) Emission

j = emission coeff = $\frac{\text{energy emitted}}{\text{time} \cdot \text{solid}\angle \cdot \text{Volume}}$

j_ν = monochromatic emission coeff = $\frac{(\text{units of } j)}{\text{freq}}$

so $$\boxed{dE = j_\nu \, dV \, d\Omega \, dt \, d\nu} \quad ②$$

more sensible units ⇒

ϵ_ν = emissivity = $\frac{\text{energy emitted spontaneously}}{\text{frequency} \cdot \text{time mass}} = \frac{\text{erg}}{\text{gm s Hz}}$

So $$\boxed{dE = \epsilon_\nu \overbrace{\rho \, dV}^{\text{mass}} \, dt \, d\nu \, \underbrace{\frac{d\Omega}{4\pi}}} \quad ③$$

↖ takes into account the fraction radiated into $d\Omega$

So ② = ③ ⇒ $$\boxed{j_\nu = \frac{\epsilon_\nu \rho}{4\pi}} \quad ④$$

And intensity added to beam by spontaneous emission:

$$\boxed{dI_\nu = j_\nu \, ds} \quad ⑤$$

$\left(\frac{dI_\nu}{ds} = j_\nu\right)$ for only that

(ii) Absorption

$$\boxed{\alpha_\nu (cm^{-1}) = \text{absorption coefficient}}$$

Convention $\alpha_\nu > 0$ for energy removed from beam, so:

Defined by:

$$\boxed{dI_\nu = -\underset{\text{has units } ^1/_s}{\alpha_\nu} I_\nu ds} \quad ⑥$$

(see R&L p. 10)

in terms of the microscopic cross-section, σ, we disussed last time:

$$\boxed{\alpha_\nu = n\sigma_\nu} \quad ⑦$$

where n = # dens of absorbers

K_ν is defined as the "mass absorption coefficient" or the "opacity" and is given by

$$\boxed{\alpha_\nu = \rho K_\nu} \quad ⑧$$

K_ν has units $\frac{cm^2}{g}$

so, obviously ⑦=⑧ $\Rightarrow \frac{\rho}{n} = \frac{\sigma_\nu}{K_\nu} \Rightarrow \overset{\text{mass per absorber}}{\mu} = \frac{\sigma_\nu}{K_\nu}$

or $\boxed{K_\nu = \sigma_\nu/\mu}$ ⑨

$$\boxed{\text{opacity} = \left(\frac{\text{x-section}}{\text{mass per absorber}}\right)}$$

Note: this K_ν is the opacity for absorption only - don't confuse it with scattering opacity

Note Further / Assumed here:

1. $\sigma_\nu^{1/2} \ll d \sim n^{-1/3}$ cross. secn << dist btwn ptcls
2. absorbers independent & randomly distributed (usually OK in ISM)

(ii) continued .. a weird point about α_ν

this "absorption" coefficient INCLUDES stimulated emission, since it has same $\propto I_\nu$ & just opp. sign

- i.e. $\alpha_\nu < 0$ corresponds to energy <u>added</u> to beam

α_ν is really $\sum \left[(\text{abs}) + (\text{stim em}) \right]$ & $\sum \lessgtr 0$

(iii) Scattering (we'll return to this later) [illegible] ..

["Full" Equation of Radiative Xfer (w/o scattering)]

$$\boxed{\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu} \quad ⑩$$

combining ⑤ & ⑥

of course, the trick is finding <u>ALL</u> the relevant contributors to α_ν & j_ν !! can be <u>many</u> choices

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Note:
When scattering is included this gets complicated, because things are <u>not isotropic w/in the beam</u> anymore. Often, numerical integration req to include scatt.

To write solution to ⑩

Define Optical Depth, τ_ν

$$\boxed{d\tau_\nu = \alpha_\nu \, ds} \quad (11a)$$

so ⑥ ⇒ for just abs $dI_\nu = -d\tau_\nu I_\nu$

$$\boxed{\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s') \, ds'} \quad (11b)$$

✓ note this is a funcn of ν & CAN vary along s

AY208: Meeting #7

Last Time: Radiative Transfer "Review" & Einstein Coefficients

Today: ① Absorption & Emission in Terms of Einstein Coeff.
② Radiative Transfer in Terms of Einstein Coeff.

End of Last Time.

$$\boxed{\frac{dI_\nu}{d\tau} = -I_\nu + S_\nu} \quad (12)$$

where $\boxed{S_\nu \equiv \frac{\overset{\text{emission coeff}}{j_\nu}}{\underset{\text{absorption coeff}}{\alpha_\nu}}}$ (13) = source funxn

$$(11b)\quad \boxed{\tau_\nu = \int_{s_0}^{s} \alpha_\nu(s')\,ds' = \underbrace{\alpha_\nu s}_{\text{for T indep of s}}}$$

$(\alpha_\nu = \rho \underset{\text{opacity}}{\kappa_\nu})$

$\int$ (12) $\Rightarrow$

$$\boxed{I_\nu(\tau_\nu) = \overset{\text{orig. ints.}}{I_\nu(0)}\, \overset{\text{abs}}{e^{-\tau_\nu}} + \int_0^{\tau_\nu} \overset{\text{abs term}}{e^{-(\tau_\nu - \tau_\nu')}}\, \overset{\text{source}}{S_\nu(\tau_\nu')}\, d\tau_\nu'} \quad (14)$$

NEW

If S_ν is independent of τ_ν' (e.g. a blob of uniform comps'n, T, n) then

$$\boxed{\begin{aligned} I_\nu(\tau_\nu) &= I_\nu(0)e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu}) \\ &= S_\nu + e^{-\tau_\nu}(I_\nu(0) - S_\nu) \end{aligned}} \quad (15)$$

called "constant source funxn" approx'n

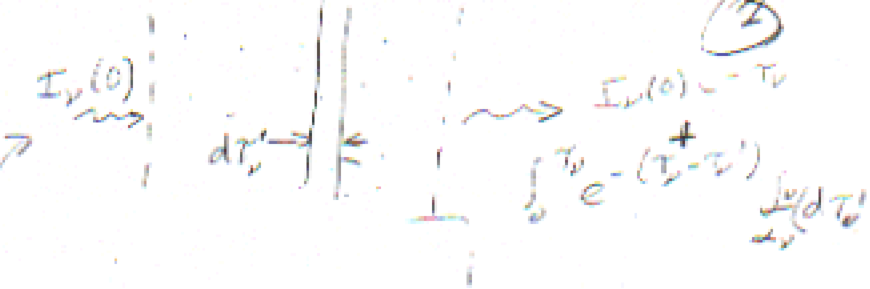
Interesting Limits: $\tau > 1$ optically thick, opaque
$\tau < 1$ " thin, transparent

and very thick $\tau_\nu \to \infty$ $\quad I_\nu \to S_\nu$ (e.g. a rock)

very thin $\tau_\nu \to 0$ $\quad I_\nu(\tau_\nu) \to I_\nu(0) + S_\nu \tau_\nu$ (15a)

since $e^{-\tau_\nu} \to 1 - \tau_\nu$ for small τ

Think for a minute.
What does all this mean?



(5a) ⇒ For spontaneous emission alone intensity increases $\left(I_\nu \propto \int j_\nu ds\right) \sim$ linearly

(6a) ⇒ For absorption alone intensity decreases exponentially $I_\nu \propto I_\nu(0)e^{-\tau_\nu}$.

If τ is large then specific value doesn't matter $I_\nu \rightarrow S_\nu$

If τ is small then $e^{-\tau} \rightarrow 1-\tau$ and exponential behavior → linear behavior

---

Now let's connect ~~Rad Xfer~~ formalism to previous discussions

Thermo. Eq

Einstein A, B coeff → masers (more from LJG)

then we'll add scattering...

THERMAL RADIATION (Emitting & absorbing material in T.E.)

$$\boxed{S_\nu = B_\nu(T)}$$ or, using 13, $$\boxed{\overset{\text{emission}}{j_\nu} = \overset{\text{absorption}}{\alpha_\nu} B_\nu(T)} \quad ⑯$$

"Kirchoff's Law"

note: this is not the same as saying $I_\nu = B_\nu(T)$, which is only true (see 15a) when $\tau \rightarrow \infty$

> "Thermal radiation becomes blackbody radiation only for optically thick media."

In general (re-writing 15) for thermal radiation @ one T

$$\boxed{I_\nu = I_\nu(0)e^{-\tau_\nu} + B_\nu(T)\left[1-e^{-\tau_\nu}\right]} \quad ⑰$$

isothermal stuff

Actually, we assumed one T for whole emitting region to write ⑰, more generally T can be, in essence, T(τ) which makes integ more complicated

---

Recall from meeting #4: Brightness Temperature, T_B

$$\boxed{I_\nu = B_\nu(T_B) = \frac{2h\nu^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT_B}\right) - 1}} \quad ⑱$$

↑ by definition of T_B

radiate xfer can be expressed as propagation of T_B in R-J regime; because $I_\nu^{RJ} \propto T_B$, as follows:

Recall

for $h\nu \ll kT$ ⑱ reduces to

$I_\nu \propto \nu^2 T$ ← observed!! eg opt thick HII reg

$$\boxed{I_\nu^{RJ}(T) = \frac{2\nu^2}{c^2} kT} \quad ⑲$$

"Rayleigh-Jeans limit"

for $h\nu \gg kT$

$$\boxed{I_\nu^{W} = \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{kT}\right)} \quad ⑳$$

"Wien limit"

So, using ⑲ eq. ⑰ becomes:

R-J only

$$\boxed{T_B = T_B(0)\, e^{-\tau_\nu} + T\left(1 - e^{-\tau_\nu}\right)} \quad ㉑$$

useful in radioastro

note 8: T_B does not depend on ν for a true blackbody — by definition

as $\tau \to \infty$ $T_B \to$ true temperature of the material

Absorption & Emission Coefficients in Terms of Einstein Coeff

Spontaneous Emission: j_ν

→ each transition produces $h\nu$ distributed 4π steradians

(Recall: A_{ul} = trans prob per unit time)

$$\therefore \quad \boxed{j_\nu = \frac{h\nu}{4\pi} n_u A_{ul} \phi(\nu)} \quad ㉒$$

Absorption: α_ν

by similar reasoning

$$\boxed{\alpha_\nu = \frac{h\nu}{4\pi} n_l B_{lu} \phi(\nu)} \quad ㉓$$

(uncorrected for stimulated emission)

Stimulated Emission: ∝ "Negative Absorption" (depends on $\bar{J}$ as does Abs)

So this adds a "$-\frac{h\nu}{4\pi} n_u B_{ul} \phi(\nu)$" term to 23, giving

Total absorption coefficient:

$$\boxed{\alpha_\nu = \frac{h\nu}{4\pi} \phi(\nu) (\underset{\uparrow \text{ Abs}}{n_l B_{lu}} - \underset{\uparrow \text{ Stim Em}}{n_u B_{ul}})} \quad ㉔$$

Radiative Transfer in Terms of Einstein Coefficients

Eq. $\boxed{⑩}$ ㉒, ㉔ give

becomes

transfer eqn

$$\boxed{\frac{dI_\nu}{ds} = \underbrace{\frac{-h\nu}{4\pi}\left(n_\ell B_{\ell u} - n_u B_{u\ell}\right)\phi(\nu)}_{\alpha_\nu} I_\nu + \underbrace{\frac{h\nu}{4\pi} n_u A_{u\ell}\,\phi(\nu)}_{j_\nu}} \quad ㉕$$

Recall that $S_\nu = \frac{j_\nu}{\alpha_\nu}$ (13), so, using ㉒ & ㉔ gives

$$\boxed{S_\nu = \frac{n_u A_{u\ell}}{n_\ell B_{\ell u} - n_u B_{u\ell}}} \quad ㉖$$

Remember Einstein relations

$$\boxed{\begin{aligned} g_\ell B_{\ell u} &= g_u B_{u\ell} \\ A_{u\ell} &= \frac{2h\nu^3}{\boxed{c^2}} B_{u\ell} \end{aligned}} \quad ㉗$$

4.2×10^{-20}

only need one of $A_{u\ell}$, $B_{u\ell}$, $B_{\ell u}$ to get other two

So we can write α_ν & S_ν in terms of just $B_{\ell u}$ & stat wts:

$$㉔ \Rightarrow \boxed{\alpha_\nu = \frac{h\nu}{4\pi} n_\ell B_{\ell u}\left(1 - \frac{g_\ell n_u}{g_u n_\ell}\right)\phi(\nu)} \quad ㉘$$

$$㉖ \Rightarrow \boxed{S_\nu = \frac{2h\nu^3}{c^2}\left(\frac{g_u n_\ell}{g_\ell n_u} - 1\right)^{-1}} \quad ㉙$$

& we get a VERY GENERAL Q.M. description of absorption & emiss. proc that can be propagated using ∫⑩ to give "macroscopic" results!!

"Examples" (Interesting cases of eqs 28, 29)

Ⓐ "LTE"

$$\boxed{\frac{n_\ell}{n_u} = \frac{g_\ell}{g_u} \exp\left(\frac{h\nu}{kT}\right)} \quad (30)$$

locally holds

(28) ⇒

$$\boxed{\alpha_\nu = \frac{h\nu}{4\pi} n_\ell B_{\ell u}\left[1-\exp\left(\frac{-h\nu}{kT}\right)\right]\phi(\nu)} \quad (31)$$

(29) ⇒ (gives back eq 16)

$$\boxed{S_\nu = B_\nu(T)} \quad \text{(Kirchoff's Law)} \quad (32)$$

$$= \frac{j_\nu}{\alpha_\nu}$$

(Definition of)

abs & em are simply related since Th. Eq.

just need one Einstein coeff, line prof, & T

Ⓑ "Non-Thermal Emission" $\frac{n_\ell}{n_u} \neq \frac{g_\ell}{g_u}\exp\left(\frac{h\nu}{kT}\right)$

More later in course!

eg "Synchrotron, scattering present, etc.

Ⓒ "Inverted Populations & Masers"

(Normally:) $\frac{n_u g_\ell}{n_\ell g_u} = \exp\left(\frac{-h\nu}{kT}\right) < 1$ $\quad e^{-x} \to 1-x$

which means $\frac{n_\ell}{n_u} > \frac{g_\ell}{g_u}$

& $\frac{g_\ell}{g_u}$ usually <1 so $\frac{n_\ell}{n_u} > 1$

Inverted Masing

causes: $\frac{n_\ell}{n_u} < \frac{g_\ell}{g_u}$ (33)

usually $g_\ell < g_u \overset{so}{\Rightarrow} n_\ell << n_u$

⇒ much emission

⇓

$\alpha_\nu < 0$ intensity increases along a ray!

$\tau < 0$

e.g. $\tau = -100$

amplification 10^{43}

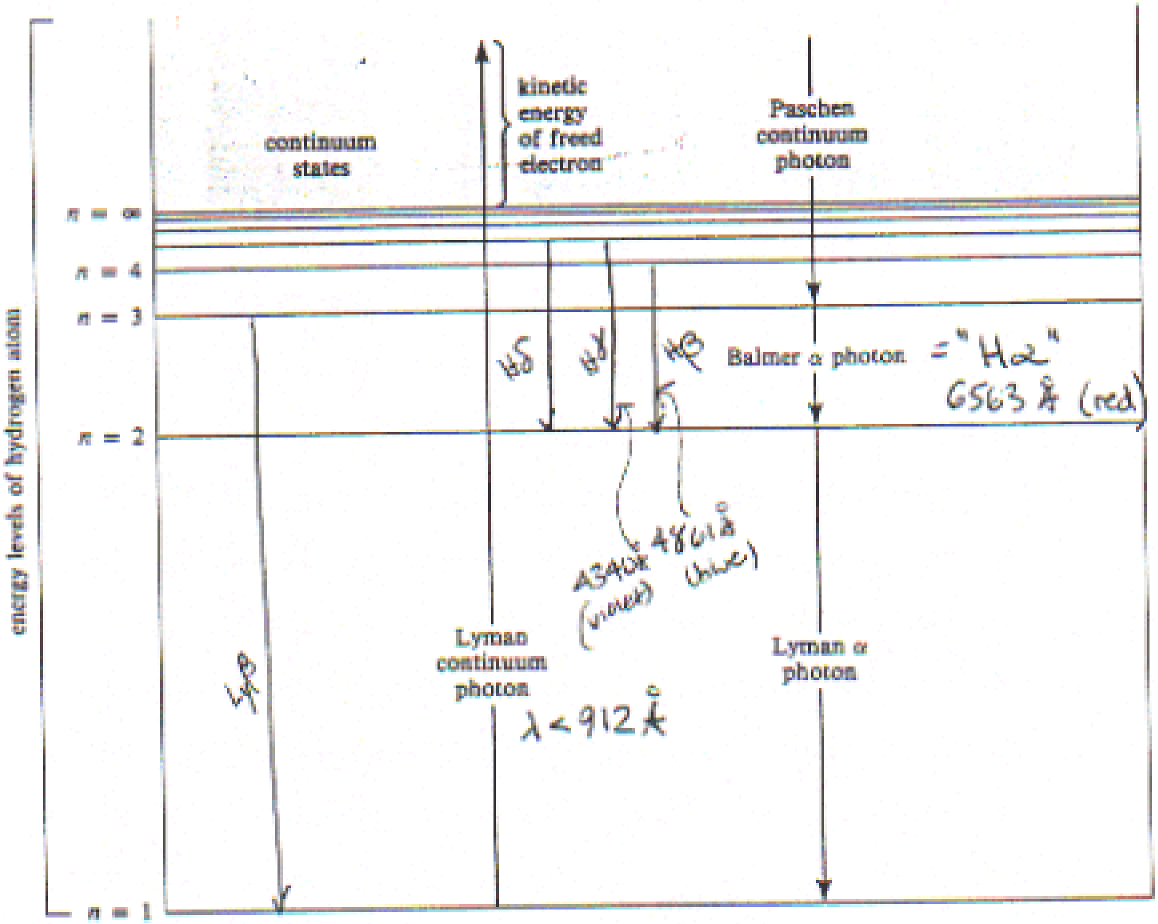


Figure 3.20. Radiative transitions to and from the continuum. Bound states of hydrogen are represented by the horizontal lines labeled $n = 1, n = 2, \ldots, n = \infty$, with the height from the ground state ($n = 1$) proportional to the energy difference of the states. Above the last bound state, $n = \infty$, exists a continuum of free states (ionized hydrogen). An electron can be freed into the continuum from the ground state by the absorption of a Lyman continuum photon. Conversely, a free electron may be captured, say, into the level $n = 3$ with the emission of a Paschen continuum photon. The electron in $n = 3$ may then cascade to the ground level by the subsequent emission, say, of a Balmer α photon and a Lyman α photon.

Note:
Recombination lines, e.g. H109α @ 6cm occur btwn very high bound states (e.g. n= 109).

emission or in absorption, would constitute spectroscopic evidence for the presence of hydrogen.

Problem 3.7. For atomic hydrogen show that equation (3.8) results in the expression

$$\lambda = 4\pi\left(\frac{\hbar c}{e^2}\right)\left(\frac{n^2\hbar^2}{m_e e^2}\right)\left(\frac{n'^2}{n'^2 - n^2}\right),$$

where n' and n are, respectively, the principal quantum numbers associated with the upper and lower levels. The combination $\hbar c/e^2$ is known as the inverse of the "fine-structure constant" and has an approximate value of 137. The combination $n^2\hbar^2/m_e e^2$ is the size of the hydrogen atom in the lower state. Thus, if the last term is of order unity, the hydrogen atom typically emits and absorbs radiation with wavelength roughly 10^3 times its own size. (This fact leads to an important approximate method for calculating radiation probabilities, but it lies outside

Shu, The Physical Universe, 1982

Astronomy 208 Meeting #8

Part 3 of the Course: "The ISM of the Milky Way"

3.1 Multi-phase Paradigm

3.2 The "Cold" ISM

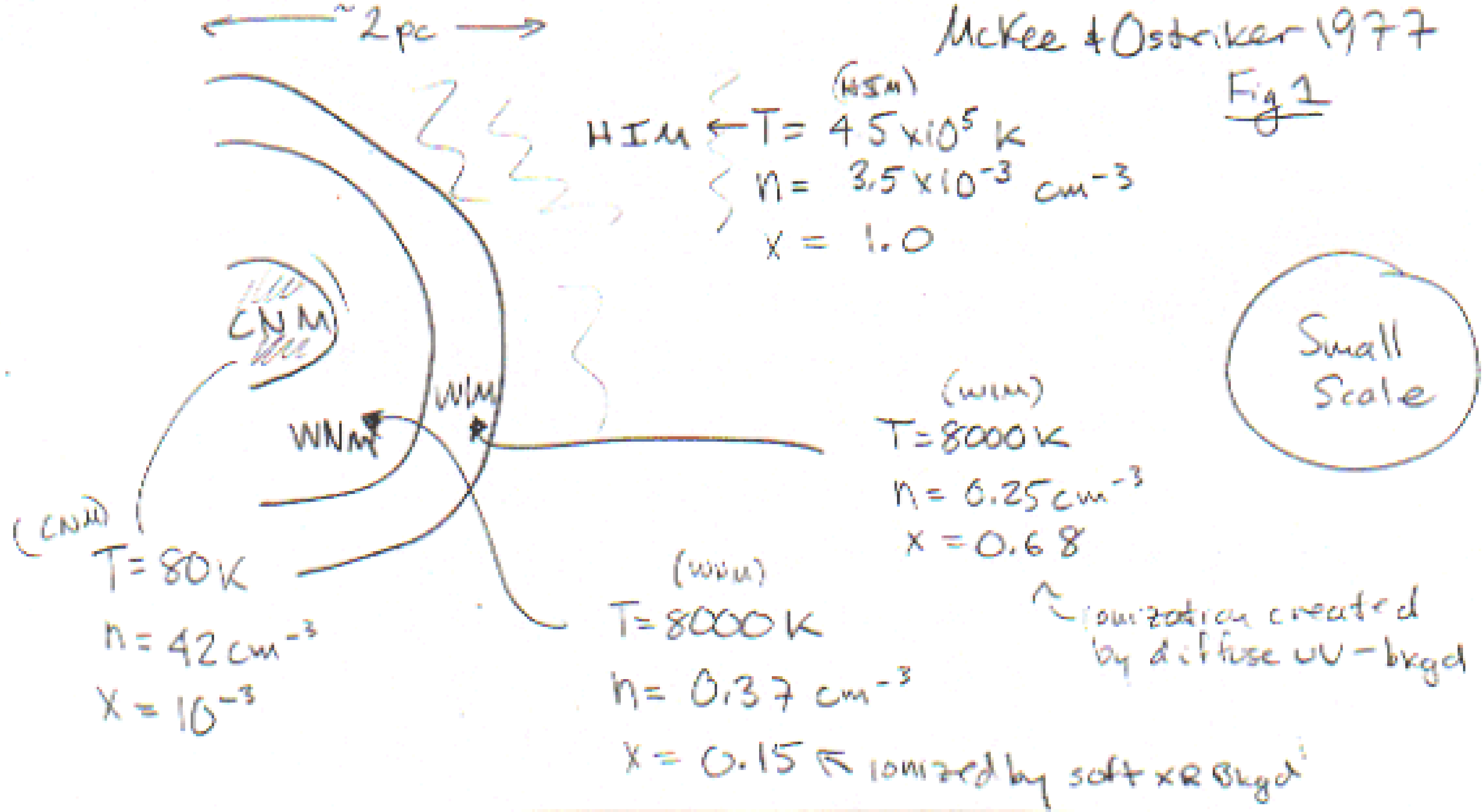
a) Atomic Gas

Origin of 21-cm Line Spin-Flip

21-cm Line Surveys

see http://cfa-www.harvard.edu/~agoodman/HI.html

Multi-Phase Paradigm in Brief:



McKee & Ostriker 1977 Fig 1

"Cross-section of Characteristic Small Cloud"

(Fig 1)

Fig 2

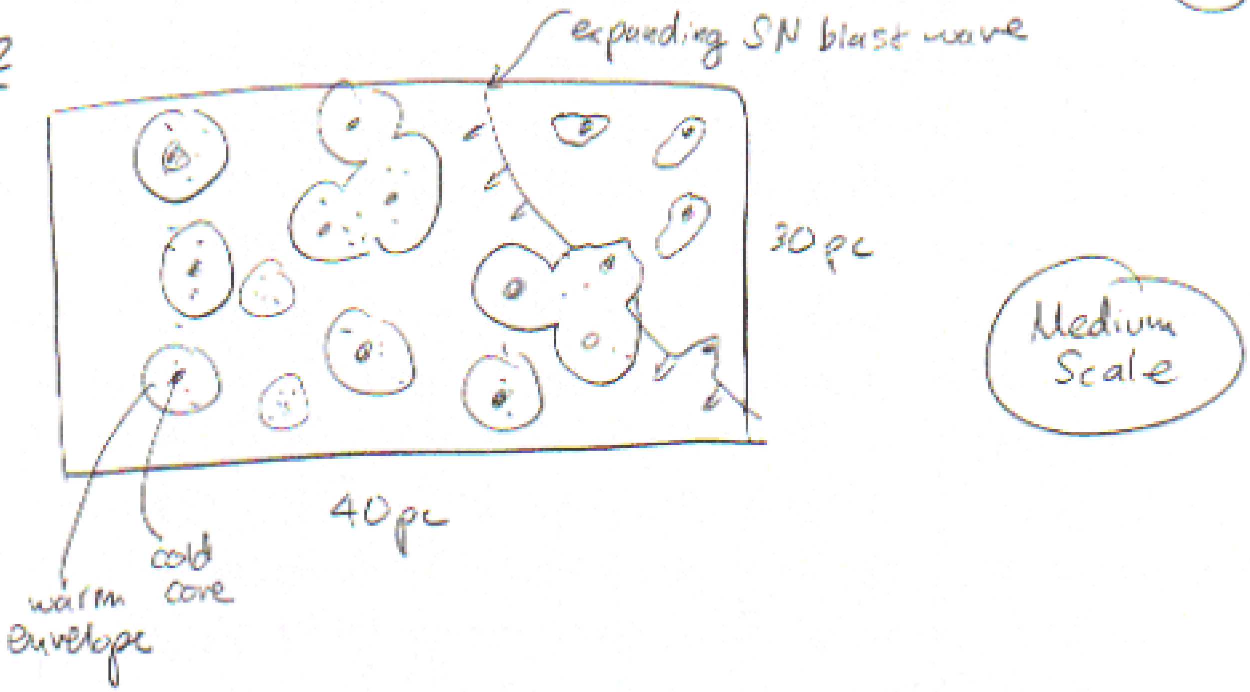
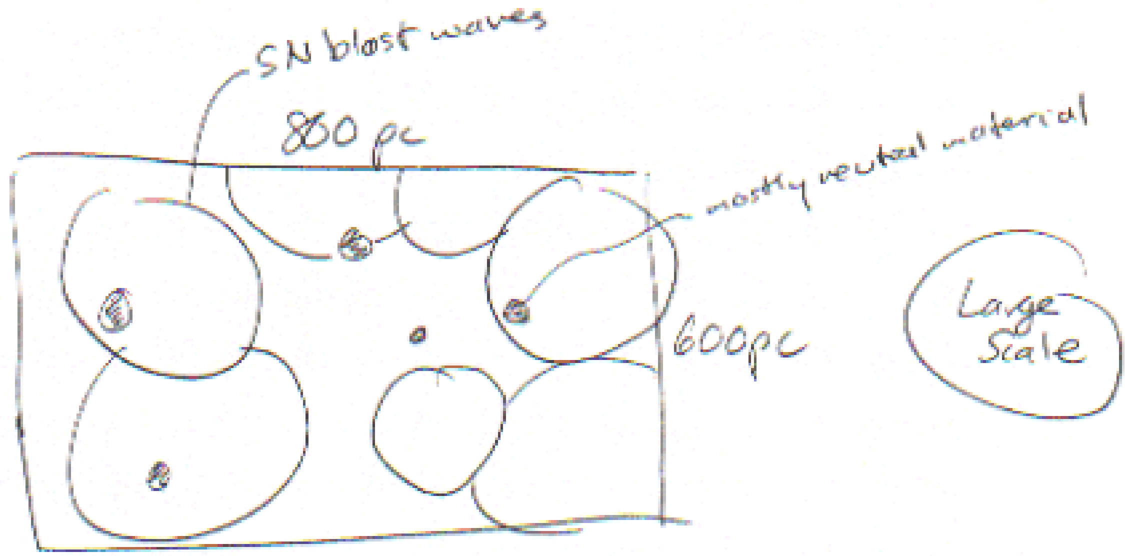


Fig 3



$f_{HIM} = 0.7 - 0.8$

$f_{CNM} = 0.02 - 0.04$

$f_{WIM} \sim 0.2$

+3 weeks from now — Reconsider This picture...

