Astronomy 208 Introduction to Meeting #5

Goal for today (& next time) ...

What distribution of observable photons are produced / absorbed etc. by "reactions" in the ISM, under specific conditions?

Points to Remember (Very Important)

1 → Except in H II regions, we're NOT usually talking about electronic transitions (e.g. Bohr Hydrogen)

2 → "Upper level" and "lower level" just refer to two different QM states of an atom or molec.
$E_{upper} > E_{lower}$

3 → "transitions" can be induced by
- photons
- "interactions" w/ cosmic rays
- "collisions" w/ atoms, molec, ions
- "interactions" w/ electrons

4 → "levels" can refer to electronic, rotational, vibrational, spin, or "magnetic" states

5 → Need to know:
- Chemical Composition (Atoms, Molec, Ions, Elec, c rays, e^-)
- Photon Bath (Radiation Field)
- Velocity Distribution (from μ & T)

We'll discuss Collisions & Transition Probabilities Today
n-n, n-i A ul, B ul, B lu etc

(2.1b non-equilibrium ~~states~~ distributions e.g. masers – later)

2.2. EXCITATION PROCESSES

note nice calc of t to estab Maxwellian dist

{Much is determined by collisions among various species...
{ion – ion or ion – e^- are essentially Coulomb forces (see Spitzer 2.1)

Short – Range Forces amongst neut-ion or neut-neut; inherently [QM'l]

[Neutral-Neutral:] Very weak interaction until electron clouds overlap ⇝ behave like "hard spheres", so given $r_{atom} \sim 1\,\text{Å}$

$$\boxed{\sigma_{nn} \approx \pi(\underset{\text{atom 1}}{r_1^2} + \underset{\text{atom 2}}{r_2^2}) \sim 10^{-15}\ \text{cm}^2} \quad (1)$$

What collision rate does that imply?

$$\boxed{\text{m.f.p.} = l_c \approx (n_n \sigma_{nn})^{-1} = \frac{10^{15}\ \text{cm}}{n_4}} \quad (2)$$

That's about 3×10^{-4} pc

length = [(# density) × (area)]$^{-1}$

∴ In gas at a temperature T, mean atomic velocity given by

$$\boxed{\frac{3}{2} m_n v^2 = kT} \quad (3)$$

m_n = mass of a neutral

So

$$\boxed{\frac{1}{\tau_{nn}} \approx \frac{v}{l_c} \approx \left(\frac{2kT}{3m_n}\right)^{1/2} n_n \sigma_{nn} = 7\times10^{-12}\, n_n\, T^{1/2}\ \text{s}^{-1}} \quad (4)$$

$[\tau = 4.5\times10^{3}\, n^{-1} T^{-1/2}\ \text{yrs}]$

So, if $n_n = 1$ and $T = 80$ K ⇒ $\boxed{\tau_{nn} = 500 \text{ years}}$!!

(core) $n_n = 10^4$ and $T = 10$ K ⇒ 1.7 months

(hot/ave ISM) $n_n = 1$ $T = 10^4$ K ⇒ 45 years

Remember 3.2×10^7 s/yr

[Density Matters More than T $\frac{1}{\tau} \propto n T^{1/2}$]

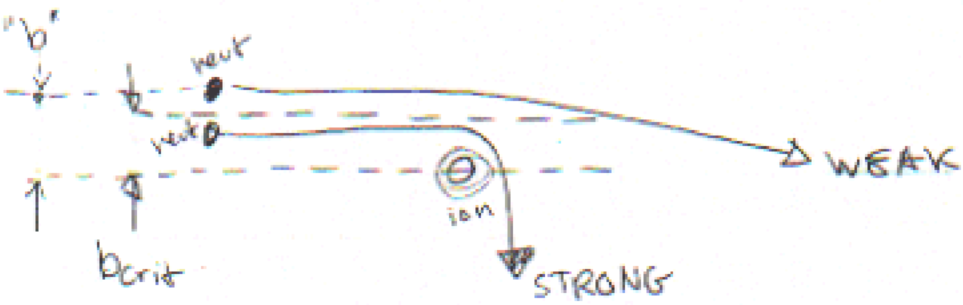
What about $\boxed{\text{Ion-Neutral}}$ Collision Rates?

Neutral is "polarized" by $\vec{E}$ field of Ion so that interaction energy

$$\boxed{U(r) \cong \vec{E}\cdot\vec{p} = \underbrace{\overbrace{\frac{Ze}{r^2}}^{\text{E field of ion}}}_{}\Big(\overbrace{\alpha\,\frac{Ze}{r^2}}^{\text{E field created by}}\Big) = \frac{\alpha Z^2 e^2}{r^4}} \quad (5)$$

$\vec{E}$-fld due to polarization (dip)

α = polarizability of neutral $\approx a_0^{\,3}$ (a_0 = Bohr radius = 0.529 Å)



Two types of interactions depending on masses & rel velocity

μ = reduced mass = $\dfrac{m_1 m_2}{m_1 + m_2}$

Impact parameter = b

(6a)

WEAK: $\dfrac{\alpha Z^2 e^2}{r^4} \ll \dfrac{\mu v^2}{2}$	STRONG: $\dfrac{\alpha Z^2 e^2}{r^4} \gg \dfrac{\mu v^2}{2}$
interaction < kinetic	interaction energy > kinetic

Boundary corresponds to "critical impact parameter" = b_{crit} given by

$$\boxed{\frac{\alpha Z^2 e^2}{b_{crit}^{\,4}} = \frac{\mu v^2}{2}} \quad (6b)$$

$\therefore$ Effective Collision X-sectn, σ_{ni} given by re-arranging 6b

$$\boxed{\sigma_{ni} \cong \pi b_{crit}^{\,2} = \pi Z e \left(\frac{2\alpha}{\mu}\right)^{1/2} \frac{1}{v}} \quad (7)$$

Since $n_i \neq n_n$ necessarily, we can't just say (eq. 2 & 4)

$$\frac{1}{\tau_{ni}} \approx \frac{v}{\ell_c} = v\, n