Key ideas: Pressure Balance - Momentum, Energy Deposition
Time Dependence
(Outflows??)

History

- spin-flip transition predicted to be detectable by van de Hulst in ~1944

~• 1951 Ewen & Purcell detect 21 cm line out window @ Jefferson Lab

(Dutch)
Muller & Oort were very close to doing it first, but they had a fire – they succeeded soon after, as did Australians (Christansen & Hindman)

new quest is analogous line in Deuterium @ 327 MHz not yet (quite) detected... despite some claims

21-cm line surveys

Principal Results: (see Kulkarni & Heiles 1988 in Galactic & Extragalactic Radio Astronomy, Verschuur & Kellermann ed.

HI in Milky Way ~ $4.8 \times 10^9 M_\odot$ = 4.4% of visible matter
(H_2 " " " ~ 25 to 90% as much as HI)

HI *not* really in "clouds" like H_2

HI filling factor ~ 20 to 90% depending where & who you believe

Some Important Definitions

"Permitted Transition" transitions which occur relatively quickly (high rate of spontaneous emission)

→ "allowed" electric dipole transitions

many selection rules are only exact in the absence of spin-orbit coupling and/or external fields and/or collisions

"Forbidden Transition" very low spontaneous emission rate

→ come from magnetic dipole or electric quadrupole transitions

collisionally excited

note: 21-cm line is a "forbidden" transition!

Note: This happens often in ISM where density is very low

"Critical Density"

$$n_{crit} = \overset{1/sec}{A_{ul}} \Big/ \underset{\uparrow\, cm^3/sec}{\gamma_{ul}}$$

$\gamma_{ul} =$ collisional rate coeff

$= \langle \sigma_{ul} v \rangle \quad cm^2 \cdot \frac{cm}{sec} = \frac{cm^3}{sec}$

indicates density at which collisions can keep up with spontaneous radiative transitions

for $n < n_{crit}$ line flux $\propto n^2$

for $n > n_{crit}$ " " $\propto N$

show formally later

3.2 The Cold ISM

a) Atomic Gas

Origin of 21-cm line : "spin-flip"

interaction of
mag mom of proton
w/spin-orbit magmom
of e^-

transition is btwn two hyperfine levels of $n=1$ electronic state $(1\,^2S_{1/2})$

Upper level
parallel spins

$g_u = 3$ ← note: this degeneracy removed in Zeeman splitg.

Lower level
antiparallel spins

$g_\ell = 1$

$\nu_0 = 1420.406$ MHz $\qquad \frac{h\nu_0}{k} = 0.068\,K$ very low energy!

$A_{u\ell} = 2.869 \times 10^{-15}\, s^{-1}$ (Very small!)

τ_r = rate of spontaneous decay after being excited to upper state $= \frac{1}{A_{u\ell}} \sim 10^7$ yrs!

but recall $\tau_{coll} = \frac{1}{n_H \sigma v} \cong 4 \times 10^3$ yr for $\left(\begin{matrix} n_H = 1 \,;\, \sigma \text{ geometric} \\ V \sim 1 \text{ km/s} \end{matrix} \right)$

Thus for HI in ISM

$\tau_{coll} << \tau_r$ ... see K&H handout...

HI is primarily
collisionally
excited in CNM,
but in WNM, Lyα
plays role see
K&H & Field 1959

Optical Depth Effects

characterizes level populations

$\downarrow$

$T_S = T_{ex}$ for spin-flip

$\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} \exp\left(\frac{h\nu_{u\ell}}{kT_{ex}}\right)$

Recall

$$\boxed{T_B(\nu) = T_{bg}(\nu) e^{-\tau_\nu} + T_s(1-e^{-\tau_\nu})} \quad ①$$

(for an isolated homogeneous cloud where τ & T don't vary inside it)

Actual measurements give $\boxed{\Delta T_B = T_B - T_{bg}}$ ②

so ① & ② →

$$\boxed{\Delta T_B = (T_s - T_{bg})(1-e^{-\tau_\nu})} \quad ③$$

Note: FYS $\tau \propto \frac{N}{T_s}$ ④ can derive this (see next page)

$\boxed{\tau << 1} \rightarrow \boxed{T_B = T_s \tau = N_\nu / C}$ ⑤ (assuming $T_{bg} \rightarrow 0$)

T_B gives column density

more stuff → more emission, linearly

$\boxed{\tau >> 1}$ $\boxed{T_B = T_s}$ ⑥ only see to $\tau \approx 1$ surface

Observed brightness temp is independent of column density — only depends on temperature

Recall from eq. ㉛ of Meeting #7 for LTE

⑦
$$\boxed{\begin{aligned}\alpha_\nu &= \frac{h\nu}{4\pi} n_\ell B_{\ell u}\left[1-\exp\left(\frac{-h\nu}{kT_{ex}}\right)\right]\phi(\nu)\\ &= \frac{h\nu}{4\pi}\phi_\nu n_\ell B_{\ell u}\left(\frac{h\nu}{kT_{ex}}\right) \quad \text{for} \quad h\nu << kT_{ex}\end{aligned}}$$

(Kirchoff)

$S_\nu = B_\nu(T) = \frac{j_\nu}{\alpha_\nu}$

(RJ)

and ⑧
$$\boxed{\tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\,ds'}$$

So then as long as T_{ex} is independent of s 7+8 ⇒

⑨
$$\tau_\nu = \frac{(h\nu)^2}{4\pi}\phi_\nu B_{\ell u}\frac{1}{k}\int \frac{n_\ell}{T_{ex}}\,ds'$$

and ⑩ $\boxed{N = \int n\,ds}$ = column density

then
$$\tau_\nu = \frac{(h\nu)^2}{4\pi k} B_{\ell u}\phi_\nu \frac{N_\ell}{T_{ex}}$$

Note: for HI

$A_{u\ell} = 2.9\times10^{-15}\ s^{-1}$

$B_{u\ell} = \frac{c^2}{2h\nu^3} A_{u\ell}$

$B_{\ell u} = \frac{g_u}{g_\ell} B_{u\ell} = 3B_{u\ell}$

$\left[B_{\ell u} = 7.1\times10^{19} A_{u\ell} = 2.1\times10^{5}\right]$

for HI groundstate

recall $\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} e^{-h\nu/kT_{ex}}$ but $\frac{h\nu}{kT_{ex}} << 1$ so $\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell} = 3$

So $N_u = 3N_\ell$ and ⑪ $\boxed{N_{TOTAL} = 4N_\ell}$

And ⑫
$$\boxed{\tau_\nu = 5.49\times10^{-14}\,\frac{N_{TOTAL}\,\phi(\nu)}{T_{ex}}}$$

$\phi(\nu)$ in units of $cm^{-1}s$

$\int \phi(\nu)\,d\nu = 1$

$$\tau_\nu = \frac{N_{TOT}}{C\times T_s} \qquad C = 1.82\times10^{13} \quad \text{for cgs}$$

AY208

Handout on Collisional Excitation
to accompany
Meetings 8 & 9

Collisional Excitation

for transitions $u \rightleftarrows \ell$ triggered by collisions:

(C1) $$\boxed{C_{u\ell} \equiv C_{u\to\ell} = n\gamma_{u\ell} = n\langle\sigma_{u\ell}v\rangle}$$

(C2) $$\boxed{C_{\ell u} = C_{\ell\to u} = C_{u\ell}\frac{g_u}{g_\ell}\exp\left(\frac{-h\nu}{T_k}\right)}$$ for LTE

$\sigma_{u\ell}$ = xsecn for coll $u\to\ell$ @ velocity v

$\gamma_{u\ell}$ [$\mathrm{cm}^3\,\mathrm{s}^{-1}$] = overall rate coeff $u\to\ell$

for Maxwellian vel dist @ T_k $$\boxed{\gamma_{u\ell} = \frac{4}{\sqrt{\pi}}\left(\frac{\mu}{2kT_k}\right)^{3/2}\int_0^\infty dv\,\sigma_{u\ell}(v)\,v^3\cdot\exp\left(\frac{-\mu v^2}{2kT_k}\right)}$$ (C3)

for neut-neut $\gamma_{u\ell} \sim 10^{-11}$ to 10^{-10}

neut-ion $\sim 10^{-9}$

(often uncertain by ~×2)

μ = reduced mass

Given $\overset{\text{rate coeff}}{\downarrow}{\gamma_{ul}}$, A_{ul}, T_K & n one can get

n_u & n_l from detailed balance ($n = n_{tot} = n_u + n_l$)

$$\left[\, n_l(\gamma_{ul} n) = n_u(\gamma_{ul} n + A_{ul}) \,\right] \quad \text{Ⓒ4}$$

$$\left[\, \frac{n_u}{n_{tot}} = \frac{\frac{g_u}{g_l}\exp\left(\frac{-h\nu}{kT_K}\right)}{1 + \frac{g_u}{g_l}\exp\left(\frac{-h\nu}{kT_K}\right) + \frac{n_{crit}}{n}} \,\right] \quad \text{C5}$$

$$\left[\, n_{crit} = A_{ul}/\gamma_{ul} = \text{critical density} \,\right] \quad \text{C6}$$

Using (C5), we see that for $n >> n_{crit}$ u, l thermalized@ T_K

so

$n >> n_{crit}$

$$\left(\frac{n_u}{n_l}\right)_{\text{thermal}} = \left(\frac{g_u}{g_l}\right)\exp\left(-\frac{h\nu}{kT_K}\right) \quad \text{C7}$$

but for $n << n_{crit}$ spontaneous > collisions

⇒ each collision l → u gives emission

$$\left[\, \frac{n_u}{n_l} = \left(\frac{n}{n_{crit}}\right)\left(\frac{n_u}{n_l}\right)_{\text{thermal}} \leftarrow \text{from eq C7} \,\right] \quad \text{C8}$$

$$\text{Flux} \propto A_{ul}\int n_u dz \left\langle \begin{array}{lll} \int n_{tot}\, n\, dz & \propto n^2 & n < n_{crit} \\ \int n_{tot}\, dz & \propto N & n > n_{crit} \end{array} \right] \quad \text{C9}$$

Astronomy 208 Meeting # 9 Cold ISM, cont'd

Last time a) Atomic Gas

Recall: $\tau_\nu = 5.49 \times 10^{-19} \frac{N_{Total}\,\phi(\nu)}{T_{ex}}$

See handout for

$\tau(v) = \frac{N(v)}{C \times T_s}$ $C = 1.82 \times 10^{18}$

(units on board may have been confusing last time)

e.g. for $\tau = 1$ @ $T_s = 1000\,K$

$1 = \frac{N(v)}{1.82 \times 10^{18} \cdot 1000} \Rightarrow N \simeq 2 \times 10^{21}$

(1 mag A_v)

& for $N >> 2 \times 10^{21}$ $\tau >> 1$

$N << 2 \times 10^{21}$ $\tau << 1$

Three more points about H I

(i) Line shapes

(ii) Radial & Vertical Dist'b'n.

(iii) Using Absorption & Emission Together to get τ, T_s

(i) Line Shapes

keep in mind.

for a photon emitted by an atom @ rest

$$\Delta E \Delta t \geq \hbar \qquad \text{(Heisenberg)}$$

$$2\pi \Delta\nu \Delta t \geq 1$$

$\Delta t = 1/A_{ul} \propto$ occupation time in upper state

e.g. for HI $A_{ul} \cong 3\times10^{-15}\ s^{-1} \Rightarrow \Delta\nu_{nat} = \frac{A}{2\pi} = 5\times10^{-16}$ Hz

In velocity, @ $\nu = 1420$ MHz (21cm) this gives:

$$\Delta v = \Delta\nu \frac{c}{\nu} \cong 10^{-19}\ \text{km/s} \lll \text{any obsv'd } \Delta v$$

In 21-cm (& all molec. line obs.) "natural" linewidth inconsequential

Actual Lineshape determined by:

① distribution of material as function of velocity (Doppler profile)

② temperature distribution along l.o.s.

→ (radiative xfer gives overall profile)

Gross example of ②

If $T_{bg} > T_s \Rightarrow$ absorption

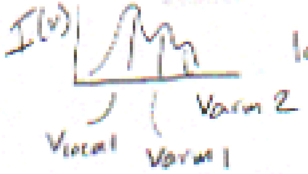
$T_{bg} < T_s \Rightarrow$ emission

$T_{bg} = T_s$ NOTHING

e.g. what if extragal HI is @ 3K?

(ii) Radial & Vertical Distribution of HI

radial: spiral arms can be identified from radial velocities of peaks



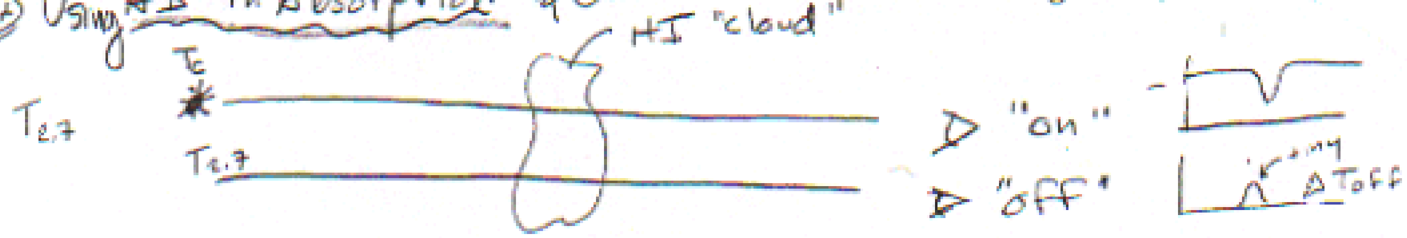
Idealized profile

{actually hard to see all 3 along same ℓ}

vertical: can model as a plane-parallel atmosphere, but HI not isothermal w/ z, so this doesn't work perfectly

see handout for empirical determination

(iii) Using HI in Absorption & Emission Together to get τ, T_s



$$T_{on} = T_c e^{-\tau} + T_{2.7} e^{-\tau} + T_{HI}(1-e^{-\tau}) \qquad \text{(R-J)}$$

$$- \quad T_{off} = T_{2.7} e^{-\tau} + T_{HI}(1-e^{-\tau})$$

$$\boxed{T_{on} - T_{off} = \Delta T = T_c e^{-\tau} \quad \Rightarrow \text{gives } \tau \text{ directly}} \; ①$$

$$\boxed{\Delta T_{off} = (T_{HI} - T_{2.7})(1-e^{-\tau}) = T_{off} - T_{2.7}} \; ②$$

observation of off pos'n alone

So ① gives you τ, sub into ② $\xrightarrow{\text{gives}}$ T_{HI}

spin temp of HI in absorbing region

Note: We'll talk more about abs lines later in course e.g. hot ISM

§ 3.2 b. Molecular Gas

Why can't we just observe H_2?

(We can: UV Interstellar Absorption Lines (of H_2) is Big Business – but you need a background source: see Spitzer p. 96-98 & for ISM §)

In emission:

center of mass ≡ center of charge ⇒ no dipole radiation possible

(most radio transitions come from ro-vib dipole radiation.. see CO, below)

some H_2 vib emission from shocks – later

Observe "Trace" Species Instead

- choice of tracer depends on $\bar{n}$ for R.O.I & "n_{crit}" for tracer

e.g.

In PRACTICE for cold gas $0 \lesssim T \lesssim 100K$

type	n in cm^{-3}	tracer
low	$10 \lesssim n \lesssim 500$	^{12}CO
"dark"	$300 \lesssim n \lesssim 5 \times 10^3$	^{13}CO, OH
dark/dense	$10^3 \lesssim n \lesssim 10^4$	$C^{18}O$, CS
dense	$5 \times 10^3 \lesssim n \lesssim 10^6$	NH_3, CS
very dense	$n \sim 10^8$	OH masers
very, very, dense	$n \sim 10^{10}$	H_2O masers

recall $\boxed{n_{crit} = \frac{A_{ul}}{\gamma_{ul}}}$

– density @ which collisional excitation beats spontaneous emission rate

– leads to continuing emission from the tracer (otherwise, all just go to lower state & stay there) see handout

trouble is – some tracers emit fine @ $n < n_{crit}$ due to radiative trapping, etc.

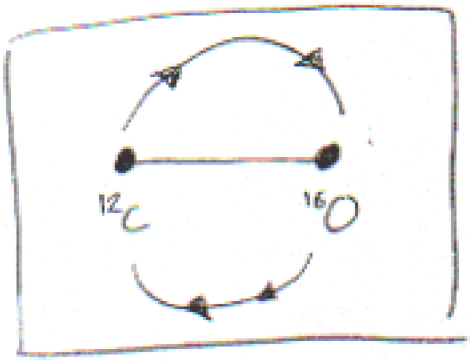
e.g. CS

Energy ↓ electronic
vibrational
rotational

⑤

CO as Tracer of H_2

(next most abundant after H_2, but down by $\mathcal{O}(10^{-5})$)

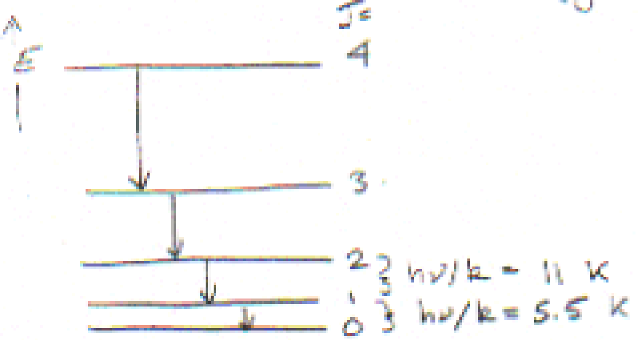


- has dipole moment (center of mass ≠ center of charge)
- rotational states are quantized →
- important coolant of molecular gas
- key tracer of molecular gas

Note: ~~Tex~~ for these rotational trans. = "T_{rot}"

~ Rotational Ladder ~

(Vibrational ground state v=0)



Energy levels for linear molec are those allowed by Q.M. for a "rigid rotator"

spacing of lines is equidistant in frequency

$$\boxed{\nu_{J\to J-1} = 2BJ}$$ (3a) (3b)

since
- $E = hBJ(J+1)$, $J = 0,1,2$

and
- $\Delta J = \pm 1$ (3c) (selection rule)
- $B = h / 8\pi^2 I$ (3d)

(for CO: $B = 5.75\times10^{10}$) →

^{12}CO

$$\boxed{\begin{aligned} \nu(J=1-0) &= 115.271\ \text{GHz} \\ \lambda(J=1-0) &= 2.7\ \text{mm} \\ A_{10} &= 6\times10^{-8}\ s^{-1} \end{aligned}}$$

$= 2BJ$ $(J=1)$

Easily excited @ low temperatures ... 1^{st} excited state only 5.5 K above ground ($kT_{1st\ state} = 8.6\times10^{-5} \frac{eV}{deg}\ 5.5 deg = 4.7\times10^{-4} eV$)

Maximum emission in a rotational ladder @ T_{rot} occurs @

$E_J = hBJ(J+1) \approx kT_{rot}$, so $$\boxed{J_{max} \approx \sqrt{T_{rot}/hBk}}$$ ④

↑ from 3b

Which ^{12}CO transition is strongest where?

T_{rot}	J	λ	
10 K	2 → 1	1.3 mm	
50 to 100 K	4 ⋯ 7	sub-mm	← SMT !!
> 1000 K	20 ⋯ 40	far-IR	

⇝ relative strengths of these ^{12}CO lines can give T_{rot} (but absolute level of emission depends on collision rates, too thus various combinations of n & T_{rot} can give same flux ... see Genzel Fig. 8 (on website)

AG

10/24/2000

Charlie Lada

1/2

Interstellar Dust

1) William Herschel 1738 - 1822
Caroline 1750 - 1848

"ein Loch im Himmel" = hole in the heavens

many "holes" near star clusters ← catalogue of dark patches ex: (Ophiucus)

30" telescope → sky survey; star counts → map of structure of universe (stars = std. candles)

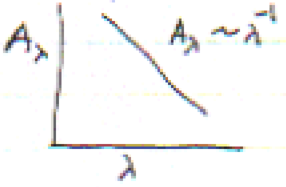
2) E.E. Barnard 1857 - 1923 ⇒ CLOUDS photographs of sky (a photographer)

3) V.M. Slipher 1912 ⇒ Reflection Nebula ⇒ small ~~[illegible]~~ particles (Rayleigh scatt.)
spectra of Pleides → match stellar spectra

λ-dep of absorption

4) R. Trumpler (1886 - 1956) (see below)

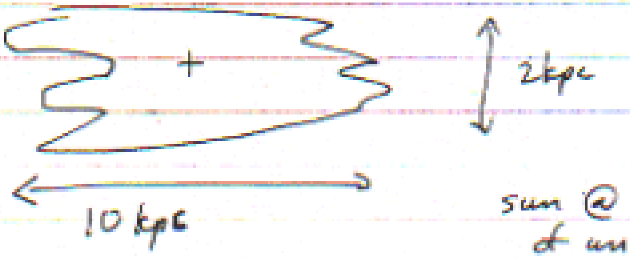
$L = \frac{L}{4\pi D^2}$



Absorption / Extinction due to dust
(not covered: polarization, spectra, emission/reradiation)
recommended book: (Whitter (?))

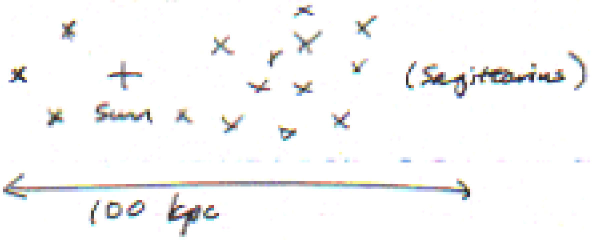
Kapteyn (1851 - 1922)
star counts:

(map similar to Herschel — added scale)



sun @ center of universe

Harlow Shapley - globular clusters



g.c's concentrated near Sag.

(Shapley - Curtis debates)

Trumpler - photometric distances (get Abs mag. fr. spectral type)

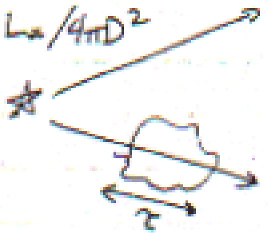
$L = \frac{L}{4\pi D^2}$

⇒ intervening medium absorption
0.7 mag/kpc

zone of avoidance – spiral nebulae appear uniformly except for plane of M.W.

why study dust? it just gets in the way!

$F_\lambda = L_\lambda/4\pi D^2$



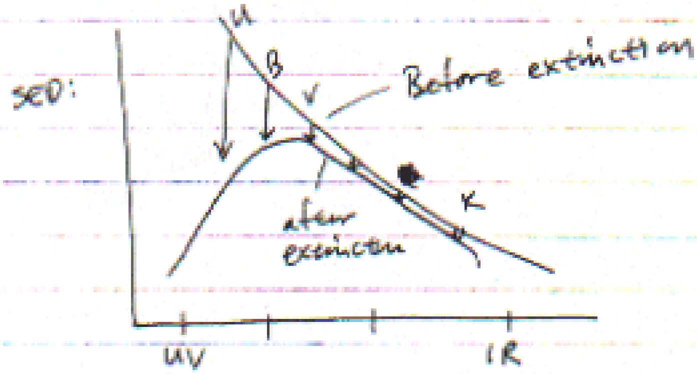
$\triangleright \quad F_\lambda \Rightarrow m_\lambda = -2.5\log\frac{F_*}{F_{vega}}$

$\triangleright \quad F_\lambda e^{-\tau} \Rightarrow m_\lambda' = -2.5\log\frac{F_* e^{-\tau}}{F_{vega}}$

$$= \underbrace{-2.5\log F_*}_{m_{obs} = m_\lambda} + \underbrace{1.086\,\tau_{dust}}_{A_\lambda}$$

$$m_v - M_v = 5\log d - 5 + A_\lambda$$

SED:



$B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5}\left[e^{hc/\lambda kT} - 1\right]^{-1}$

Wien: $\lambda << \frac{hc}{kT}$

$B(T) \sim e^{-c/\lambda}$

$A_\lambda \sim \frac{1}{\lambda} \Rightarrow \tau_D \sim \frac{1}{\lambda}$

$F_{obs} = F_\lambda e^{-1/\lambda}$ ⇒ mimics Wien law

↳ Reddening

↳ effectively shifts the BB

extinction more impt @ short wavelengths

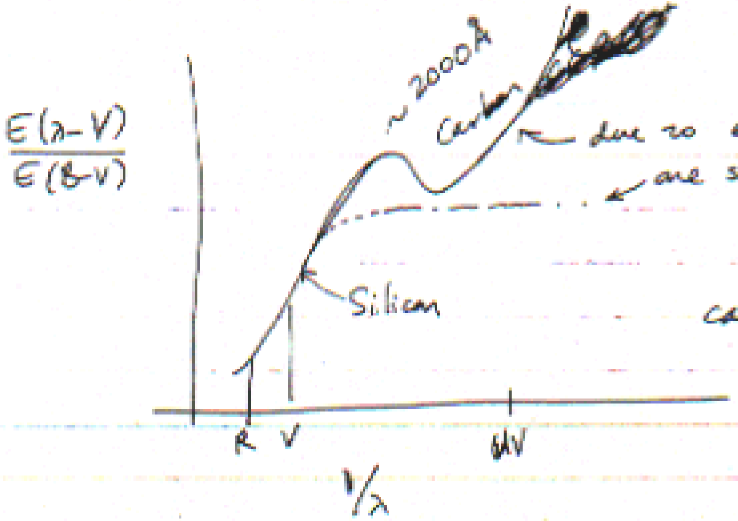
Color Excess

$$E(m_{\lambda_1} - m_{\lambda_2}) = \underbrace{(m_{\lambda_1} - m_{\lambda_2})_{obs}}_{\text{essentially a flux ratio} \rightarrow \text{steepness of SED}} - (m_{\lambda_1} - m_{\lambda_2})_{intrinsic}$$

$$= -2.5\log\left[\frac{(F_{\lambda_1}/F_{\lambda_2})e^{-(\tau_1-\tau_2)}}{F_{\lambda_1}/F_{\lambda_2}}\right] = 1.086(\tau_{\lambda_1} - \tau_{\lambda_2})$$

$$= A_{\lambda_1} - A_{\lambda_2}$$

10/24/2000



← due to ensemble of particle sizes

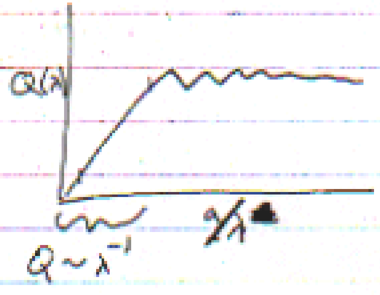
← one size of particle

$n(a) \sim a^{-3.5}$

can learn about nature of particles fr. shape of curve

$$\tau_D = N_D \underbrace{Q_{ext}(\lambda)}_{Q_{abs}(\lambda) + Q_{scatt}(\lambda)} \pi a^2$$

Theory (scattering)



10/26/2000

David Wilner

Molecular Clouds H_2

most impt tracer = CO
- high abundance ~5×10^{-5} CO/H_2 molecules
- (next highest abund = dust)
- favorable excitation ~5.5K (J=1→0)
- chemically robust

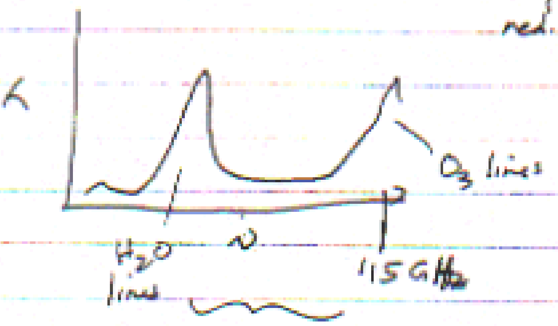
$$h\nu = hB\,2(j+1)$$

rotational constant $= \dfrac{h}{8\pi^2 I}$ (units of freq.)

I ↑ MR^2 ~10

red. mass of C+O ⇒ radio freq.

$\nu_{1-0} = 115.27$ GHz



obs. window in Earth atmosphere

Use CO to measure N, T

$$\Delta I^{12} = \left(B(T_{ex}^{12}) - B(T_{bg})\right)\left(1 - e^{-\tau_{12}}\right) \qquad (^{12}CO)$$

T_{bg} = 2.7 K

$$\frac{n_2}{n_1} = \frac{g_1}{g_2} e^{-h\nu/kT_{ex}}$$

LTE ⇒ $T_{ex} = T$

$$B = \frac{2kT\nu^2}{c^2}$$

$$\Delta I^{13} = \left(B(T_{ex}^{13}) - B(T_{bg})\right)\left(1 - e^{-\tau_{13}}\right) \qquad (^{13}CO)$$

if $\tau_{12}, \tau_{13} \ll 1$ and $T_{ex}^{12} = T_{ex}^{13}$

$$\Rightarrow \frac{\Delta I^{12}}{\Delta I^{13}} = \frac{\tau^{12}}{\tau^{13}} = \frac{N^{12}}{N^{13}} = 89 \Rightarrow \text{solar neighborhood, Sun}$$

measure w/ telescope: "solar abundance"

$$\frac{\Delta I^{12}}{\Delta I^{13}} \sim 3\text{-}5$$

$\Rightarrow$ ~~[illegible]~~ ^{12}CO optically thick

$\Rightarrow \tau_{12} \gg 1$

$$\Rightarrow \frac{\Delta I^{12}}{\Delta I^{13}} \approx \frac{1}{\tau_{13}} \quad {}^{13}\text{CO opt. thin.}$$

$$\Delta I^{12} = [B(T_{ex}) - B(T_{bg})](1) \quad \text{for } \tau_{12} \gg 1$$

$$\tau_\nu = \frac{(h\nu)^2}{4\pi k} B_{lu} \underset{\hookrightarrow \text{ line profile}}{\phi_\nu} \frac{N_l}{T_{ex}} \quad (\text{HI})$$

apply to ^{13}CO

$$N_{tot}(^{13}\text{CO}) = \underset{\substack{\uparrow \\ \text{physics}}}{(\text{const})}\,\underset{\substack{\uparrow \\ ^{13}\text{CO} \\ \text{spectrum}}}{\tau}\,\underset{\substack{\nwarrow \\ ^{12}\text{CO}}}{T_{ex}}$$

$$N_{tot}(H_2) = X\, N_{tot}(^{13}\text{CO})$$

(digression)

^{12}CO opt thick, yet measured ^{12}CO still related to mass size of cloud

$\Rightarrow$ filling factor, clumpiness

Meeting #12 AY208 v. Y2K

"Cleaning Up Dust"

Charlie Lada gave you:

History, Techniques, A_λ ← wavelength dep of extinction, $E(B-V)$

[def. of color excess

Today:

Add "Theory" of $\underbrace{\text{Abs., Scatt,}}_{\Downarrow \text{ Extinction}}$ Polarization

Give dust extra time in this course for ② reasons:

① Most important constituent of M.W. ISM for extragalactic obsv'ns, at many λ's

② AG likes it!

Astronomy 208 - Meeting #12

Interstellar Dust

The Basic Question:

"How much is there & where?"

Ⓐ Easiest Approach: Star-Counting, Extinction Studies (e.g. Barnard, Bok)

Ⓑ Less direct: Thermal Dust Emission (KAO, IRAS, SOFIA, SIRTF) + mm, sub-mm

Ⓒ Even Less direct Approach: Reddening Studies

look @ effect of dust on ★ colors

Ⓓ Least direct Approach: Measure Gas, infer N_{dust} from "assumed" dust/gas ratio

Complications:

- Not all dust grains are the same.

PROPERTIES
- sizes
- composition
- shapes

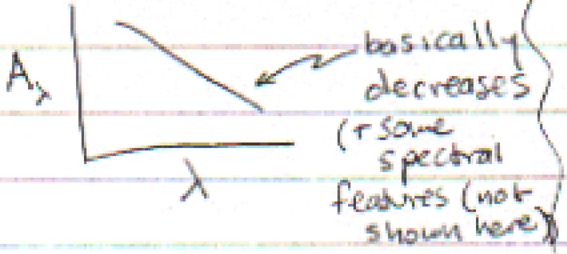
} vary w/ local conditions

- Absorption, scattering & emission efficiency depends on "PROPERTIES" above. These efficiencies also vary as a function of λ.

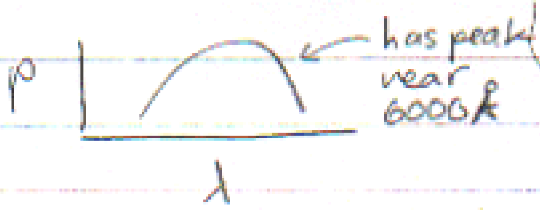
Two ways to look at the problem — they need to be connected for a full understanding...

Empirical

① • extinction vs. wavelength

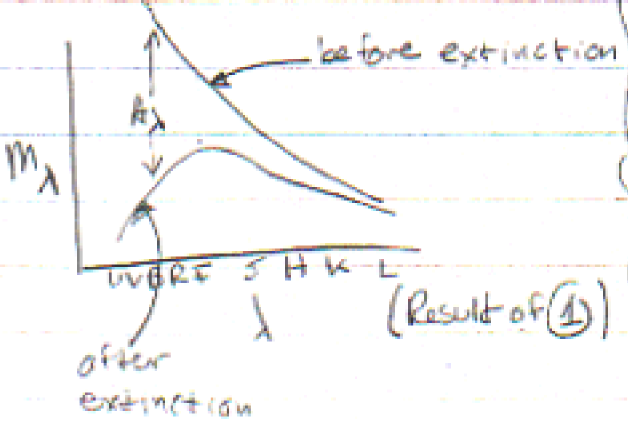


② • polarization vs. wavelength



③ • Star Color Changes as a funxn of Wavelength

aka "Selective Extinction" or "Color Excess"



(Result of ①)

Theoretical

photon encounters grain

(i) SIZE

Result of encounter depends most strongly on ratio $\frac{a}{\lambda}$.

$a \gg \lambda$ → "brick wall" all photons absorbed

$a \ll \lambda$ → very little effect photons "sneak around" grains

$a \simeq \lambda$ → need detailed e.g. scattering theory (Mie), but this is where grains have maximum effect

(ii) COMPOSITION

index of refraction determines scattering properties

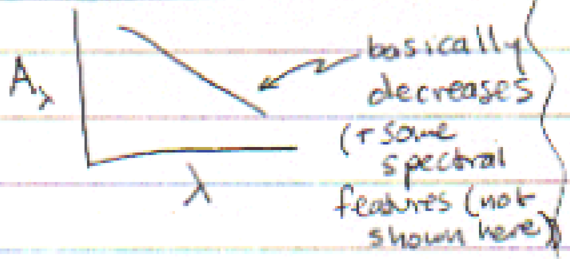
(iii) SHAPE

- "fluffy" or fractal grains have pieces w/ size $\ll a$
- strong effect on polarizing properties

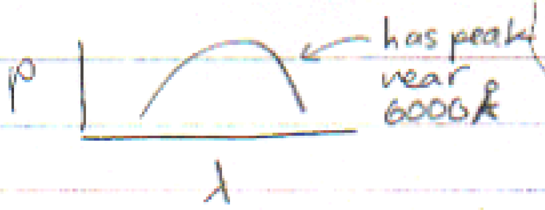
Two ways to look at the problem – they need to be connected for a full understanding...

Empirical

① • extinction vs. wavelength



② • polarization vs. wavelength



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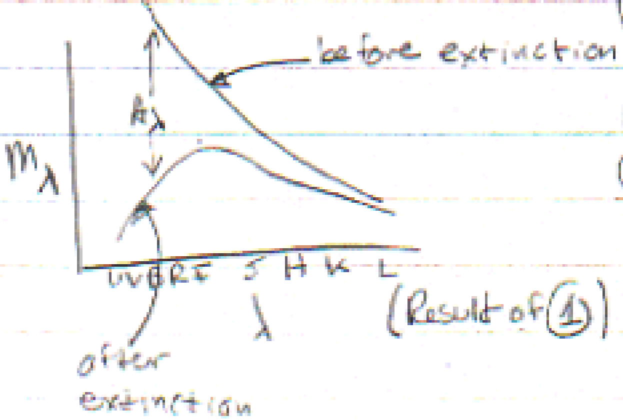

(Result of ①)

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index of refraction determines scattering properties

(iii) SHAPE

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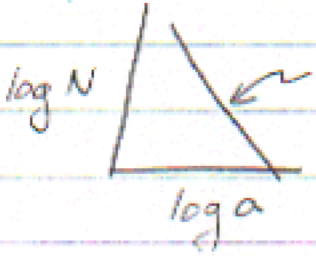
- strong effect on polarizing properties

TWO Most fundamental Empirical ⇔ Theoretical Connections

Observed effects of dust depend critically on size distribution, $N(a)$



slope ~ −3.5 (Mathis, Rumpl & Nordseik 1977 ApJ 217, 425)

See Kim & Martin 1994

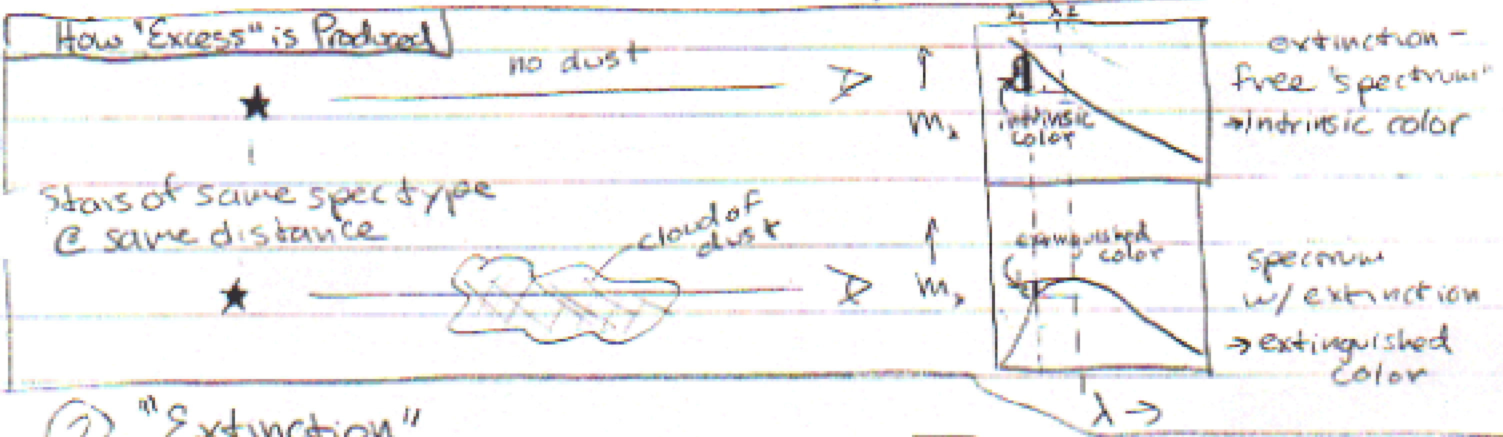
① Total # grains (in "relevant" size range) $= \int_{a_1}^{a_2} N(a)\,da$ related to magnitude of overall effects of dust (e.g. how much extinction, how much emission, etc.)

② Details of Size Distribution $(N(a))$ related to wavelength dependences of extinction, emission, scattering, polarization, etc.

Arcane Astronomical "Definitions" Related to Dust

① "Color Excess" = degree of reddening or "selective extinction"

$$\text{color excess} = \boxed{E_{\lambda_1-\lambda_2}} = \underset{\text{observed "color"}}{\overbrace{(m_{\lambda_1} - m_{\lambda_2})}^{\text{obs'd mags}}} - \underset{\text{"color" that would be observed w/o extinction}}{\overbrace{(m_{\lambda_1} - m_{\lambda_2})_0}^{\text{apparent mags w/o extinction}}}$$



② "Extinction"

$$\boxed{\underset{\substack{\uparrow\\\text{apparent mag @ }\lambda_1}}{m_{\lambda_1}} = \underset{\substack{\uparrow\\\text{absolute mag @ }\lambda_1}}{M_{\lambda_1}} + \underset{\substack{\uparrow\\\text{extinction @ }\lambda_1}}{A_{\lambda_1}} + 5\log \underset{\substack{\uparrow\\\text{distance in pc}}}{d} - 5}$$

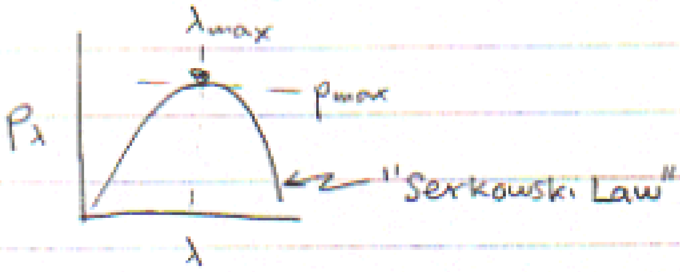
③ "Ratio of total-to-selective Extinction," R_{λ_2}

$$\boxed{A_{\lambda_2} = R_{\lambda_2} E_{\lambda_1-\lambda_2}}$$

$$\text{e.g. "}\underset{\substack{\uparrow\\\text{extinction in the V-band}}}{A_V} = \underset{\substack{\uparrow\\\text{ratio for V-band}}}{R_V}\, \underset{\substack{\uparrow\\\text{color excess where}\\\lambda_1 = B\\\lambda_2 = V}}{E_{B-V}}\text{"}$$

(~ Part of § 3.3b)

④ Wavelength of Maximum Polarization



$$\frac{p_\lambda}{p_{max}} = \exp\left\{-K \ln^2\left(\frac{\lambda_{max}}{\lambda}\right)\right\}$$

note: K actually can be a function of λ_{max}

MORE INFO: If grains all same size $\boxed{\lambda_{max} \simeq 2\pi \underset{\text{size}}{a}(\underset{\uparrow}{n}-1)}$

↑ index of refraction ≈ 1.6 for silicates

More on why later.

For now realize that this is where the grain size is ≈ wavelength & that tells you about the size distribution of (polarizing) grains.

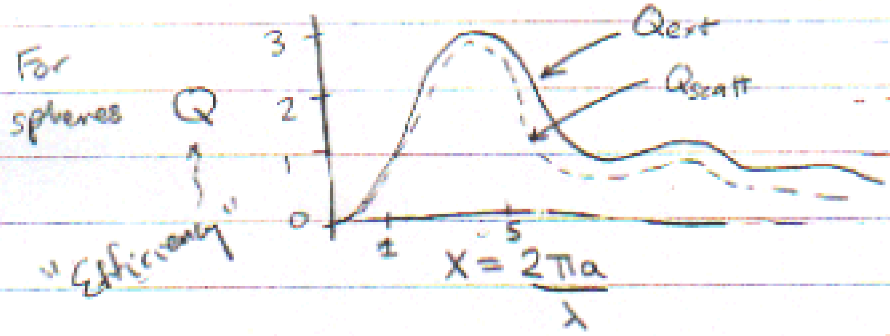
Empirical claim: (Whittet & van Breda 1978)

$$\left\{ \boxed{\underset{\uparrow}{R_V} = (5.6 \pm 0.3)\lambda_{max}} \right\} \quad \lambda_{max} \text{ in } \mu m$$

changes in this also related to changes in grain size distribution

Making the Observation – Theory Connection

All boils down to scattering theory. { Mie scattering theory / Discrete Dipole Approx (DDA)



exact values depend on complex refractive index, $m = n - ik$

(for dielectrics, $k = 0$)

n, k called "optical constants"
(see work of Draine & Lee)

Extinction

$$Q_{ext} = \overset{\text{absorption}}{Q_{abs}} + \overset{\text{scattering}}{Q_{scatt}} \text{ = photons removed from l.o.s.}$$

see Whittet '92 p.58 : $A_\lambda = 1.086\, \underset{\text{column density of dust ptcls}}{N_d}\, \pi a^2 Q_{ext}$ for one-size-fits-all dust

[illegible]

$$\boxed{A_\lambda = 1.086\pi \int a^2 Q_{ext}(a) N(a)\, da}$$ for distribution of N(a)

so if "optical constants" & N(a) are known, then A_λ can be predicted theoretically
(e.g. handout from Mathis' review; see Cardelli, Clayton & Mathis 1989 ApJ 345, 245)

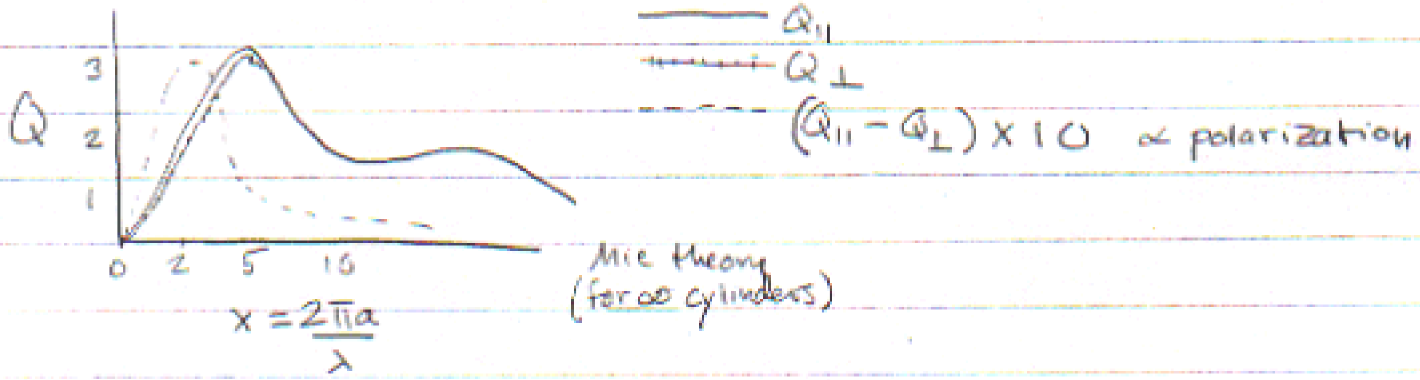
Fine Points:

$\text{Albedo} = \gamma = \frac{Q_{scatt}}{Q_{ext}}$ "How shiny?" Very relevant to KBOs

see Whittet '92 p. 61 for discussion of scattering.

Polarization

Efficiencies for extinction are ≠ ∥ & ⊥ to grain's short axis



Mie theory (for ∞ cylinders)

σ = x-secn'l area of grain in plane of wavefront $(\sim\pi \overset{\text{short axis}}{a}\,\overset{\text{long axis}}{b})$

$$A_{\parallel} = 1.086\, N_d\, \sigma\, Q_{\parallel}\ ; \quad A_{\perp} = 1.086\, N_d\, \sigma\, Q_{\perp}$$

$$\boxed{p = \text{polarization} = A_{\parallel} - A_{\perp} = 1.086\, N_d\, \sigma (Q_{\parallel} - Q_{\perp})}$$

for a given grain size

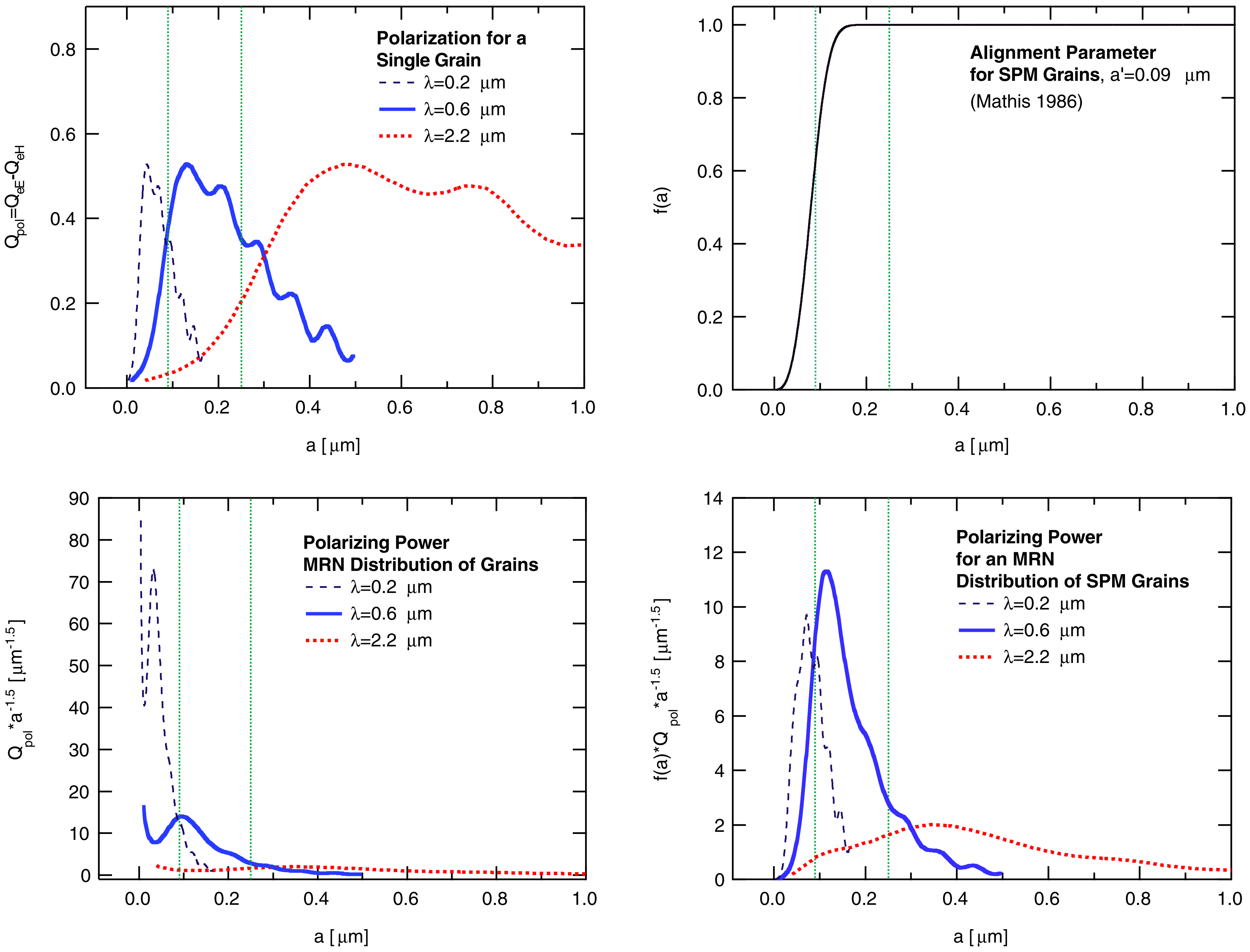
$$\boxed{\frac{p}{A} = 2\left\{\frac{Q_{\parallel} - Q_{\perp}}{Q_{\parallel} + Q_{\perp}}\right\} = \text{"polarization-to-extinction ratio"}}$$

Actual p_λ will depend on size distribution of polarizing grains.

See AG for discussion of pol.ay208.jpg

in this directory

(return to this in discussion of B-fields)


Polarization for a Single Grain
λ=0.2 µm
λ=0.6 µm
λ=2.2 µm
$Q_{pol}=Q_{eE}-Q_{eH}$
0.0
0.2
0.4
0.6
0.8
1.0
a [µm]
Alignment Parameter for SPM Grains, a'=0.09 µm
(Mathis 1986)
f(a)
Polarizing Power MRN Distribution of Grains
$Q_{pol}*a^{-1.5}$ [$\mu m^{-1.5}$]
0
10
20
30
40
50
60
70
80
90
Polarizing Power for an MRN Distribution of SPM Grains
$f(a)*Q_{pol}*a^{-1.5}$ [$\mu m^{-1.5}$]
2
4
6
8
10
12
14

Wavelength Dependence of Extinction and Polarization

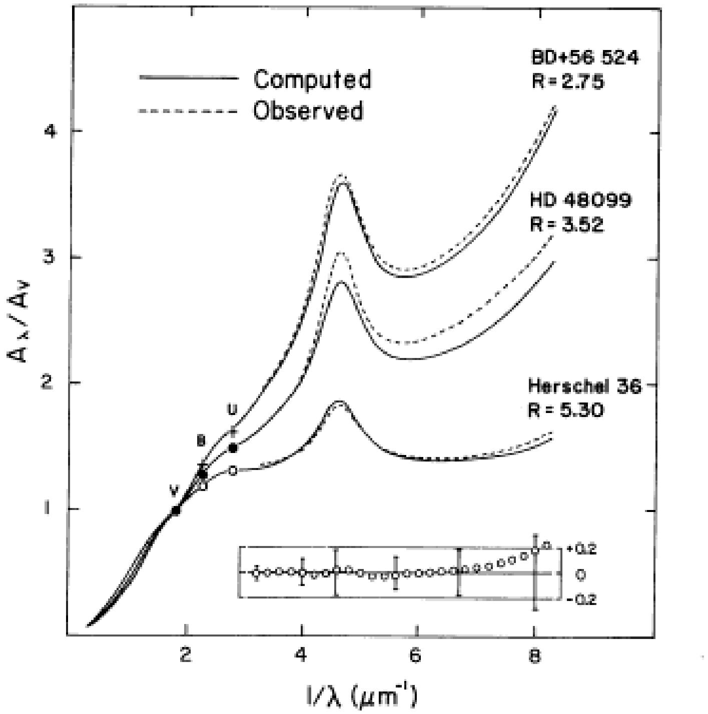


FIG. 4.—Same as Fig. 3 except for the UV portion of the mean R_V-dependent extinction law from eq. (4). The data at U, B, and V from Fig. 3 are also plotted. Again, the "error" bars in the lower inset represent the computed standard deviation of the data about the best fit of $A(\lambda)/A(V)$ vs. R_V^{-1} with $a(x) + b(x)/R_V$. The open symbols in the inset represent the difference between $A(\lambda)/A(V)$ from eq. (4) and the average curve of Seaton (1979) for $R_V = 3.2$. The only serious deviation occurs for $x > 7\ \mu m^{-1}$ (see text).

Cardelli, J. A., Clayton, G. C. & Mathis, J.S.1989, ApJ, 345, 24

$$p/p_{\max} = \exp\left[-K \ln^2 (\lambda_{\max}/\lambda)\right]$$

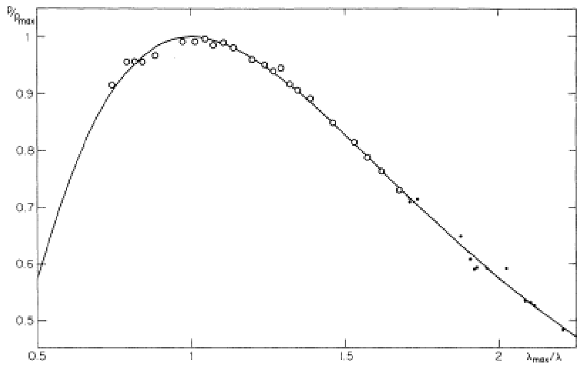


FIG. 3.—The normalized wavelength dependence of interstellar linear polarization derived from the observations with the Siding Spring multichannel polarimeter-photometer. The solid line is calculated from eq. (4) for $K = 1.15$. Every open circle is based on 20 stars, while each dot represents the observations of an individual star with a particular filter.

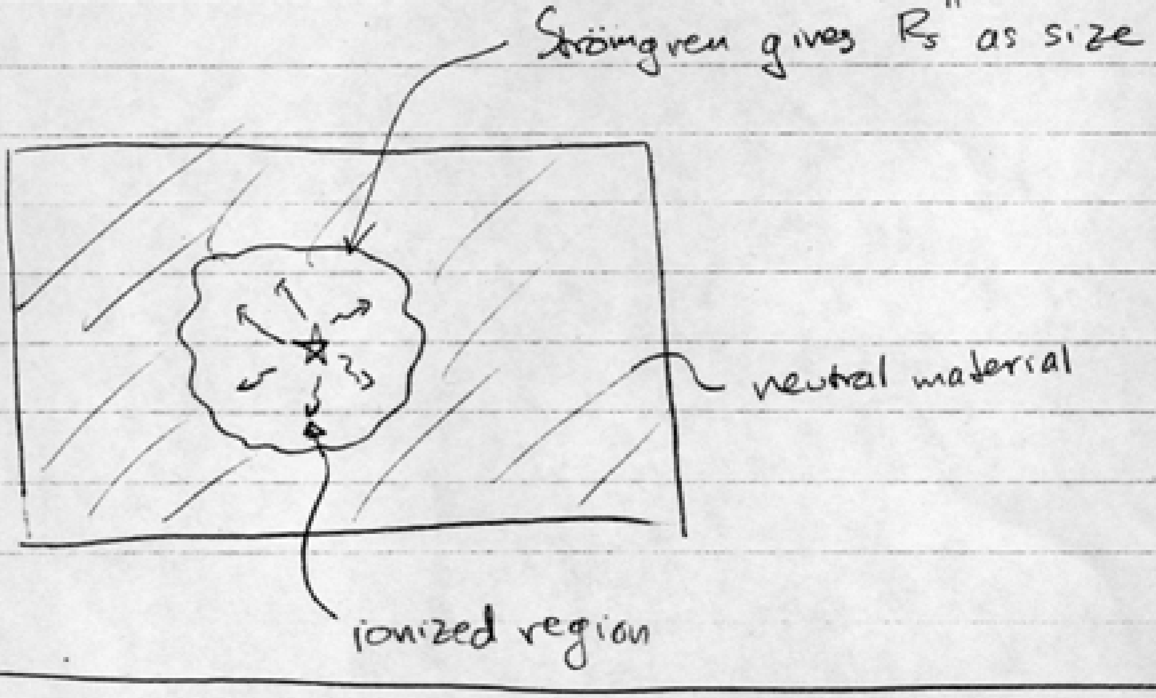
Serkowski, K., Mathewson, D. L.& Ford, V. L.1975, ApJ, 196, 261

Astronomy 208 v. Y2K Meeting #16

Order for next sections: H II Regions — Th Rodriguez-Gaspar et al
then SNR + Hot ISM — Tues Williams et al
(incl shock discussions)

Still Remaining (7 meetings after Thanksgiving −1 for Paolo?)
- more on S.F. & winds from young & evolved ☆'s
- ISM in external galaxies
- IGM / ISM at $z >> 0$

Today Basics of H II Regions



? Time evolution

? Effects of clumpiness/structure

? Effects of/on gas composition

4. Interactions of Photons with the ISM

4.1 HII Regions & PDRs

a.) <u>Strömgren Sphere</u> : "Equilibrium" where ionizations = recombns

(Strömgren, B. 1939 ApJ <u>89</u>, 529)

X = # ionizing photons/sec from ☆ ($\lambda < 912 Å$)

$\alpha n_e n_H$ = # recomb/sec / volume

$\uparrow$ = recombination coefficent ($\approx 3.1 \times 10^{-13}\ cm^3 s^{-1}$ @ 8000K)

Assume all "H" & electrically neutral H^+ = proton

so $n_e = n_p$

Total ionized volume $= \frac{4}{3}\pi R_s^3$

So   Ioniz = Recomb

$$X = \frac{4}{3}\pi R_s^3 \alpha n_e n_H$$

$$\boxed{R_s = \left(\frac{3X}{4\pi\alpha}\right)^{1/3} (n_e n_H)^{-1/3}}$$

often assumed that $n_e = n_H$ @ boundary

so $R_s = \left(\frac{3X}{4\pi\alpha}\right)^{1/3} n_H^{-2/3}$

Allows for gross estimates:

e.g.   O6☆ $T_{eff} \cong 4.5 \times 10^4 K$; $L \tilde{=} 1.3 \times 10^5 L_\odot$

Wiens Law $\Rightarrow \lambda_{max} = 640 Å$ (So assume all $\lambda < 912 Å$ & @ $640 Å$)

$X = L / {hc/\lambda_{max}} = 1.6 \times 10^{49}$ phot/sec $\boxed{\text{for } n_H = 5000 \quad R_s = 0.3 pc}$

often lower due to clustering

Refinements :

Real ☆ spectrum: # ionizing photons/sec = $n_H \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H) d\nu$

n_H = density

J_ν = mean intensity

$\frac{4\pi J_\nu}{h\nu}$ = # incident phot of freq ν

$a_\nu(H)$ = ionization x-secn for H by photons w/ $h\nu$
$\sim 6 \times 10^{-18}$ cm^{-2} for H

BUT
- J_ν should really include reduction by scattering w/in the nebula
- Multiple ionizations by one photon
- $\boxed{\text{Time Evolution}}$

Return to Time Evolution for Rest of Today
after looking @ possible morphologies

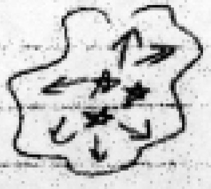
See Also

http://cfa-www.harvard.edu/~agoodman/hii.html

Types of Real HII Regions

Giant HII Regions $5-50\,pc$

(OB assn) $1-100\ cm^{-3}$



e.g. 30 Dor W3

Blister HII Regions $1-10\,pc$ (see below)

$10^2-10^3 cm^{-3}$

Compact & Ultracompact $\lesssim 1\,pc$ to $<< 1\,pc$

$> 10^3$ to $>> 10^5\ cm^{-3}$

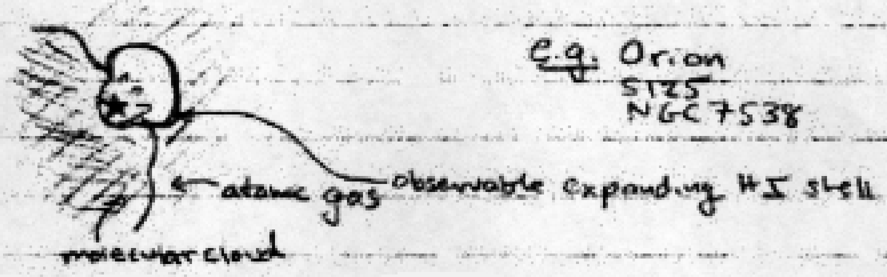
Often produce → champagne-flow

Ref: Tenorio-Tagle 1979

e.g. (many) see Garay et al article

Note Geometry: blister

usually larger than "champagne flows"



e.g. Orion
S125
NGC 7538

FORMATION OF AN HII REGION

(Approximation)
for constant n

① ★ turns on instantaneously, emitting ionizing radiation @ rate $\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu$ $\frac{\text{photons}}{\text{sec}}$

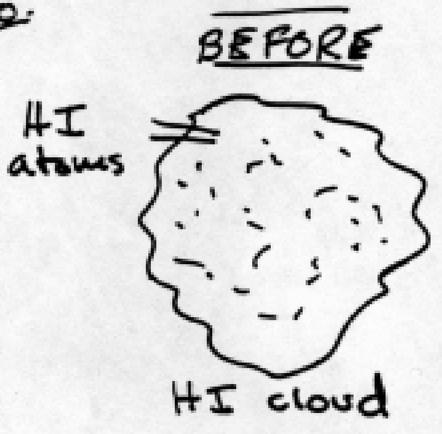
L_ν = luminosity @ ν

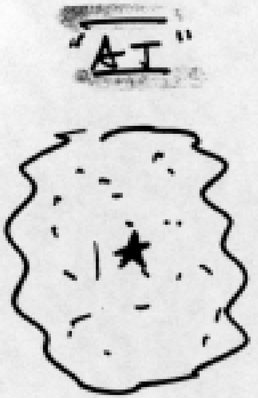
$h\nu_0$ = energy needed to ionize from ground state

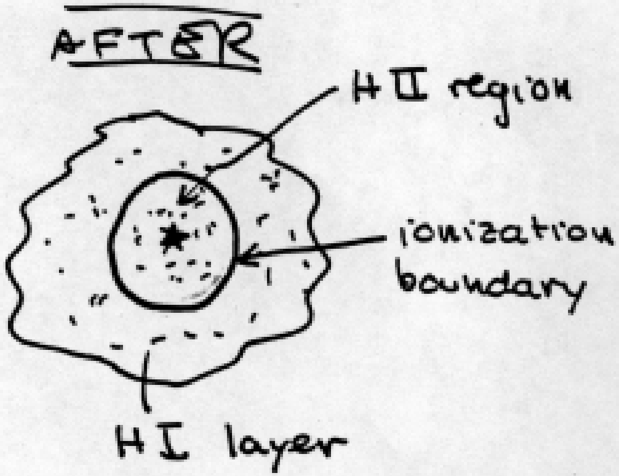
② Assume ★ is in a cloud of HI gas with n_H $\frac{H}{cc}$

③ Photons move out @ c & ionize gas

So:







In this Approximation... growth analysis:

star emits photons @ rate

$$\boxed{\frac{d}{dt}(N_i) = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu}\, d\nu} \qquad \frac{\text{photons}}{\text{sec}}$$

which ionize a spherical shell around ★ of radius R thickness dR ⟹

shell contains $\boxed{4\pi R^2 dR\, n_H}$ atoms of H I

rate of change of R given by

$$\boxed{\frac{dN_i}{dt} = 4\pi R^2 n_H \frac{dR}{dt}}$$

because each ionizing photon ionizes one n_H (ignoring recomb)

<u>In H I gas</u> m.f.p. = $\boxed{\delta = \frac{1}{n_H a}}$

a = ionization x-sectn $\approx 6\times10^{-18}$ cm^{-2} for H

$\Rightarrow \underline{\delta_H = 1.67\times10^{14}\,\text{cm} = \boxed{5\times10^{-5}\,\text{pc}}}$

photon will ionize w/in 5×10^{-5} pc

But in ionized gas, relevant x-sectn is for scattering

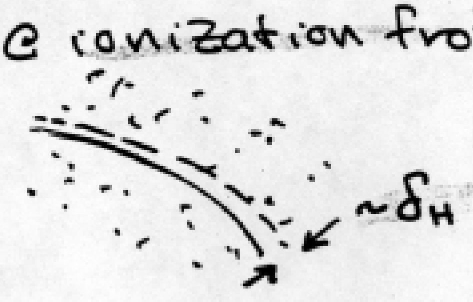
$\sigma \cong 6.7\times10^{-25}$ cm^2 = very small

$\underline{\delta_i = \frac{1}{n\sigma}} = \boxed{469\,\text{pc!!}}$ = very large m.f.p.

photon not bothered much

⟹ very sharp transition @ ionization front

ere m.f.p gets very short



In the previous growth approx, we neglected RECOMBINATION, which will slow or halt growth.

[α = recombination rate coeff [$\text{cm}^3\ \text{s}^{-1}$]
$= 4 \times 10^{-13}\ \text{cm}^3\ \text{s}^{-1}$ for H^+]

[t_{recomb} = recomb time = $\frac{1}{n_H \alpha}$ = 80 years]

Thus, including Recombinations, we find:

$$\underbrace{\frac{dN_i}{dt}}_{\text{ionizations}} = \underbrace{4\pi R^2 n_H \frac{dR}{dt}}_{\text{growth}} + \underbrace{\frac{4}{3}\pi R^3 n_p n_e \alpha}_{\text{recombinations}}$$

EQUILIBRIUM state = constant volume $\Rightarrow \boxed{\frac{dR}{dt} = 0}$

$$\Rightarrow \boxed{R_s^3 = \frac{3}{4\pi n_e n_p \alpha} \frac{dN_i}{dt}} \Rightarrow \text{"Strömgren Radius"}$$

for B0 to O5 ★'s & $n_H \sim 10^3\ \text{cm}^{-3}$ $\boxed{R_s \sim 0.1 - 1\ \text{pc}}$

Recall

To get more quantitative, we need to consider

- → stellar spectrum to give $\frac{dN_i}{dt}$ ✓ O.K.
- → radiative transfer of ionizing photons ✓ O.K.
- → structure in neutral gas too hard for today
- → composition of neutral gas assume all H today

Define:
- n_H° = volume dens. neut. H
- n_e = " " electrons (free)
- n_p = " " protons (free)
- J_ν = mean intensity of radiation field $= \frac{1}{4\pi}\int I_\nu d\Omega$
- $a_\nu(H)$ = ionization x-sectn $\underset{h\nu}{\rightsquigarrow} \cdot H$
- $\alpha(H^\circ, T)$ = recomb. rate coeff ($cm^3 s^{-1}$)

Set

$$\frac{\text{ionizations}}{\text{time}\cdot\text{volume}} = \frac{\text{recombinations}}{\text{time}\cdot\text{volume}}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

then

$$\underbrace{n_{H^\circ} \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H)\, d\nu} = \underbrace{n_e n_p \alpha(H^\circ;T)}$$

"Ionization Equilibrium Equation" for a Pure H Cloud.

In the H II region, the equ. of transfer is:

$$\frac{dI_\nu}{ds} = -n_{H^0} a_\nu I_\nu + j_\nu$$

I_ν is a bit complicated because some photons are from ★ & some from nebula itself

$$I_\nu = I_{\nu s} + I_{\nu d}$$

<u>Stellar Component</u>: ★ is only source $\Rightarrow j_\nu^\star = 0$

so we have

$$4\pi J_{\nu s} = \pi I_{\nu s}(R_\star) \times \frac{R_\star^2}{r^2} e^{-\tau_\nu} \left.\vphantom{\frac{R^2}{r^2}}\right\} \begin{array}{l} \frac{1}{r^2} \text{ dilution} \\ \tau \text{ dilution} \end{array}$$

<u>Diffuse Component</u>: source of ionizing $h\nu$ is recaptures to the <u>ground state</u> (Lyman series)

$$\text{so} \quad \# \text{ ionizing photons} = 4\pi \int_{\nu_0}^{\infty} \frac{j_\nu}{h\nu} d\nu = n_p n_e \underset{\uparrow}{\alpha_1}(H^0, T)$$

recomb to ground state coeff

<u>Two</u> Approximations Possible

1. Optically <u>Thin</u> — ignore diffuse component
   say all Ly photons escape
2. Optically <u>Thick</u> — say all Ly photons absorbed
   & re-radiated (possibly @ longer λ)

Optically thick Case (More Interesting)

source func
ionization x-sectn
mean intensity

$$\underbrace{4\pi \int^{vol} \frac{j_\nu}{h\nu}\, dV}_{\text{emitted}} = \underbrace{4\pi \int^{vol} N_{H^\circ} \frac{a_\nu J_{\nu d}}{h\nu}\, dV}_{\text{absorbed}}$$

"On-the-spot" approximation: above eq. holds locally

so that:

$$J_{\nu d} = \frac{j_\nu}{n_{H^\circ} a_\nu}$$

not too bad, since m.f.p. for abs. is short in τ high case

Substituting, we find that the equation of transfer

ionization equilibrium becomes

re-rad comp (source func) stellar comp

$$4\pi J_\nu = \frac{4\pi j_\nu}{n_{H^\circ} a_\nu} + \pi I_{\nu s}(R_*) \frac{R^2}{r^2} e^{-\tau_\nu}$$

$$\boxed{\frac{n_{H^\circ} R^2}{r^2} \int_{\nu_0}^{\infty} \frac{\pi I_{\nu s}(R_*)}{h\nu} e^{-\tau_\nu} a_\nu\, d\nu = n_e n_p \left[\alpha_{tot}(H^\circ, T) - \alpha_1(H^\circ, T)\right]}$$

allow for recomb ground state

Notice that:

recomb rate coeff to ground state

$$\alpha_{TOT} - \alpha_1 = \alpha_2 = \sum_2^{\infty} \alpha_n(H^\circ, T)$$

$\Rightarrow$ recombinations to ground state produces photons which get trapped in nebula — they don't count (don't contribute to ionization balance)

Integrating the ionization equilibrium over r gives:

$$R^2\int_{\nu_0}^{\infty}\frac{\pi I_{\nu_*}(R)}{h\nu}\cdot\int_0^{\infty}e^{-\tau_\nu}a_\nu n_H^0\,dr\,d\nu=\int_0^{\infty}n_p n_e \alpha_2 r^2\,dr$$

but recall $d\tau_\nu = n_H \alpha_\nu dr$ so

$$R^2\int_{\nu}^{\infty}\frac{\pi I_{\nu}(R)}{h\nu}\int_0^{\infty}e^{-\tau_\nu}\,d\nu\,d\tau_\nu=\int_0^{\infty}n_e n_p \alpha_2 r^2\,dr$$

which gives

$$4\pi R^2\int_{\nu_0}^{\infty}\frac{\pi I_{\nu s}(R)}{h\nu}\,d\nu = 4\pi\int_0^{\infty}n_e n_p \alpha_2 r^2\,dr$$

So IF $n_e = n_p = n_H$ for $r \le R_s$ and $n_e \approx 0 = n_p$ for $r > R_s$

we can replace ∞ radius with R_s which means

$$4\pi\int_0^{\infty}n_e n_p \alpha_2 r^2\,dr = 4\pi n_H^2 \alpha_2 \frac{R_s^3}{3}$$

and use definition of Luminosity $L_\nu = 4\pi R_*^2 \pi I_\nu(R)$

an finally(!)

$$\boxed{\int_{\nu_0}^{\infty}\frac{L_\nu}{h\nu}\,d\nu = \frac{4}{3}\pi R_s^3 n_H^2 \alpha_2}$$

R_s = Strömgren Radius!!

4.2½ Heating & Cooling in H II Regions
4.3 Ionization Fraction & Chemical Balance in PDRs ion-neur

Interlude

→ Real Spectrum of an H II Region

4.2½ "Thermodynamic State of H II Regions" (what is T_e?)

$\Gamma - \Lambda = 0$ for equilibrium

Recall: $n_H \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu(H) d\nu = \frac{\text{ionizations}}{\text{time} \cdot \text{volume}}$

Γ: Heating (due to Photoionization)

from last time

ioniz. x-sectn

$$\boxed{\Gamma_{\text{photoionization}} = n_H \int_{\nu}^{\infty} \frac{4\pi J_\nu}{h\nu} \underbrace{h(\nu-\nu_0)}_{(h\nu - h\nu_0)} a_\nu(H^\circ)\, d\nu \ (\text{erg cm}^{-3}\,\text{s}^{-1})}$$

This factor gives "x energy"

The energy above $h\nu_0$ in each ionization is $h(\nu-\nu_0)$

That extra energy → K.E. of electrons

the distribution of K.E.'s is given by a Maxwellian because the x-sectn for e-e collisions is very high

i.e. $\boxed{\sigma_{ee} \approx 10^{-13}\,\text{cm}^2 \quad >> a_\nu \approx 6 \times 10^{-18}\,\text{cm}^2}$

so ~as soon as e^- created, it will find the pre-existing distr. of e^- which have a Boltzmann dist @ T_e ("e^- are rapidly thermalized")

(initially it depends on $J_\nu a_\nu / h(\nu-\nu_0)$ but that information is quickly lost)

Recall J_ν = mean intensity of radiation field $= \frac{1}{4\pi}\int I_\nu d\Omega$

$4\pi J_\nu - 4\pi J_\nu \frac{\nu_0}{\nu} = 4\pi J_\nu \left(1 - \frac{\nu_0}{\nu}\right)$

$4\pi J_\nu \left(\frac{\nu - \nu_0}{\nu}\right)$

Λ : Cooling inside H II Regions ... radiation by...

a) Recombination b.) free-free c.) line cooling

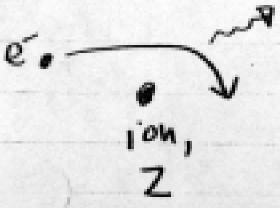
a.) $$\boxed{\Lambda_R = n_e n_p k T_e \beta_2(H^0, T)}$$

β_2 = recomb coeff averaged over Boltzmann v-dist @ T_e
(counting only recomb to $n \geq 2$)

note: for $n_e = n_p = 1$ & $T = 10^4$ K $\left[\Lambda_R \cong 10^{-25} \text{ erg cm}^{-3} \text{ s}^{-1}\right]$

b) $$\Lambda_{ff} = 4\pi \overset{\substack{\text{emissivity}\\ \text{for ff}\\ \downarrow}}{j_{ff}} = 4\pi \int_0^\infty j_\nu \, d\nu$$

(From analyzing interaxn energy as we did for ion-heat)



$$= \frac{2^5 \pi e^6 Z^2}{3^{3/2} h m c^3} \left(\frac{2\pi k T}{m}\right)^{1/2} g_{ff} n_e n_p$$

$$\boxed{\Lambda_{ff} = 1.42 \times 10^{-27} Z^2 T^{1/2} \underset{\uparrow \text{ gaunt factor}}{g_{ff}} n_e n_p} \qquad \propto T_e n_e^2$$

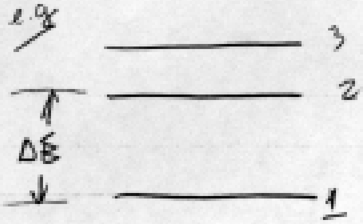
note: for $n_e = n_p = 1$ & $T = 10^4$ K $\left[\Lambda_{ff} \cong 10^{-25} \text{ erg cm}^{-3} \text{ s}^{-1}\right]$
(see handout fig 5.8)

c.) Cooling by Line Radiation from Collisionally Excited Atoms ← Most Important!!

(next page)

c.) cont'd (line cooling)

e.g.

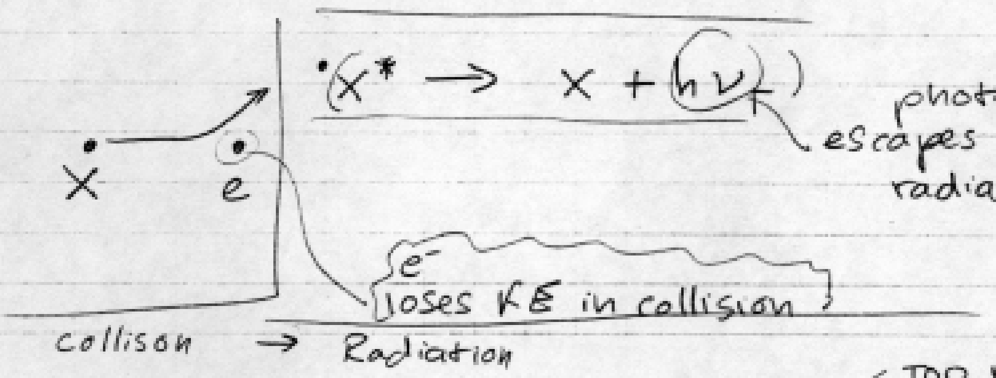


@ T~10^4K

} this is too big for H & He, usually smaller for other elements

Basic Idea! Convert KE of e^-/ion gas into escaping radiation.

Some atom w/@ least one bound e^- = "X"



$(X^* \longrightarrow X + h\nu)$

photon escapes H II region & thereby radiates away energy

X-sectns for collis 10^5 x grtr than for radiative capture

For H, $\Delta E_{21} = 13.6\text{eV} = k(\underbrace{1.6\times10^5\ \text{K}})$ ← TOO HIGH!!

For certain impurities (eg. O^+, O^{++}, N^+) $\boxed{\Delta E \approx kT_e}$

↑ important coolants (scarce: if more abundant, HII reg would cool off fast)

∴ each excited state for each element has its own e^--collision x-sectn & detailed calculations are needed...

But as an example (see Spitzer 1978 Fig 6.1)

[Λ_{OII} @ $T=10^4$K is $\sim 10^{-24}$ erg cm^{-3} s^{-1} for $n_e = n_p = 1$ this is $>> \Lambda_{ff}$ or Λ_R for just 1 line!]

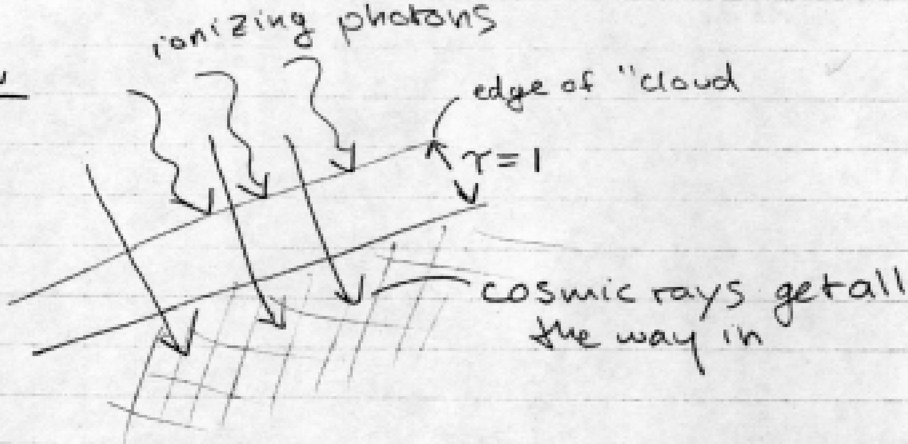
Note: w/o line cooling $T_{HII\ reg} \sim 2.6\times10^4$K
actually more like $\gtrsim 10^4$K

So, this means ultimately T_e is determined by

$$\boxed{\Gamma_{\text{photoioniz}} = \Lambda_{\text{line cooling}} + \Lambda_{\text{free-free}} + \Lambda_{R(\text{recomb})}}$$

4.3 – One Point Most – Ionization Fraction & Chemical Balance in PDRs

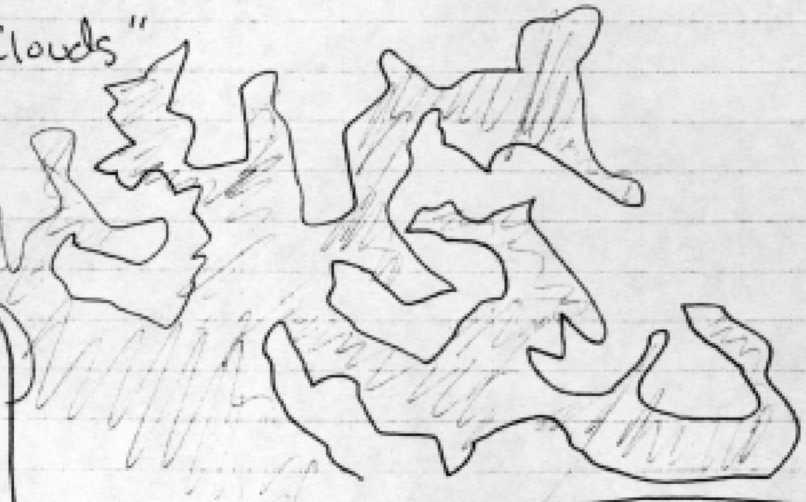
Old View



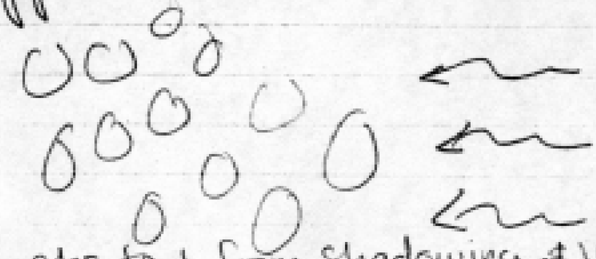
So, ionization in "exterior" layers (& hence chemistry – recall importance of ion-neutral reaxns) is "photon-dominated" – interior ionization from cosmic rays ~only.

New View "Fractal Clouds"

Where is $\tau=1$ surface??



1st approx: Billard Ball models

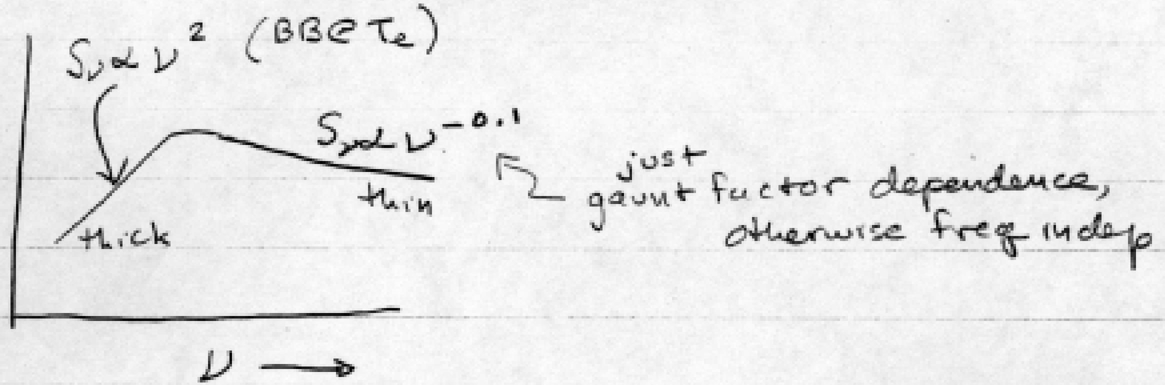


τ gets to 1 from shadowing & in densest balls

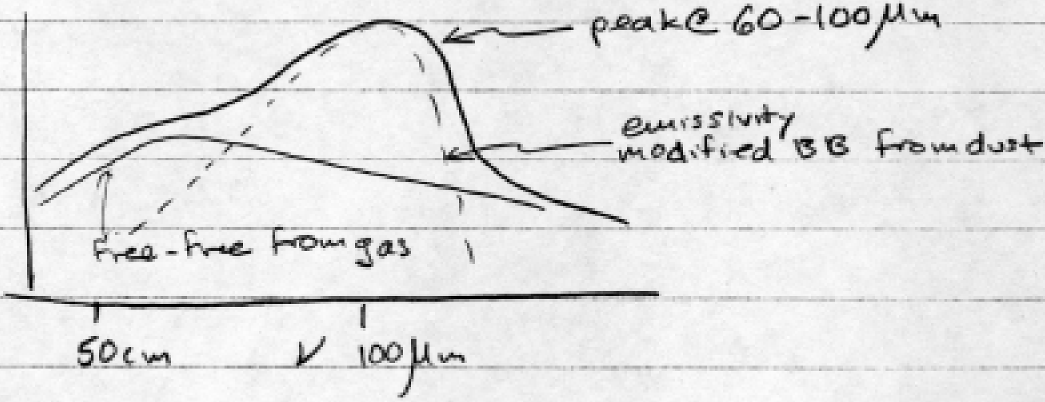
"Real" Spectrum of an H II Region

(Problem Set 4) / ... hints / handouts ...

Lowest ν dominated by free-free (Radio)



Medium ν : Dust re-radiation important

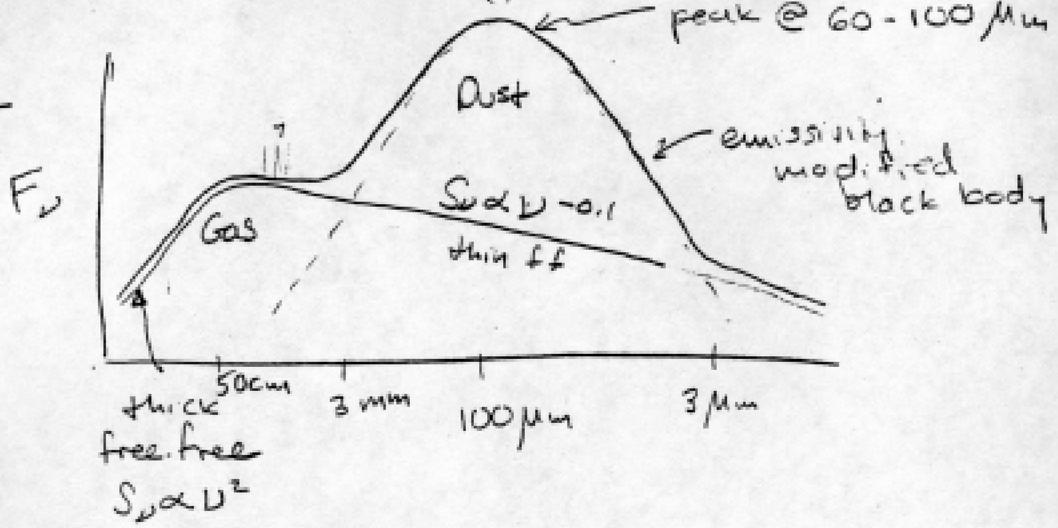


High ν

Lots of spectral lines... dominant coolants...

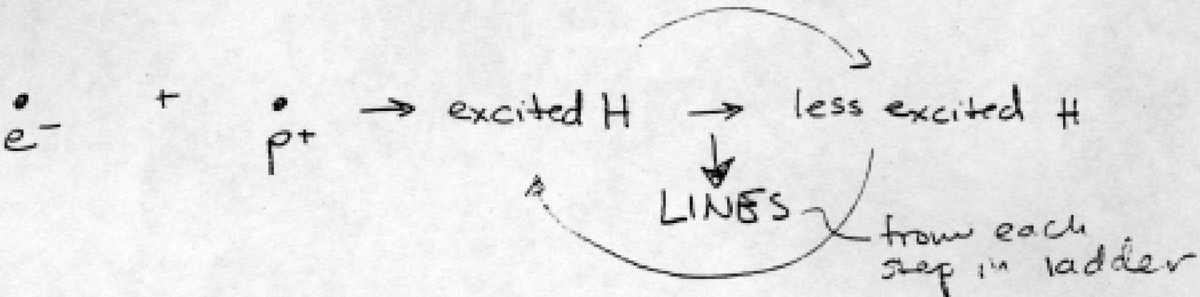
Real "Spectrum" of HII Region:

Last time:



This time: What about spectral lines superimposed on this? (consider today only those w/ origin in HII region, e.g. NOT abs lines due to foreground neut. material)

"Recombination Lines"



Note on notation:

Radio

H76α

hydrogen line

principal quantum # of lower state

Δn=1

(Recomb Lines, cont'd.)

Unfortunate optical notation.

		Radio	Optical
Balmer line of H (optical)	$n = 3 \rightarrow 2$ $(\Delta n = 1)$	H2α	"Hα" → "pink" glow of HII regions
Lymon line of H (ultraviolet)	$n = 2 \rightarrow 1$ $(\Delta n = 1)$	H1α	"Lyα"

When $n \sim 100$ & $\Delta n = 1$ frequencies are "radio"

At $n < 40$ → "optical recombination lines"

see: Spitzer 1978 § 3.3a

calculations complicated by: ① $\Delta\nu_{ul}$ > Doppler width

calc. level populations hard

② What happens to Lyα

Case A

HII rg opt thin to Lyα

hν escapes w/o reionizing anything

Case B

↓

thick to Lyα

Lyα absb'd & converted to longer λ

Significance of Recombination Lines

- ratio of $\frac{\text{energy radiatied in line}}{\text{energy in underlying continuum}}$

like a $\sim$ "partition function"

giving ratio $\frac{\text{bound}}{\text{free}}$ electrons

$\rightarrow T_{gas}$

where pressure broadening is significant, also get n_e

where Doppler profile interesting also get $T_B(v)$

Spitzer 1978

more than an order of magnitude greater. Collisional excitation of the former levels, which produces radiation in the infrared, is quite insensitive to temperature for T greater than 1000°K. Excitation of the latter, however, increases quite sharply with increasing T, since only the electrons in the tail of the Maxwellian distribution have the 1.9 to 3.3 eV of energy required, and the number of these rises sharply with T. Hence these transitions to different spectroscopic terms provide a thermostatic mechanism that tends to keep the temperature in the neighborhood of 10,000°K.

Values of $\Lambda/n_e n_p$ for low-density H II regions ($n_e < 10^2\,\mathrm{cm}^{-3}$) are shown as a function of temperature in Fig. 6.1 [4], computed on the arbitrary assumption that O, Ne, and N, the only three radiating elements considered, are each 80 percent singly ionized and 20 percent doubly ionized. The abundances relative to H are essentially those in Table 1.1. At n_e significantly above $10^2\,\mathrm{cm}^{-3}$, collisional deexcitation reduces Λ below the

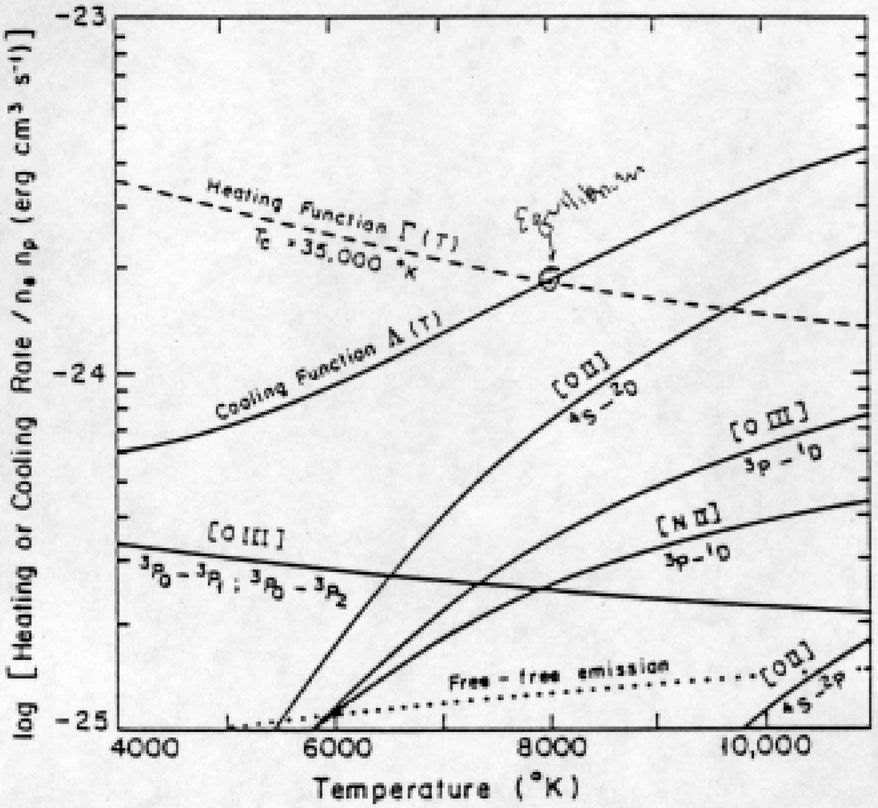
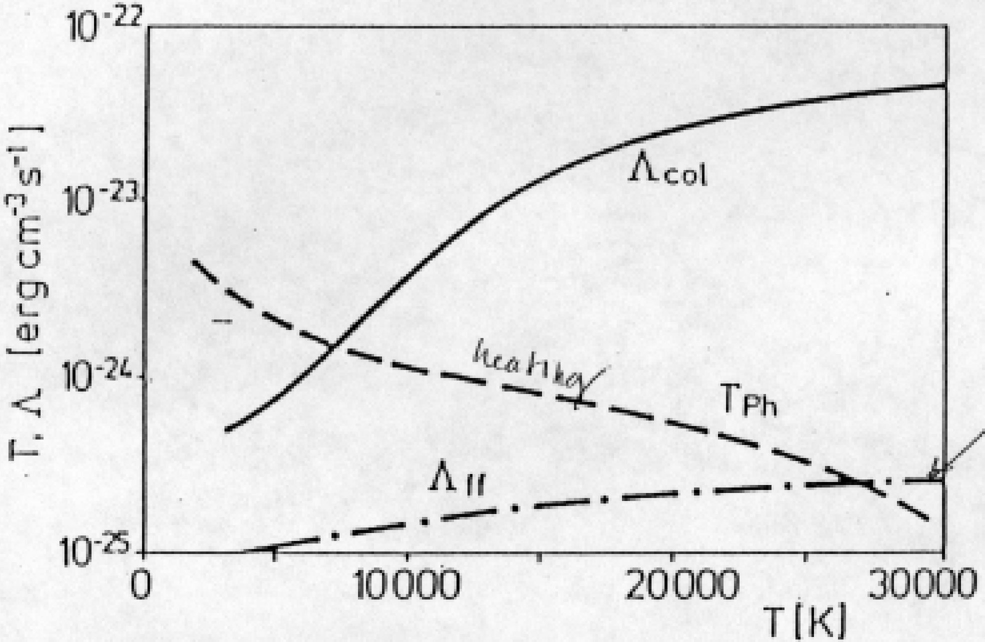


Figure 6.1 Heating and cooling functions in H II regions. Both $\Lambda/n_e n_p$ and $\Gamma/n_e n_p$ are shown as functions of the temperature T. In addition, the contributions to $\Lambda/n_e n_p$ from individual transitions in O and N ions are plotted for the low-density limit ($n_e < 10^2\ \mathrm{cm}^{-3}$) [4]. The dotted line shows $\epsilon_{ff}/n_e n_p$. The heating function represents an average for the H II region as a whole (see text), for a central star of color temperature, T_c, of 35,000° at far ultraviolet wavelengths.



as T rises ff cooling more important, but still << line radiation cooling

Fig. 5.8. Heating and cooling rates for an H II region with $N_e = N_p = 1$, caused by a st with surface temperature $T_\bullet = 32\,000$ K, as a function of the gas temperature T. For oth values of N_e and N_p ($\lesssim 10^2$ cm^{-3}) the ordinate scale denotes $\Gamma/N_e N_p$ and $\Lambda/N_e N_p$ wi dimension erg cm^3 s^{-1}. Explanation in text. [After Spitzer (1968)]

AY218 Handout of Lecture 20

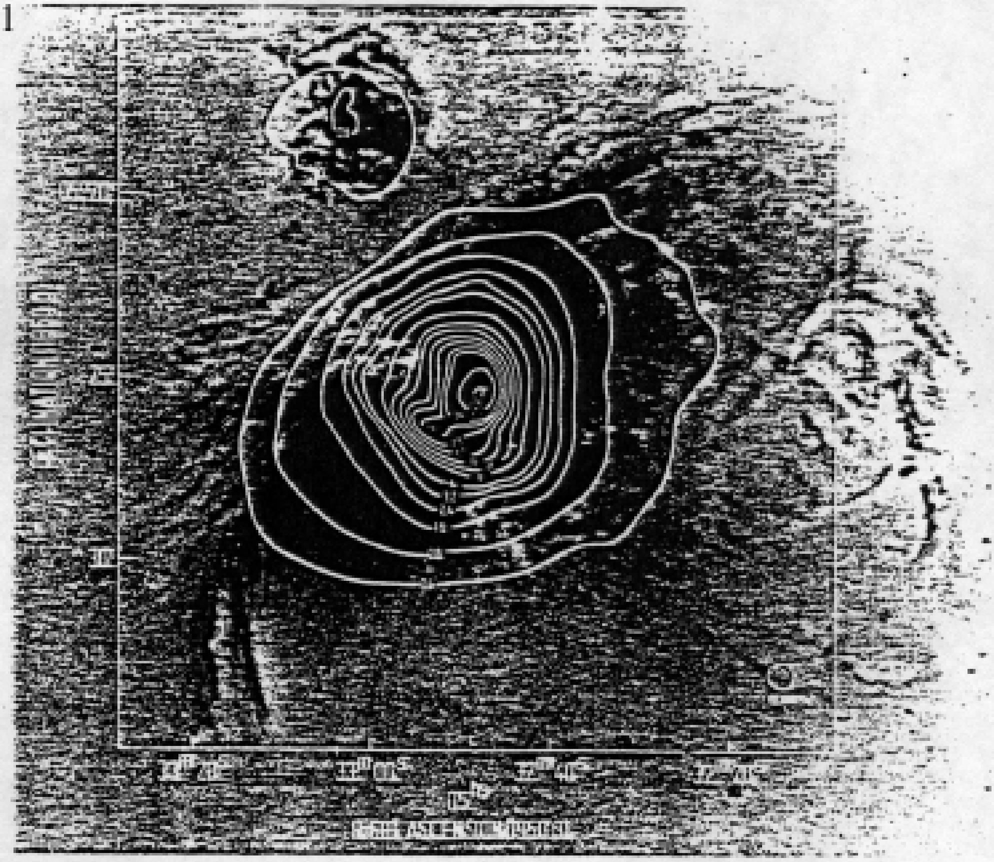


Fig. 2.3. The 23-GHz radio continuum contours, in units of main-beam brightness temperature, on an optical photo in Hα and [NII] of NGC 1976 (Orion A, M42), below, and NGC 1982 (M43), above. The angular resolution is 42", which at the distance of Orion A, corresponds to a linear resolution of 0.10 pc. (Wilson and Pauls, 1984).

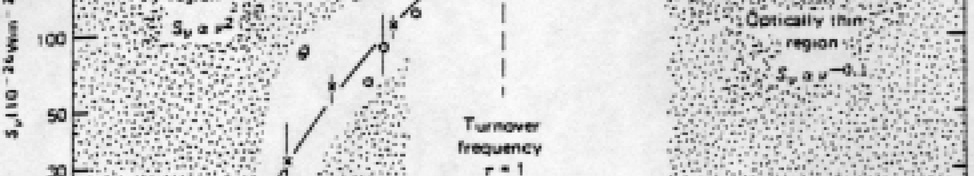


Fig. 2.2. Spectral flux density of the Orion Nebula plotted against frequency. The shaded regions mark the optically thick and thin regions of the spectrum. (Reprinted with permission by Gordon and Breach Science Publishers from: Terzian, Y. and Parrish A., *Astrophysical Letters*. Vol. 5(1970), pp. 261.

FROM: GALACTIC AND EXTRAGALACTIC RADIO ASTRONOMY
EDS. GL VERSCHUUR + K.I. KELLERMAN, 1974, SPRINGER-VERLAG

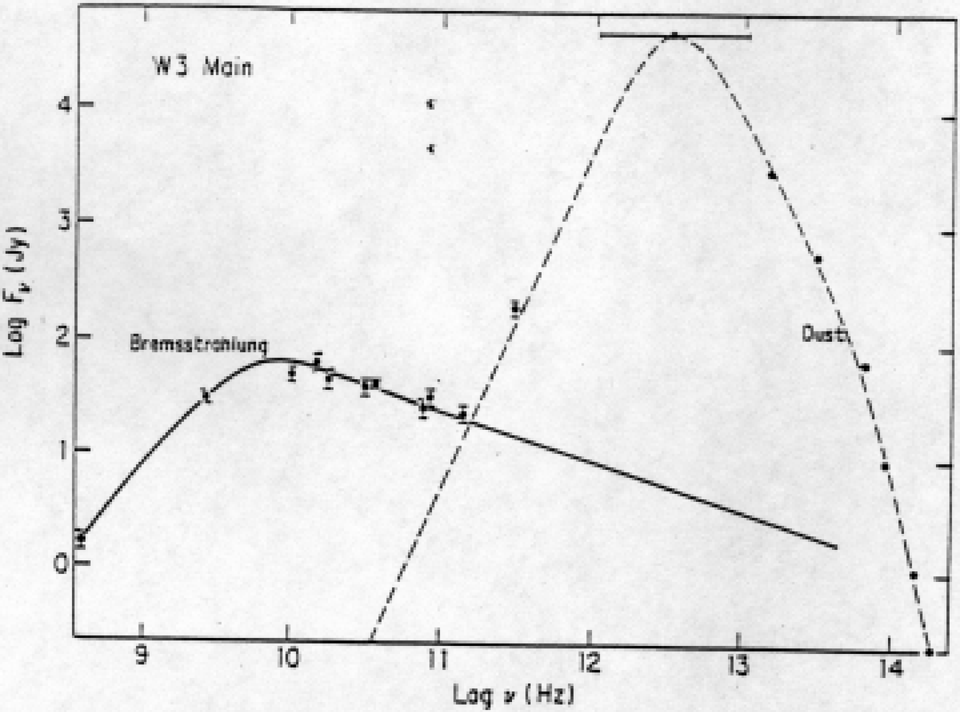


Fig. 2.4. Open circles mark observations of the integrated flux from the HII region W3 Main. Solid line: bremsstrahlung emission; dashed line: thermal emission from dust. (Malkamäki et al., 1979).

also from Gordon's contribution

Astronomy 208 v.Y2K Meeting #19

The Hot ISM & The Interaction of X-Rays (High Energy Rad) with the ISM

(Note: More on non-therm em proc next time.)

Last time, in our discussion of SNRs, we showed that

$$\sum V_{SNR} = \underset{\substack{\uparrow \\ \text{SNR in the Galaxy}}}{rate_{SN}} \cdot \underset{\substack{\uparrow \\ \text{lifetime} \\ \text{(how long does each matter)}}}{\tau_{SNR}} \cdot \underset{\substack{\nwarrow \\ \text{volume of 1 SNR} \\ \text{(average)} \\ \text{big one}}}{V_{SNR}}$$

$$= \frac{1}{100 yr} \cdot 3 \times 10^6 yr \cdot \frac{4}{3}\pi (90 pc)^3 = 2 \times 10^{10} pc^3$$

Vol of MW $\approx \pi (30 kpc)^2 * 300 pc \approx 2.7 \times 10^{11} pc^3$

Fraction of Total Vol that's in "Hot" $\underset{\substack{\uparrow \\ \text{connections, chimneys,} \\ \text{worms}}}{\text{"SNRs"}}$ $\approx 10\%$

$\approx$ McKee & Ostriker model

Recall: Energy Deposited/SNR $\approx 10^{51}$ erg

$$\frac{E}{Vol} \approx \text{pressure} \approx \frac{3}{2} n k T = \frac{10^{51}}{\frac{4}{3}\pi (90 \times 3 \times 10^{18})^3} = 1.8 \times 10^{-11}$$

$k \cong 1.4 \times 10^{-16}$

$\Rightarrow \frac{3}{2} n T \cong 1 \times 10^5$ $(= P/k)$

$n \leq 1$ $\boxed{T \approx 10^5}$

(still expanding)

[Note: P/k for ISM actually thought to be $\approx 2 \times 10^4$ cm^{-3} K]

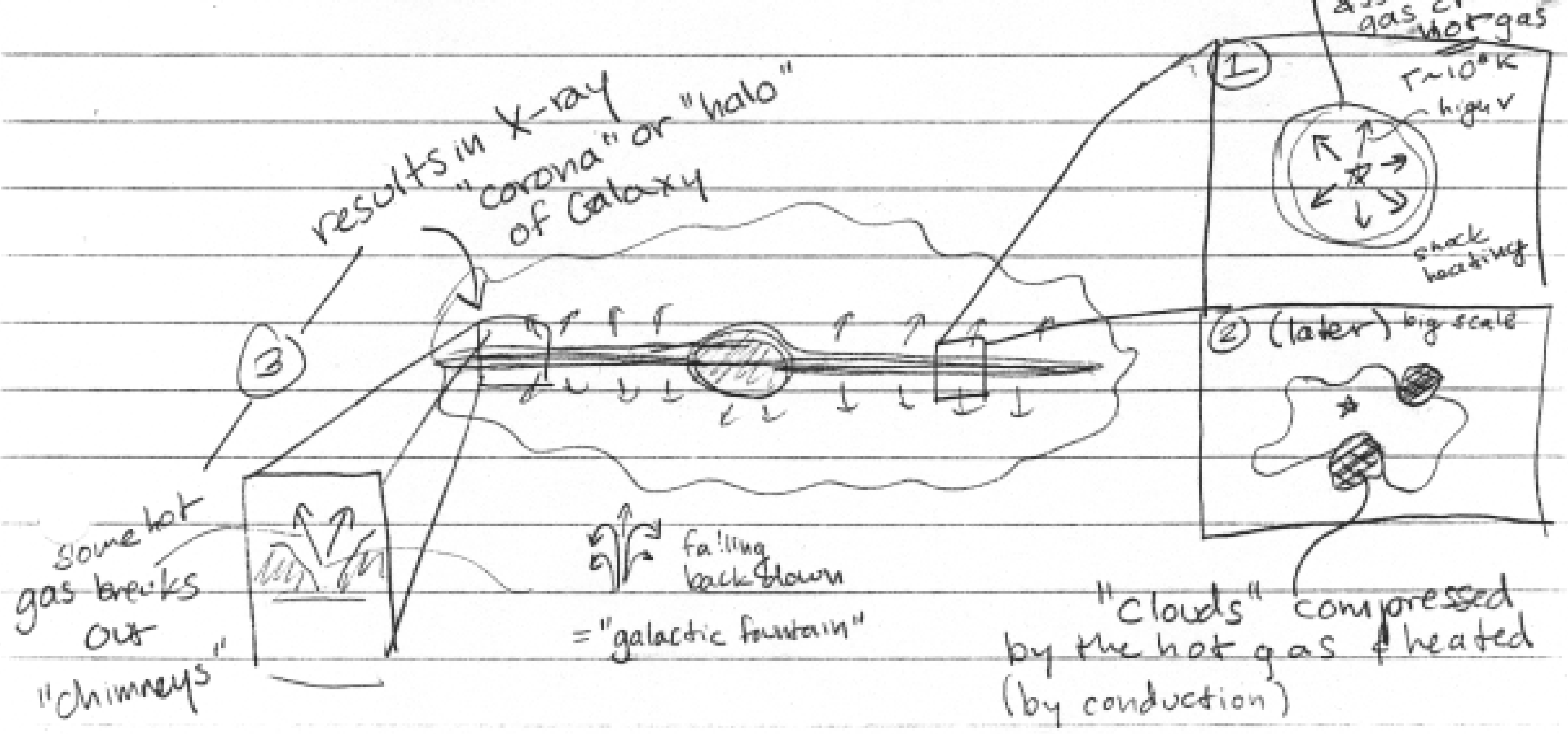
3.6 The "Hot" ISM *

Why should it exist... theories, see Spitzer 1990, ARAA, 28, 71-101.

: but here's an important quote from :

> "Understanding the processes that occur as the hot interstellar gas evolves in our Galaxy is an ambitious goal that we are far from achieving."

General Picture



History

1962 Shklovsky calculated expansion of non-radiating SNR into ISM

1940's known that T's would be very high behind SN shock, but no way available (then) to obs. this very hot gas

note: resolution of X-ray "telescopes" was very low

1968 start of rocket-borne obsv'ns of diffuse X-rays (Bowyer, Field & Mack 1968)

1973 source of diffuse X-ray emission still unclear — suggestions were about hot gas — was it galactic or intergalactic

1974 ① detection of OVI abs. lines in ISM

[Copernicus Obs. ultraviolet] $\Longrightarrow$ major support for idea of hot (soft) X-ray emitting galactic gas

soft 0.1 to 1 keV

100 to 1000 eV

$\boxed{10^6 \text{ to } 10^7 K}$

② detection of Fe XIV from Cygnus Loop, which was strong source of (thermal) free-free & mostly X-rays @ $T \sim 2 \times 10^6$ K

③ about 6 SNe shown to have soft X-ray spect consistent w/ $\underbrace{\text{thermal radiation}}$ @ $2 < \frac{T}{10^6 K} < 15$ (+ a little non-thermal) actually line emission from ionized atoms

post-1974 SNe "believed" as source of hot gas in ISM

Today "FUSE" Far Ultraviolet Spectral Explorer - launched 6/99

Where do the X-rays come from? ← on a microscopic level

- mixture of ionic emission lines &
- continuum: from the hot plasma (see J. Raymond)
  free-free + some synchrotron

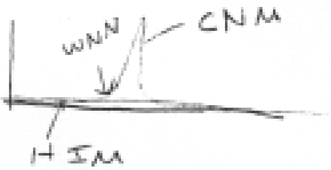
Big problem: DXRB = (local) + (distant) + (extragalactic)
these could be prod. by different processes
e.g. Xgal could be inverse compton btwn relivstc e^- & CMB ← see handout

In general, though:
X-ray emission modelling → $\boxed{T \simeq 10^6 \quad n_e = 3\times10^{-3}\ \mathrm{cm}^{-3}}$

$\frac{3}{2}nT \cong 5\times10^3$

$$\Delta V_{HI}(T=10^6\mathrm{K}) = 2.35\sqrt{\frac{kT}{1\,\mathrm{amu}}} \cong \underline{\underline{200\ \mathrm{km/s}}}$$

very low density & high-ionization makes this ~impossible to see in HI



- easier to look @ X-rays directly —
  ROSAT has made a large contribution
  CHANDRA to more
  XMM wil do spectroscopy!

note: T from X-rays consistent w/ T needed for OVI UV absorption

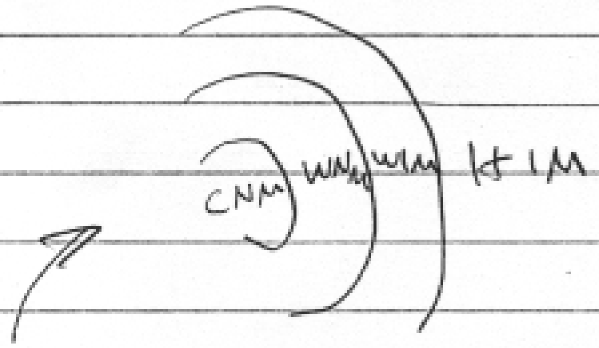
Why does HIM stay hot?

- Density very low collisions infrequent
- Ionization states → mostly just nuclei, no bound e⁻ – not even possible to cool by line radiation (some free-free matters)

WIM (T~8000K) represents transition zone

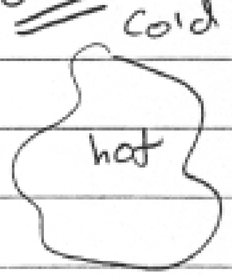
What is Overall Distribution (even just in the Galaxy) of HIM?

McKee & Ostriker: individual clouds



Convexity/concavity problem: Rosen Bregman & Kelson '96.

or?



soft X from nearby
harder from farther

The general picture is that the <u>Dif XRB</u> is produced in a "Local Bubble" of hot gas w/in ~100 pc of $\odot$ $T \sim 10^6 K$

Absorption lines OVI in the disk } more extended gas
CIV & NV in halo } @ ~lower T

↗ this also produces CIV & O<u>III</u> emission lines

? Shadows or Holes?

We're back to Barnard!

How exactly would shadows be made?

~ How do these high hν photons actually <u>interact</u> with "regular" ISM?

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(1)

4.4. "Effect(s) of High Energy Photons" ($ X-ray shadows)

when $\nu \sim 10^{20}$ then $h\nu \approx m_e c^2$

FYI **An example**

Consider: $h\nu = \underline{3\,\text{keV}}$ $= k(3.5\times10^7\,\text{K})$

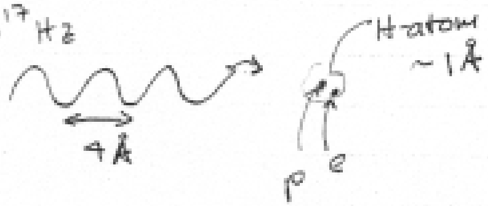
$= 4.8\times10^{-9}$ erg / photon!!

$\left(\frac{h\nu}{m_e c^2} = \underline{\underline{6\times10^{-3}}} \right)$

(recall OVI 114 eV was uv)

$\Rightarrow \nu = 7\times10^5$ THz $= 7\times10^{17}$ Hz

$\lambda = 4$ Å



This photon hardly sees the atom as a unit any more...

? How would/do such short-λ photons interact w/ISM?

Not exactly like ionizing photons @ shorter λ...

<u>At high energies</u> (e.g $h\nu \gtrsim 3$ keV) a_ν,

the "standard" <u>bound-free abs. x-sectn becomes vanishingly small</u> (see eq 2.4 and fig 2.2 of Osterbrock

but, a Compton scattering term becomes important

(see handout)

Cross sections for this

$$\boxed{a_\nu \doteq a_T \left(\frac{h\nu}{E_0}\right)^{-s}} \quad h\nu > E_0$$

r_0 = classical radius of e^-

$s \sim 3$

$\left.\begin{matrix} s \\ a_T \end{matrix}\right\}$ tabulated in Table 11.6 of Osterbrock

---

Note: At <u>very</u> high energies ($\nu \gtrsim 10^{20}$ Hz) ~ MeV photons

$\frac{h\nu}{m_e c^2} \gtrsim 1$ and yet more complications arise

In Summary, the Whole process by which ~keV X-rays interact w/ ISM:

1. incident X-ray w/ $h\nu \gtrsim 3$ keV removes an *inner* electron of some heavy element, X (*1e⁻* ejected, photon $\lambda \downarrow$)

2. X undergoes re-distribution of e⁻ by *Auger* transition → produces no photon instead ejects *another* very energetic *e⁻*

3. e⁻'s from ① & ② interact w/ surrounding gas, causing production of *additional* photons, those photons can still go back & start from ① again

4. Ultimately 1 energetic X-ray produces *many e⁻*

Approximations are possible
e.g. Joan Najita tells me ~ *30e⁻* per *keV* of photons

Where does one care about X-rays interacting w/ ISM ?

① X-ray shadows

- very low E x-rays can be absorbed in the usual way (processes involving valence electrons)

- higher energy ($\gtrsim 3$ keV) x-rays scattered (& ultimately diluted) by scattering, primarily off inner e^- in heavy elements

② Near X-ray sources

ⓐ AGNe accretion disks

ⓑ (smaller than) black hole & other compact object accretion disks

ⓒ Young star accretion disks & "envelopes"

↑ e.g. WTTS detected in X-rays

① "Thermal" vs "Non-Thermal" (Definitions)

history: $\boxed{\text{2 broad classes}}$ do whole page

"Thermal"

a (quasi-) equilibrium process with an associated Temperature

e.g. Early obs of the ☉ thermal emission at cm-λ's is from ionized solar atmosphere's free-free = bremsstrahlung

Note: various components can have their own T's eg T_e T_i

"Non-thermal"

non-equilibrium processes not easily characterized by a temperature

e.g. meter-λ ☉ emission in big outbursts - spectrum cannot be modeled as "thermal"

also: SNRs, radio jets, etc.

often power-laws...

Examples & Origins

Thermal

Continuum rad'n from dust e.g. in a dark cloud characterized by T_{dust}

Free-free (bremsstrahlung) from ionized gas characterized by T_e

e.g. H II Region

Non-thermal

Emission from transiently-heated dust, not characterized by a real "T."

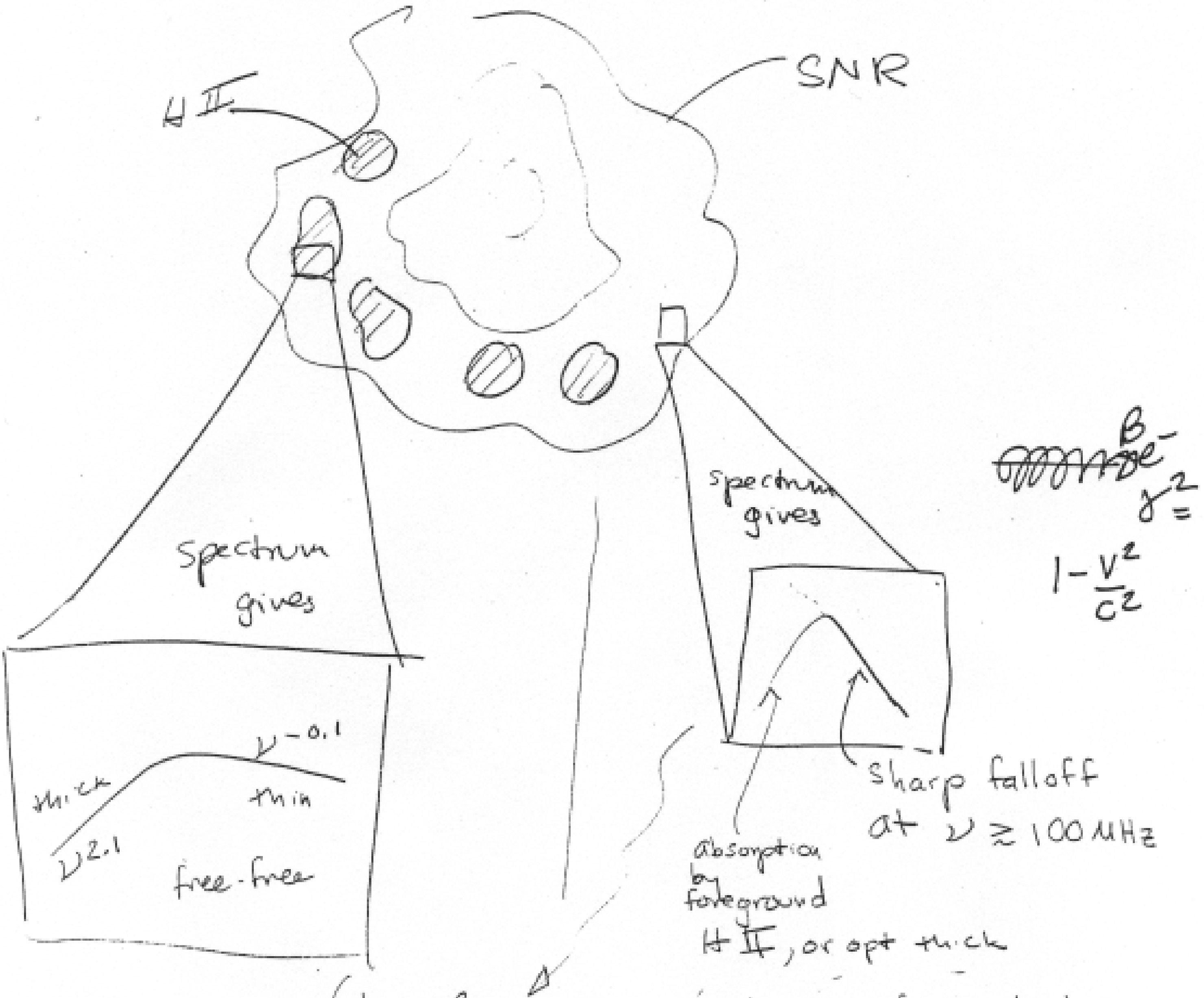
Synchrotron emission from ionized gas w/ relativistic electrons

e.g. SNR

see Kassim & Weiler 1990 example for "some of each"

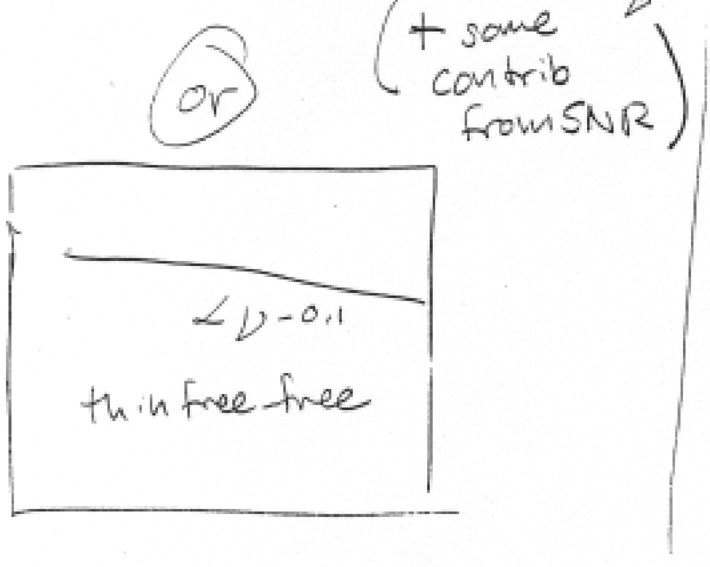
b) "Mixed" Cases

e.g. SNR + H II Regions see (Kassim & Weiler handout)



$\gamma^2 =$

$1 - \frac{v^2}{c^2}$



typical of synchrotron

γ = spectral energy index

$N(\varepsilon) \propto \varepsilon^{-p}$

then $S \propto \nu^{(1-p)/2}$

e.g. $\gamma = 2.4$

$S_\nu \propto \nu^{-0.7}$

A+ $\boxed{100}$

This is how the solution set — thanks! AC

Ay 208 - ISM   Problem Set 4   5/12/00

Jonathan Mackey

The spectrum I found is plotted in Mathematica (see over)
I plotted $\log(\nu F_\nu)$ vs $\log(\nu)$.

The spectrum is of the region within 5 arcminutes (radius) of the Trapezium cluster.
Region is roughly centred on J2000 (R.A. $5^h 35^m 15^s$; Dec $-05°23'30"$) which is the centre of the trapezium cluster.

I define a number of regions in the spectrum as follows:

Radio	$\nu < 10^{11}$ Hz
IR	10^{11} Hz $< \nu < 10^{14}$ Hz
Optical /UV	10^{14} Hz $< \nu < 10^{16}$ Hz
X-ray	10^{16} Hz $< \nu$

There are a number of upper limits on gamma ray emission from the Orion nebula, but no detections so I have not put in any gamma ray points.

From what we have learned in class, I expect free-free emission in radio, Dust in IR, stars & reflected/scattered starlight in optical/UV, and not sure what in X-ray. This is indeed what I found, with a high temperature plasma dominating X-ray emission.

Although line emission is important for cooling the HII region, I have not found a single reference that claims line emission dominates over continuum in any waveband, so I have not plotted any lines. The important lines are of course the Hydrogen lines $H\alpha$, $H\beta$ etc, along with the cooling lines of ionized oxygen (eg OIII) and sulphur (SIII), among many others.
Also prominent is the H_2 line at 2.1μ (Hyland et al, 1986)

In the following discussion I will take each of the 4 sections defined above & discuss them separately. I will list references used in each section, rather than all together at the end.

Note on collaboration:

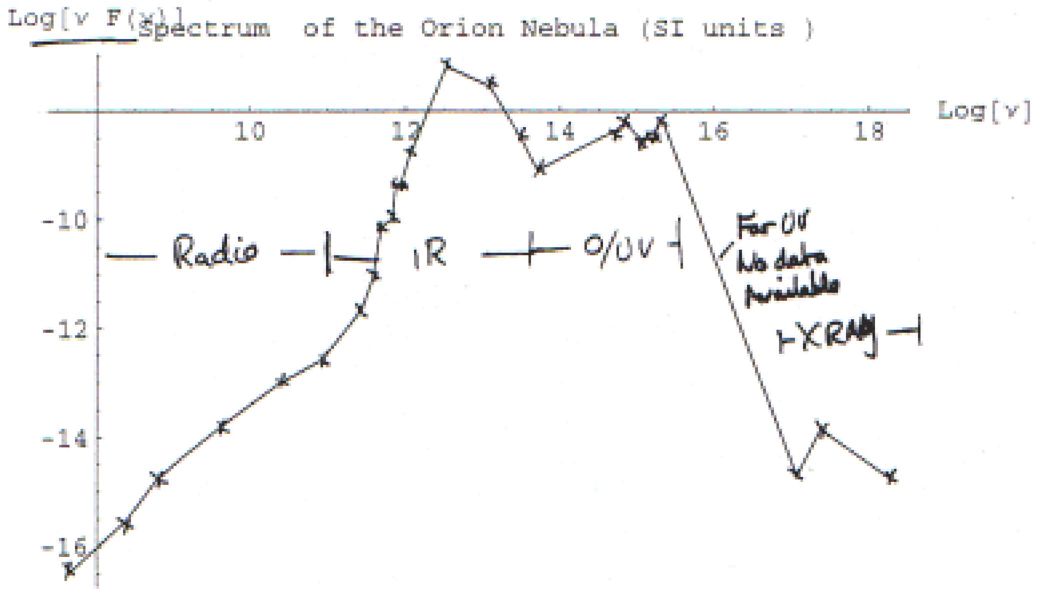
References were exchanged fairly frequently with other students; however all other work is my own unless otherwise stated (ditto when I get numbers from someone's paper...).

Ref Hyland, A.R., Allen, D.A., Barnes, P.J., Ward, M.J.
1984, MNRAS, 206, 465.

Spectrum Of The Orion Nebula.
(Central 5 ArcMinutes)

- **Data points are ones taken from the literature as described in the notes accompanying these graphs.**

```
In[75]:= linear = {{42*^6, 74}, {240*^6, 120}, {600*^6, 268},
            {1.4*^9, 324}, {5*^9, 380}, {23*^9, 400}, {86*^9, 307}, {273*^9, 800},
            {380*^9, 2136}, {480*^9, 14000}, {670*^9, 16000}, {750*^9, 60000},
            {790*^9, 50000}, {850*^9, 48000}, {1.1*^12, 0.13*^6}, {3.3*^12, 2*^6},
            {1.2*^13, 0.24*^6}, {3*^13, 11000}, {5*^13, 1620}, {5.45*^14, 810},
            {6.8*^14, 910}, {1.24*^15, 215}, {1.33*^15, 230}, {1.64*^15, 190},
            {2.14*^15, 333}, {1.2*^17, 1.6*^-6}, {2.41*^17, 5.8*^-6}, {1.8*^18, 1*^-7}};

In[76]:= nufnulin = linear;

In[77]:= Do[nufnulin[[i, 2]] = linear[[i, 2]] * linear[[i, 1]] * 1*^-26, {i, 1, 28}]

In[78]:= nufnulog = N[Log[10, nufnulin]];

In[79]:= ListPlot[nufnulog, PlotJoined → True,
          PlotLabel → "Spectrum  of the Orion Nebula (SI units )",
          AxesLabel → {"Log[ν]", "Log[ν F(ν)]"}]
```



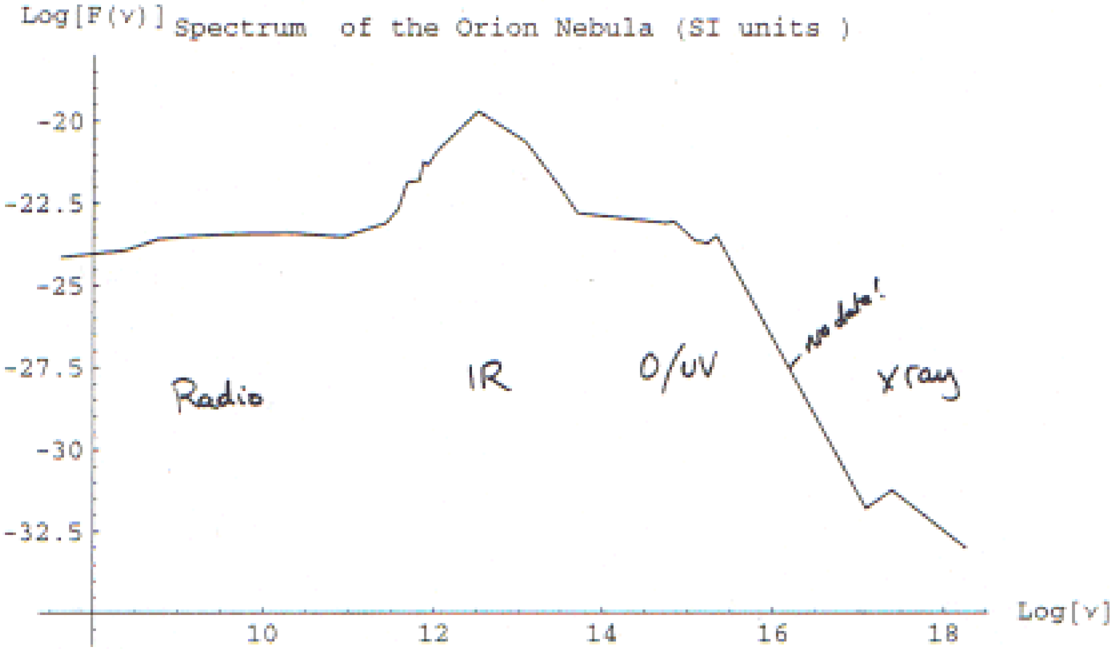
the units used are : F[W m^-2Hz^-1]
ν[Hz]

The straight lines are of course artifacts of the poor resolution I have in some parts of the spectrum.

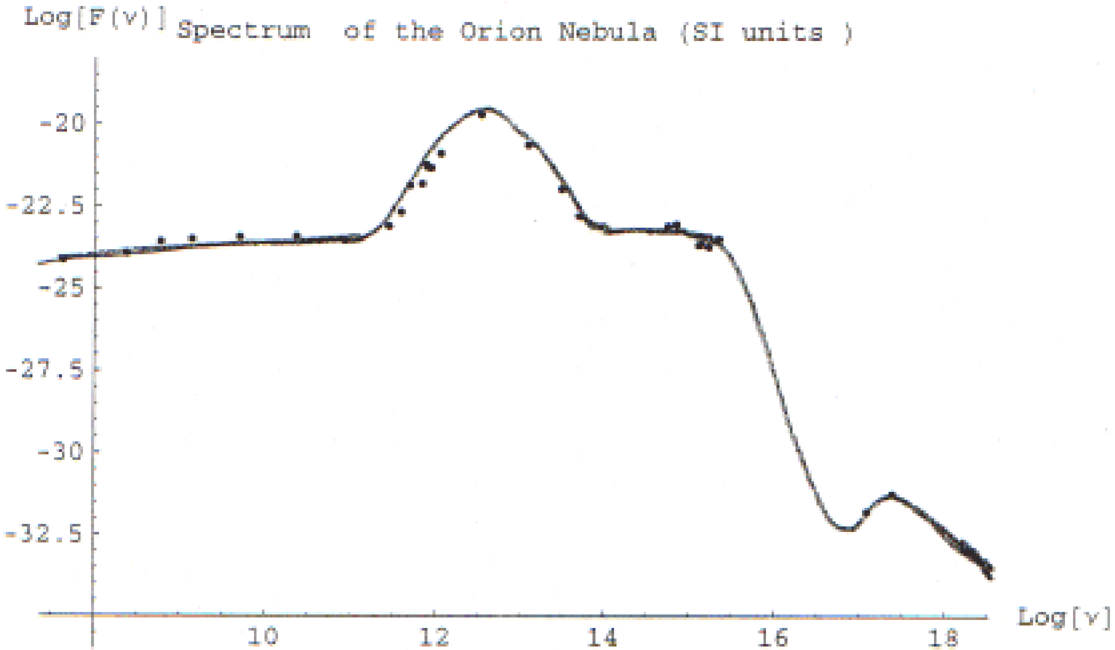
```
In[80]:= fnulin = linear;

In[81]:= Do[fnulin[[i, 2]] = linear[[i, 2]] * 1*^-26, {i, 1, 28}]

In[82]:= fnulog = N[Log[10, fnulin]];
```

P.T.O. for $\log(F_\nu)$ vs $\log(\nu)$

```
In[83]:= ListPlot[fnulog, PlotJoined → True,
          PlotLabel → "Spectrum  of the Orion Nebula (SI units )",
          AxesLabel → {"Log[ν]", "Log[F(ν)]"}, PlotRange → {-36, -18}]
```



```
In[84]:= ListPlot[fnulog, PlotJoined → False,
          PlotLabel → "Spectrum  of the Orion Nebula (SI units )",
          AxesLabel → {"Log[ν]", "Log[F(ν)]"}, PlotRange → {-36, -18}]
```



```
Out[84]= -Graphics-
```

1) Radio $\nu \lesssim 10^{11}$ Hz

This is the simplest region of the spectrum in terms of continuum emission. There are literally millions of lines all over the spectrum, but they contribute negligibly to the total flux (anything less than "comparable" is treated as "negligible" in the approximations made for this homework set).

The spectrum is essentially flat for all observed ~~wavelengths~~ frequencies less than 10^{11} Hz. This is characteristic of bremsstrahlung emission, and all authors I have found attribute all of the flux to bremsstrahlung.

The first & primary reference is Goudis (1975), who in table 1 lists all the observations of radio emission from the Orion Nebula (M42) up to 1975. The spectra do include emission from M43, but M42 completely dominates the emission.
The beam sites of the telescopes used are not given, & some (especially at longer wavelengths) are undoubtedly larger than our 5' radius region, but there are no other nearby sources of free-free to contaminate the signal, so it is ok to use the data.
I basically used this table to fill out gaps in the spectrum where I couldn't find more recent references.

I used data from Felli et al (1993) at 20cm (1.4 GHz).
They took 100m Effelsberg telescope data & integrated the flux over the whole nebula.
I did not use their other data from the VLA because interferometers filter out large angular scale power, which is important, as the authors point out.

I used Gordon et al (1987) data at 3.5mm (86 GHz). They integrated over a 10'×10' field centred on BN/KL nebula ($\lesssim 1'$ from trapezium cluster). This is larger than a cirle of radius 5', but only by 25%, so it shouldn't make much difference. The data was taken with the Kitt Peak 12-m mm-wave telescope.

Wilson & Pauls (1984) also used the Effelsberg 100m telescope to map the Orion Nebula at 23 GHz (1·3 cm). They also give an integrated flux density over the whole map. From inspection of their figure 2, almost all of the emission is within 5′ of the central region of the nebula.

Other points are from Goudis (1975) & references therein. As can be seen from the $\log F_\nu$ vs $\log \nu$ plot, the spectrum is very flat at all wavelengths/freq. $\nu < 10^{11}$ Hz. This indicates that the HII region is not becoming optically thick to free-free even at the longest wavelengths of $\lambda \sim 10$m.

Radio Refs

Felli, M., Churchwell, E., Wilson, T.L., Taylor, G.B. 1993, A&AS, 98, 137.

Gordon, M.A., Jewell, P.R., Kaftan-Kassim, M.A., Salter, C.J., 1986 Ap.J., 308, 288.

Goudis, C., 1975, Ap.&S.S., 35, 409.

Wilson, T.L., & Pauls, T, 1984, A.&A., 138, 225.

2) Infrared

From the νF_ν plot, it is clear that this wavelength region emits most of the energy from the Orion Nebula. This is thermal emission from dust in the Orion nebula & in the surrounding molecular gas.
For dust at $T \sim 40$ K, the peak of the BB spectrum (in ν) is at $\sim 2 \times 10^{12}$ Hz.
For $T = 100$K, the peak is closer to 10^{13} Hz.

Bally, Langer & Liu (1991) give IRAS data for M42. In Figure 8 they plot the intensity of emission in a cut through the nebula in declination.
Unfortunately the 60μ & 100μ integrations are saturated & so cannot be used to calculate flux.
In the end I decided not to use the 12μ & 25μ data either as they are only 1-D emission curves and I do not know the beam shape or orientation of IRAS.

Keene, Hildebrand & Whitcomb (1982) report a 400μ measurement integrated over their 3' beam of 6000 Jy.
Our region is 5' radius & so has area $\pi(5')^2 \simeq 80\,\square'$
A 3' diameter beam has area $\sim 7\,\square'$.
To convert between different beam sizes in the Far-IR I just multiplied by the ratio of the beam areas. This is effectively assuming that emission is uniform over the whole region. While this is a very bad assumption, it is not going to make a factor of more than 3 or 4 difference.
Also in the FIR, dust is pretty extended over the whole Orion molecular cloud, ~~although~~ it is very clumpy & wispy.
So I multiplied the Keene et al point by 10 to get 60 kJy.

Lis et al (1998) use the SHARC camera on the CSO to image the OMC. They give the integrated flux over a 12'×12' region centred on the nebula as 60 kJy. I divided this by 1·25 to reduce the area.

Goldsmith, Bergin & Lis (1997) also image a 12′×12′ region centred on M42 at 450μ, 790μ, and 1100μ. They also give integrated fluxes, which I again divide by 1·25 to reduce the area somewhat. I should really divide by about 1·8 to get to 80 □′, but I am weighting the outer regions less than the inner ones here, & so excluding them reduces the flux by less than the geometric area reduction. Admittedly I do this reduction in a somewhat arbitrary manner...

True, but an OK guess -

At 1100μ, the authors estimate about 20% of the flux is from free-free emission while the rest is thermal dust emission kicking in. Incidentally they find $\beta \simeq 1{\cdot}8$ for the orion Nebula.

Ristorcelli et al. (1998) use a 3·5′ beam for their flux results. This has about 10 □′, so I multiply their results by 8 to get total fluxes, again under the uniform emission hypothesis. They have data at 90μ, 210μ, 270μ, 380μ, 630μ, and I have used all of these data points on my plot.
The peak of the dust spectrum seems to be somewhat higher frequency than 3·3THz ~~(90μ)~~ (90μ).

For the nearer infrared – higher ν than the dust BB peak – I used the data of Simpson et al (1998) who took MSX spectra of the nebula.
Their beam was ~6′×9′, so I didn't correct their results for area – they are close enough.
I used Fig. 2, where they plot F_λ against λ.
I convert to F_ν via $F_\nu = F_\lambda \left(\frac{\lambda^2}{c}\right)$. They plot I_ν, so I convert this to a flux by multiplying by the solid

angle I am using i.e. 80 □'.
I picked 3 points on their graph 6μ, 10μ, and 25μ, corresponding to 5×10^{13} Hz, 3×10^{13} Hz and $1{\cdot}2\times10^{12}$ Hz.

These points, taken altogether, show the dust spectrum very well with a clearly defined peak around 7×10^{12} Hz, indicating a dust temperature in the region of $60\text{K} \lesssim T_{\text{DUST}} \lesssim 100\text{K}$.

Near infrared (1μ–3μ) information was hard to come by and I couldn't find any satisfactory references. There are many references to line emission, but this is not important to the overall flux.
OK one paper – Hyland et al (1984). They look at small regions in Orion & conclude that lines contribute at most 16% of the flux in the 2μ window, even counting the prominent H_2 line.
Their spectrum in Fig 1, shows some continuum increasing to longer wavelength.

Mostly point-
source surveys
in near IR –
hard to map extended
emission & correct
properly for bkgd – but
can (& is) be(ing) done.
See L-band work of
(Haisch et al. (2000x)

IR References

Bally, J., Langer, W.D., Liu, W. 1991, Ap.J., 383, 645.

Keene, J., Hildebrand, R.H., Whitcomb, S.E.,
1982, Ap.J., 252, L11.

Lis, D.C., Serabyn, E., Keene, J, et al.
1998, Ap.J., 509, 299.

Goldsmith, P.F., Bergin, E.A., Lis, D.C.,
1997, Ap.J., 491, 615.

Ristorcelli, I., Serra, G., Lamarre, J.M., et al.
1998, Ap.J., 496, 267.

Simpson, J.P., Witteborn, F.C., Price, S.D., Cohen, M.
1998, Ap.J., 508, 268.

Hyland, A.R., Allen, D.A., Barnes, P.J., Ward, M.J.
1984, MNRAS, 206, 465.

3) Optical / UV

I found B & V data for emission from the nebula in Greve, Van Genderen & Augusteijn (1993) - this was surprisingly hard to find.
They give the Johnson Visual (V) & Blue (B) magnitudes of the nebula as a function of distance from the centre (trapezium). They use a radial brightness profile which they fit with their data.
To a radius of 300" they find V = 3·67 mag B = 3·97 mag. For reference the trapezium stars (summed) have V = 4·69 mag, so the nebula far outshines the stars. See Table 3 for more details.
I converted this to flux using ~~the~~ Zombeck's [illegible] HSA&A p100, which lists the flux from Vega in Johnson V & B filters. Vega is defined to have mag = 0.

So $m_1 - m_2 = 2.512 \log f_2/f_1$ $\qquad 2 = \text{vega} \quad m_2 = 0$

$\rightarrow f_1 = f_2 \exp\left[-\frac{m_1}{2.5}\right]$

- f_2 is from the table in Zombeck
- m_1 is from Greve et al data.

In UV, I found Bohlin et al's (1982) treatment to be the most comprehensive & helpful, especially Table 3. They give the integrated flux from the nebula in 4 wave bands for lots of annuli around the centre of the nebula. To get the total flux out to some radius you just sum the ~~columns~~ annuli out that far. They also give the stellar flux from θ_1 Ori and θ_2 Ori combined, & this is larger than the nebulous flux out to $r = 5'$.
So for the first time, the stellar spectra are now dominating the spectrum.
I integrated nebulous emission out to 4·4' (the nearest contour to 5') and added this to the stellar fluxes to get the total signal. Bohlin et al. get the stellar fluxes from Bohlin & Savage (1981).

For an O star with $T_{eff} = 40,000$ K, the peak of the BB curve is at 710 Å (peak in wavelength) so it is not surprising that the flux is increasing to shorter wavelengths. ~~[illegible] to get [illegible]~~ (Bohlin et al look at $1820 Å < \lambda < 2420 Å$).

Again to get to F_ν I use $F_\nu = F_\lambda \frac{\lambda^2}{c}$

Optical/UV references

Greve, A., Van Genderen, A.M., Augusteija, Th., 1993, A&AS, <u>99</u>, 577.

Bohlin, R.C, Hill, J.K., Stecher, T.P., Witt, A.N. 1982, Ap.J., <u>255</u>, 87.

Bohlin, R.C., & Savage, B.D., 1981, ApJ., <u>249</u>, 109.

Zombeck, M. V. 1990 "Handbook of Space Astronomy & Astrophysics" p100.

4) XRay

Useful for getting an idea of the region is Garmire et al. (2000). They took Chandra ACIS data of the nebula & explain well what's going on. Their resolution is just <u>too good</u> though.

Much more useful is Yamauchi et al. (1996). They took ASCA data of the orion nebula & integrated over a 6' region around the trapezium. This is close enough to my radius, and X-ray data is uncertain enough, that there is no need to do an area correction.

Page 733 figure 6(e) is the one I used to get spectra. Unfortunately this spectrum, while nicely integrated over the area, is convolved with the detectors frequency response. To deconvolve, I used a Chandra utility on the www which will convert ASCA counts into a flux in erg $cm^{-2} s^{-1}$ over a specified energy range (thanks to Craig Heinke for this).

URL is http://asc.harvard.edu/toolkit/pimms.jsp

I input a Raymond-Smith source model, $N_H = 2 \times 10^{21} cm^{-2}$, $T = 3 \times 10^7 K$, & the count rate from various parts of the curve in fig. 6(e). These are the parameters that Yamauchi et al either input to their best fit model or derive from that model.

As expected the flux is much lower in X-ray than in O/IR. This flux seems to be dominated by a very hot plasma at $T \sim 3 \times 10^7 K$, due to shocks from stellar winds, outflows or ionization produced shocks which are known to be present in Orion.

X-Ray References

Garmire, G., Feigelson, E.D., Broos, P., et al.
2000, A.J., 120, 1426.

Yamauchi, S., Koyama, K., Sakano, M, Okada, K.,
1996 P.A.S.J., 48, 719.

Heinke, C. (private communication)

Summary

Overall the region looks much like I expected when I drew a "guessed" spectrum last week (in 30 minutes.):

- Free-Free dominates radio emission
- Thermal dust emission dominates submm/IR
- Starlight o scattered starlight dominates Optical o Near IR.
- UV is dominated by stellar emission
- X-ray is mostly high temperature plasma.

I expected the optical o UV light to dominate though, whereas the IR contains most of the power output. This is because of the BN/KL nebula which is an obscured region of massive star formation. It is invisible at short wavelengths and almost all its luminosity comes out in the IR. It is located within 1' of trapezium, so it is all contained in my field of view.

Line cooling is of course important in the HII region, o some spectra I saw have huge $H\alpha$ and OIII and SIII emission lines, but I have not plotted these as they do not contribute much overall power as compared to the continuum. Also, there are so many lines on such a wide range of frequency, that there's no way I could even put in a representative sample.

My spectrum looks like a cross section of one of my back teeth o gum...

Appendix A : Spectral Coverage

λ	ν	Band	Coverage			
1m	300MHz	radio				
			20cm	324Jy		
10cm	3GHz	radio	Gaudio points			
1cm	30GHz	CM	23GHz	400 Jy	d = 10'	
			3.5mm	307Jy	10'×10'	
1mm	300 GHz	mm	1.1mm	989 Jy	12'×12'	
			790μ	2671	12×12	
			↕ good coverage!			
			210	35500	d=3.5'	
100μ	3THz	FIR	90μ	250 kJy	d=3.5'	
			25μ	18 GJy/sr at peak	? ✓	→ r = 5'
			25μ			
10μ	3×10^{13}	MIR	12μ	5GJy/sr at peak	? ✓	
			10μ			
			6μ			
1μ	3×10^{14}	NIR	B= 4400Å V=5500Å		? ✓	
		optical	2420Å 2240Å 1820Å 1400Å	Bohlin et al	? ✓	
0.1μ=100nm	3×10^{15}	UV				
100Å	3×10^{16}	0.1keV	?			
10Å	3×10^{17}	1keV	$L_X \sim 10^{33}$ erg/s in 0.5–10keV		✓	
			better – 3 data points			

Appendix B: Data used

Band	Freq (Hz)	F_ν	νF_ν
Radio	42×10^{6}	74×10^{-26}	
	240×10^{6}	120×10^{-26}	
	600×10^{6}	268×10^{-26}	
(20cm)	1.4×10^{9}	324×10^{-26}	
	5×10^{9}	380×10^{-26}	
(1cm)	23×10^{9}	400×10^{-26}	
	86×10^{9}	307×10^{-26}	
(1.1mm)	273×10^{9}	800×10^{-26}	
	380×10^{9}	2136	
SubMM	480	14000	
	670	16000	
FIR	750	60000	
	790	50000	
	850	48000	
	1.1×10^{12}	0.13×10^{-20}	
MIR (90μ)	3.3×10^{12}	2.0×10^{-20}	
25μ	1.2×10^{13}	?? (IRAS) points, no good ~~8200×10^{-26}~~	2.4×10^{-21}
~~12μ~~	~~2.5×10^{13}~~	~~1410×10^{-26}~~	
10μ	3×10^{13}	11000×10^{-26}	
5μ	5×10^{13}	1600×10^{-26}	
V	5.45×10^{14}	510×10^{-26}	
B	6.8×10^{14}	910×10^{-26}	
UV	1.24×10^{15}	815×10^{-26}	
	1.33×10^{15}	230×10^{-26}	
	1.64×10^{15}	190×10^{-26}	
	2.14×10^{15}	333×10^{-26}	
X 0.5keV	1.2×10^{17}	1.6×10^{-32}	
1keV	2.41×10^{17}	5.5×10^{-32}	
5keV	1.8×10^{18}	1.0×10^{-33}	

Appendix C: Excerpts from various sources

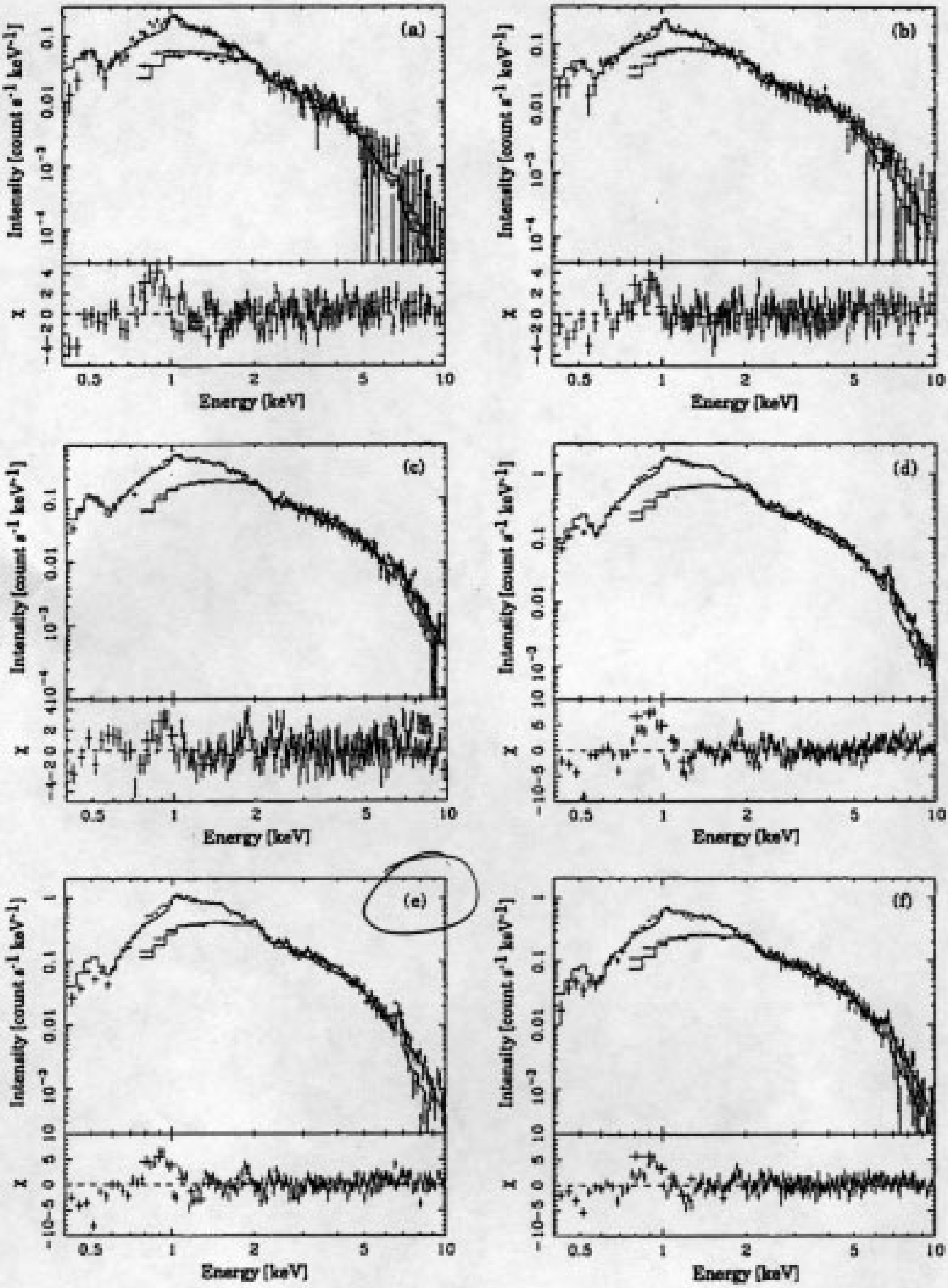


Fig. 5. Composite spectra extracted from large areas with the best-fit 1-temperature Raymond-Smith model (solid lines). (a) "field 1 north" including NGC 1977, (b) "field 1 south," (c) "field 2 north" including OMC-2, (d) "field 2 south" including Orion Trapezium, (e) "Trapezium region" extracted from a 6′ diameter region centered on the Trapezium, and (f) "outer Trapezium region." Data from SIS 0 and SIS 1 were combined, as were GIS 2 and GIS 3.

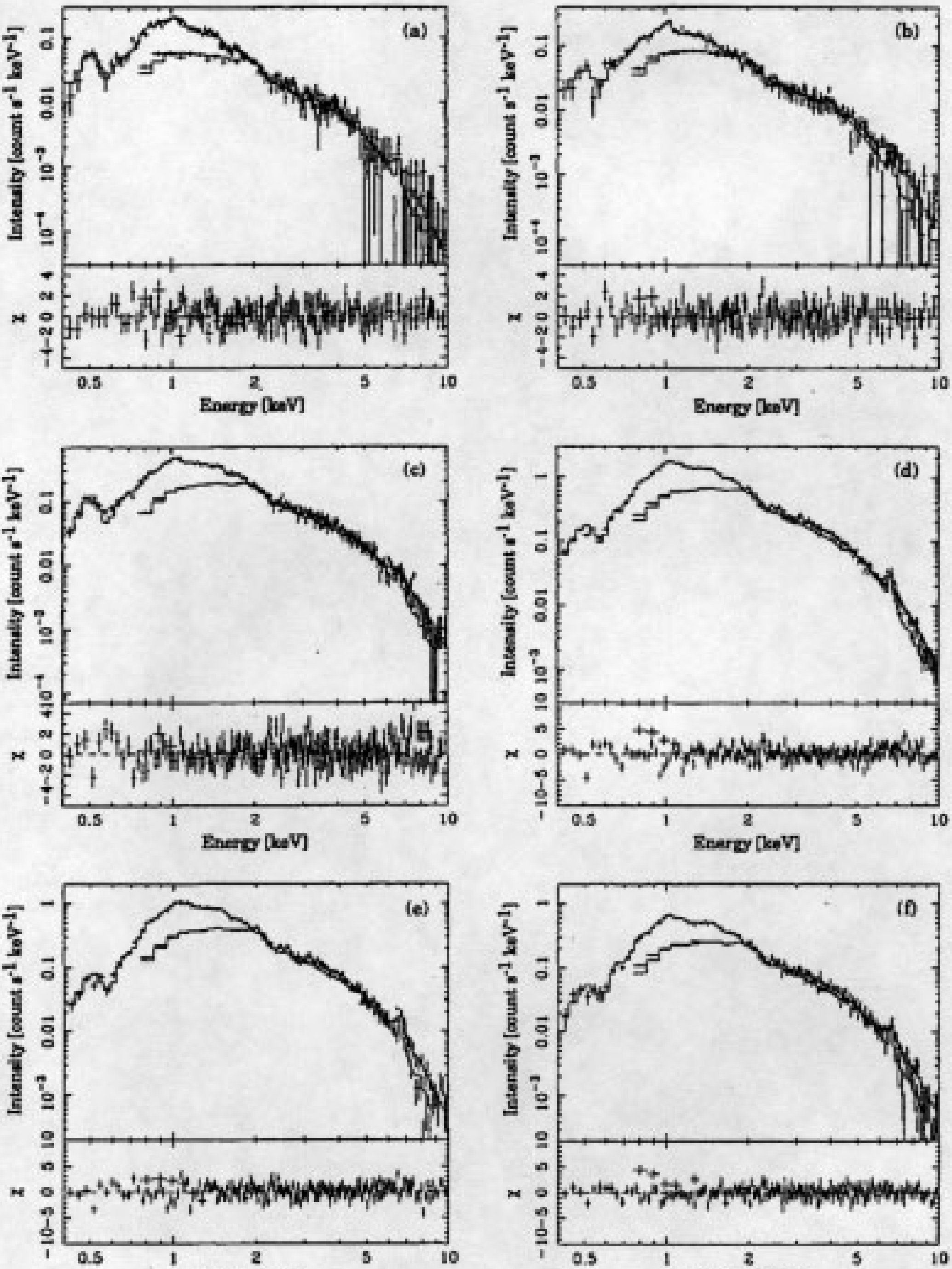


Fig. 6. Same as figure 4, but with the 2-temperature Raymond-Smith model.

1991A 383..645B

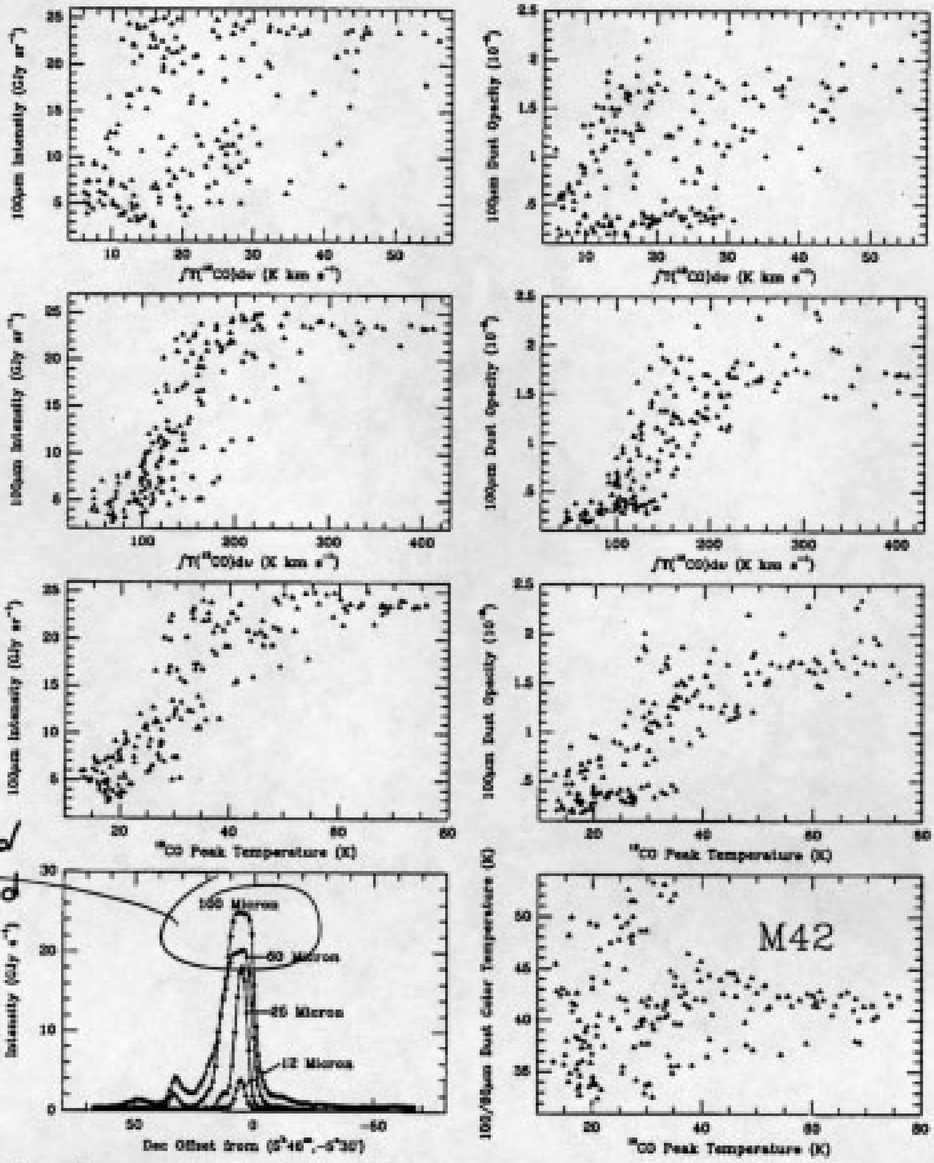
seen in all regions containing luminous IR sources, it is most vious in the Mon R2 region.

3.4.4. *Orion Nebula (M42) and NGC 1977*

The H II region M42, the most luminous region in our study, is located in the midsection of a 1° long north-south filament of high density [on average, $n(H_2) > 10^4$ cm^{-3}] gas in the northern portion of the Orion A molecular cloud (see the photographic illustration of the ^{13}CO emission in Bally et al. 1987a). The H II region forms a blister on the near side of the molecular ridge which contains a high-luminosity embedded source, IRc 2, located at a projected distance of only 1′ north of the Trapezium cluster (which is responsible for ionizing the visible nebula). The infrared morphology of M42 is very different from that of the molecular cloud; all four *IRAS* bands show M42 to consist of a very bright core surrounded by a ring of emission located just outside the optical boundary of the H II region.

A less luminous and more evolved H II region, NGC 1977, is located at the northern end of the molecular ridge. Just like M42, this region is also surrounded by a ring of infrared emission, a morphology very different from that of the associated molecular cloud.

In a 15′ × 12′ region which contains the core of the Orion A cloud and the Orion Nebula (M42), neither the 100 μm intensity nor opacity appears to correlate well with $I(^{13}CO)$ (Fig. 8). However, these quantities do correlate with $I(^{12}CO)$ up to about 200 K km s^{-1} corresponding to I(100 μm) of about 20 GJy sr^{-1}. At higher values of $I(^{12}CO)$, the dust opacity and 100 μm intensity do not vary. These high values correspond to the BN-KL source and the high-velocity outflow surrounding IRc 2. The north-south cuts through the BN-KL source in all four bands show that the peaks in 100 and 60 μm emission are flat, while 25 and 12 μm emission exhibit sharp peaks, suggesting saturation in the long-wavelength bands.



Saturated
⇒ no good

FIG. 8.—Comparison of dust and gas properties near the Orion Nebula (M42 region). Slices of intensities in all the four bands from north to south across the infrared peak of the H II region are also shown.

1975 AP+SS λ Goudis → vol 35:409

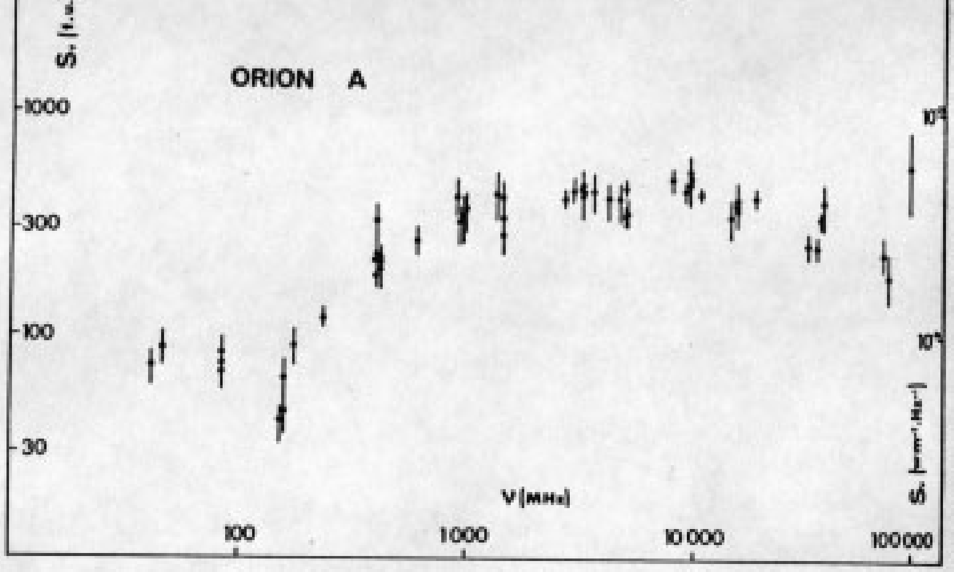


Fig. 1. The radio spectrum of the Orion nebula (M42 and M43).

TABLE I

Orion nebula – Radio spectrum

1 No.	2 Frequency ν (MHz)	3 Wavelength λ (m, cm or mm)	4 Flux density S_ν (fu) (1 fu = 10^{-26} W m^{-2} Hz^{-1})	5 References
1	42.2	7.10 m	73.7 ± 15	Haynes and Hamilton (1968)
2	47.3	6.35 m	89.0 ± 18	Haynes and Hamilton (1968)
3	85.5	3.55 m	69 ± 14	Mills *et al.* (1958)
4	86.0	3.50 m	85.0 ± 17	Risbeth (1958)
5	86.0	3.50 m	74 ± 15	Mills *et al.* (1956)
6	153	1.96 m	41.6 ± 9	Hamilton and Haynes (1967)
7	159	1.90 m	65 ± 13	Risbeth (1958)
8	159	1.90 m	45 ± 9	Edge *et al.* (1959)
9	178	1.80 m	90 + 18	Conway *et al.* (1963)
10	240	1.25 m	120 ± 12	Menon and Terzian (p.c.) (1964)
11	400	75 cm	230 ± 46	Seeger *et al.* (1956)
12	400	75 cm	325 ± 65	Seeger *et al.* (1961)
13	400	75 cm	220 ± 44	Howard *et al.* (1965)
14	405	74 cm	188 ± 9	Menon and Terzian (p.c.) (1965)
15	408	73.5 cm	200 ± 40	Long *et al.* (1963)
16	408	73.5 cm	213 ± 17	Mills and Shaver (1968)
17	408	73.5 cm	213 ± 43	Parkes Catalogue*
18	600	50 cm	268 ± 40	Piddington and Trent (1956)
19	910	33 cm	420 ± 84	Denisse *et al.* (1957)
20	960	31 cm	342 ± 10	Harris and Roberts (1960)
21	960	31 cm	360 ± 72	Wilson and Bolton (1960)
22	960	31 cm	343 ± 68	Conway *et al.* (1963)
23	1370	22 cm	430 ± 86	Westerhout (1958)
24	1410	21.3 cm	289 ± 58	Parkes Catalogue*
25	1420	21 cm	420 ± 84	Hagen *et al.* (1954)
26	1420	21 cm	331 ± 50	Hagen *et al.* (1954)
27	2700	11 cm	411 ± 41	Altenhoff *et al.* (1961)
28	2930	10.3 cm	454 ± 55	Sloaneker and Nichols (1960)
29	3130	9.6 cm	412 ± 62	Kuzmin *et al.* (1960)
30	3200	9.4 cm	426 ± 43	Haddock *et al.* (1954)
31	3200	9.4 cm	450 ± 90	Pariskii (1961)
32	3200	9.4 cm	460 ± 12	Medd (p.c.) (1964)
33	3600	8.3 cm	450 ± 90	Pariskii (1961)
34	4170	7.2 cm	410 ± 82	Yokoi *et al.* (1966)
35	4700	6.4 cm	410 ± 82	Golnev *et al.* (1965)
36	5000	6 cm	342 ± 23	Mezger and Henderson (1967)
37	5000	6 cm	470 ± 33	Baars *et al.* (1965)
38	5000	6 cm	344 ± 69	Gardner and Morimoto (1968)
39	8000	3.7 cm	502 ± 60	Menon (1961)
40	9360	3.2 cm	450 ± 30	Lazarevskii *et al.* (1963)
41	9400	3.2 cm	540 ± 108	Kaidanovskii *et al.* (1955)
42	9400	3.2 cm	480 ± 96	Zakharenkov *et al.* (1963)
43	9520	3.15 cm	480 ± 96	Haddock and McCullough (1955)
44	10700	2.8 cm	434 ± 20	McLeod and Doherty (1968)
45	14500	2 cm	343 ± 69	Baars *et al.* (1965)
46	15350	1.95 cm	390 ± 79	Terzian *et al.* (1968)
47	15350	1.95 cm	400 ± 80	Schraml and Mezger (1969)
48	15550	1.94 cm	365 ± 21	Gordon (1969)
49	18750	1.6 cm	420 ± 42	Kuzmin and Salomonovich (1963)
50	31400	9.55 mm	253 ± 37	Johnston and Hobbs (1969)
51	35000	8.5 mm	250 ± 30	Tolbert (1965)
52	36500	8.2 mm	330 ± 30	Sorochenko and Berulis (1970)
53	37000	8.1 mm	500 ± 100	Kuzmin and Salomonovich (1963)
54	67800	4.3 mm	233 ± 40	Hobbs *et al.* (1969)
55	72800	4.1 mm	182 ± 45	Kaifu *et al.* (1973)
56	94000	3.2 mm	585 ± 200	Tolbert (1965)

* Parkes Catalogue = Bolton *et al.* (1964); Price and Milne (1965); Day *et al.* (1966).
(p.c.) = private communication cited in Baars *et al.* (1965).

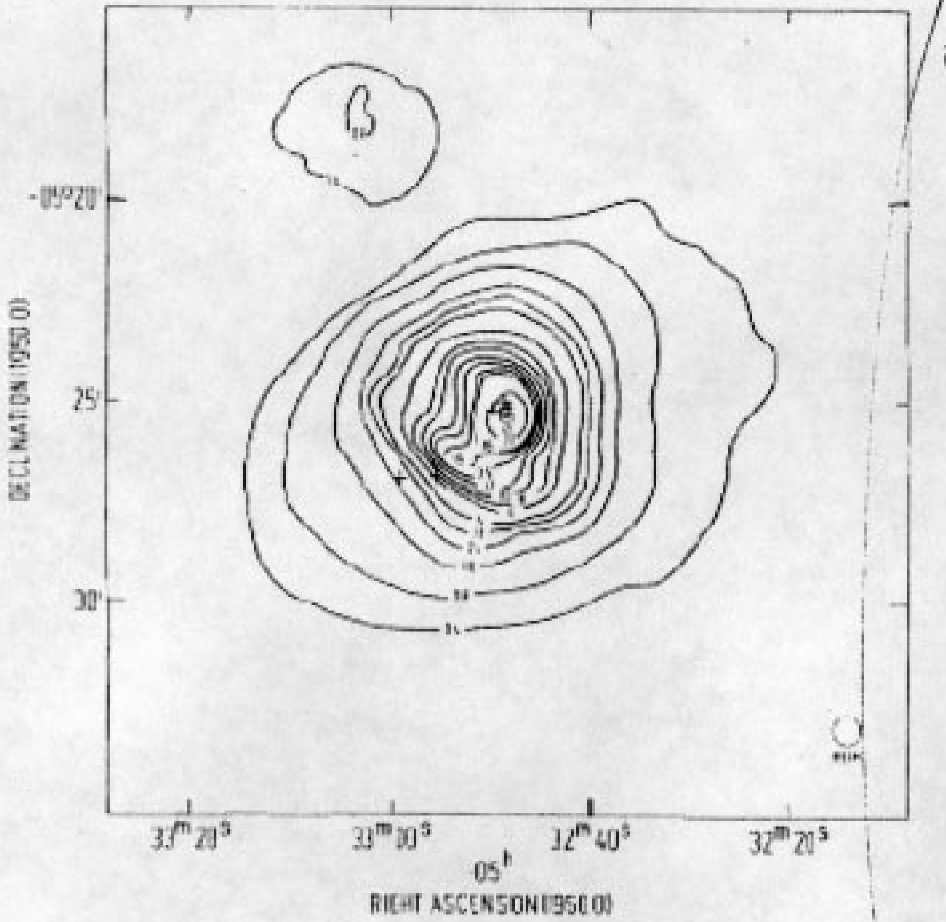


Fig. 2. The radio continuum map, as in Fig. 1, with crosses showing the positions of the 4 Trapezium stars and the two $^2\theta$ Orionis stars

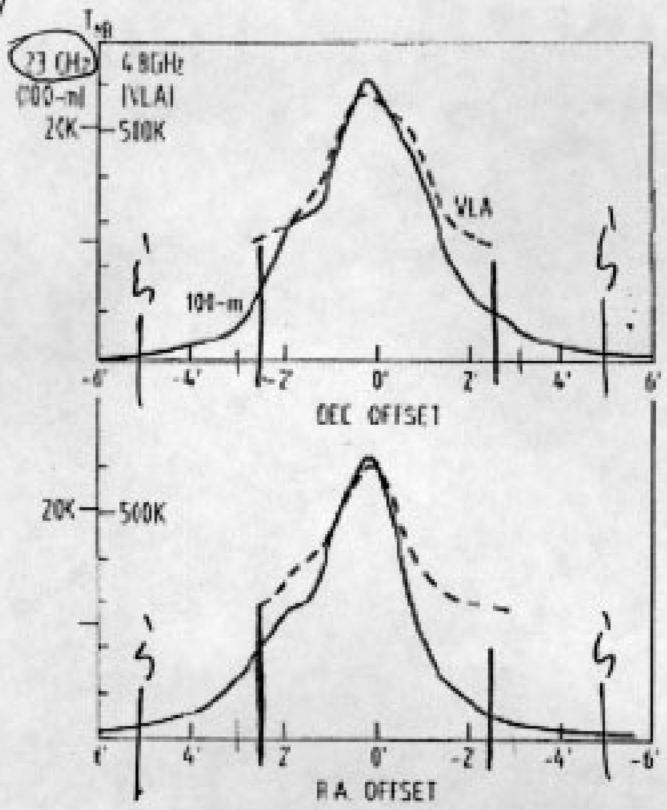


Fig. 3. Cuts in RA and Dec through the peak of the continuum (RA = $05^h32^m48^s.2$, Dec = $-05°25'30''$ (1950.0) are shown as solid lines. The dashed curves are taken from the VLA map of Johnston et al. (1983), made at 4.8 GHz and smoothed to an angular resolution of 28". The main beam brightness temperature scale, T_B, on the far left (marked 23 GHz) refers to our 100-m data. To the right, marked 4.8 GHz, is the VLA temperature scale in T_B. The temperatures are arranged to agree if the continuum were an optically thin plasma

Bohlin et al (1982)

TABLE 3

TOTAL NEBULAR FLUXES AND RELATIVE SURFACE BRIGHTNESS RATIOS

Bin	Limiting Values of H(1820 Å)	F_{1400}[a]	F_{1820}[a]	F_{2240}[a]	F_{2620}[a]	$\frac{(S/F_*)_{1400}}{(S/F_*)_{1820}}$	$\frac{(S/F_*)_{2240}}{(S/F_*)_{1810}}$	$\frac{(S/F_*)_{2620}}{(S/F_*)_{1820}}$	$\langle r \rangle$ (arcmin)
1	≤ 8	2.48[b]	1.40[b]	0.927[b]	0.910	1.06[b]	1.41[b]	1.75[b]	22.7
2	9–16	1.32	0.744	0.414	0.485	1.06	1.41	1.75	17.6
3	17–32	1.61	0.933	0.590	0.532	1.03	1.34	1.53	13.6
4	33–64	2.39	1.37	0.812	0.742	1.04	1.26	1.46	10.4
5	65–128	2.51	1.32	0.791	0.704	1.14	1.38	1.44	7.5
6	129–256	3.22	1.61	0.972	0.773	1.19	1.28	1.29	6.0
7	257–512	6.47	3.04	1.69	1.34	1.27	1.18	1.19	4.4
8	513–1024	4.94	2.42	1.24	0.945	1.22	1.09	1.05	2.6
9	1025–2048	5.19	2.33	1.21	1.04	1.33	1.10	1.20	2.0
10	2049–4096	4.69	1.98	0.987	0.781	1.42	1.06	1.06	1.8
11	> 4096	2.82:[c]	1.19:[c]	0.592:[c]	0.469:[c]	...	...	...	...
Total		37.6	18.3	10.1	8.72	1.23	1.20	1.28	
Trapezium		13.9	8.31	3.91	3.09				
Total/Trapezium		2.7	2.2	2.6	2.8				
$\theta^1 + \theta^2$ Ori		27.9	16.1	8.06	6.22				
Total/$\theta^1 + \theta^2$		1.33	1.14	1.28	1.40				

[a] In 10^{-10} ergs cm^{-2} s^{-1} Å^{-1}.

[b] For bin 1 ($H_{1820} < 8$), F_{1400}, F_{1820}, and F_{2240} were determined by requiring that their ratios to F_{2620} be the same as for bin 2.

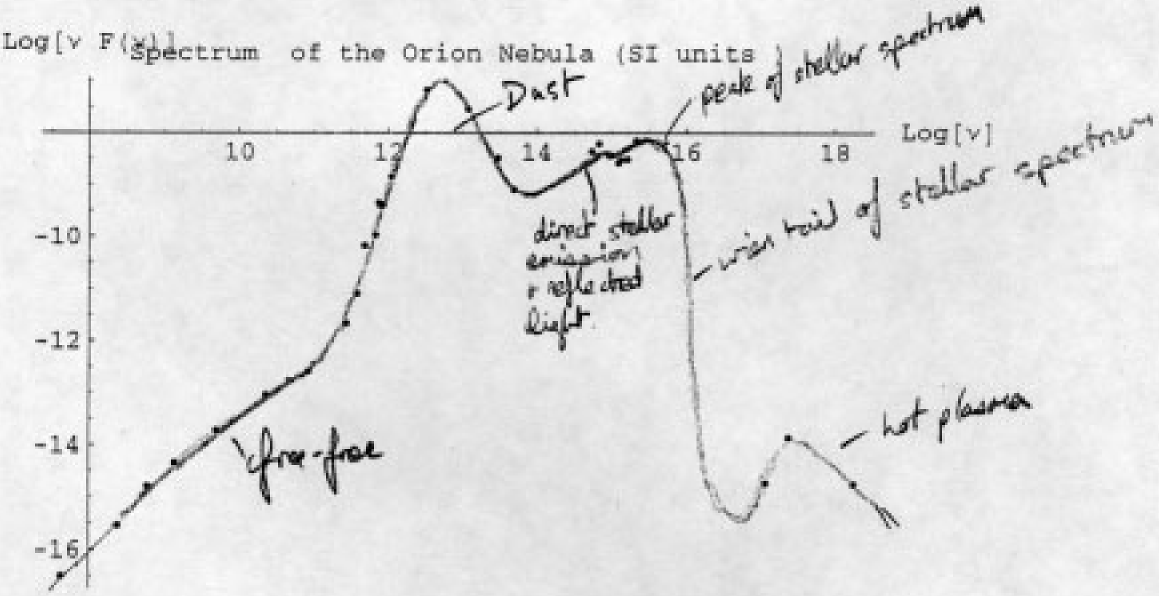
[c] For bin 11 ($H_{1820} > 4096$), F_{1400}, F_{1820}, F_{2240}, and F_{2620} were determined by requiring that the mean surface brightness be the same as for bin 10.

Spectrum Of The Orion Nebula.
(Central 5 ArcMinutes)

Curves fit by eye...

- **Data points are ones taken from the literature as described in the notes accompanying these graphs.**

```
In[75]:= linear = {{42*^6, 74}, {240*^6, 120}, {600*^6, 268},
             {1.4*^9, 324}, {5*^9, 380}, {23*^9, 400}, {86*^9, 307}, {273*^9, 800},
             {380*^9, 2136}, {480*^9, 14000}, {670*^9, 16000}, {750*^9, 60000},
             {790*^9, 50000}, {850*^9, 48000}, {1.1*^12, 0.13*^6}, {3.3*^12, 2*^6},
             {1.2*^13, 0.24*^6}, {3*^13, 11000}, {5*^13, 1620}, {5.45*^14, 810},
             {6.8*^14, 910}, {1.24*^15, 215}, {1.33*^15, 230}, {1.64*^15, 190},
             {2.14*^15, 333}, {1.2*^17, 1.6*^-6}, {2.41*^17, 5.8*^-6}, {1.8*^18, 1*^-7}};

In[76]:= nufnulin = linear;

In[77]:= Do[nufnulin[[i, 2]] = linear[[i, 2]] * linear[[i, 1]] * 1*^-26, {i, 1, 28}]

In[78]:= nufnulog = N[Log[10, nufnulin]];

In[85]:= ListPlot[nufnulog, PlotJoined -> False,
          PlotLabel -> "Spectrum  of the Orion Nebula (SI units )",
          AxesLabel -> {"Log[v]", "Log[v F(v)]"}]
```



```
Out[85]= - Graphics -
```

the units used are : F[W m^-2Hz^-1]
ν[Hz]

The straight lines are of course artifacts of the poor resolution I have in some parts of the spectrum.

```
In[80]:= fnulin = linear;

In[81]:= Do[fnulin[[i, 2]] = linear[[i, 2]] * 1*^-26, {i, 1, 28}]

In[82]:= fnulog = N[Log[10, fnulin]];
```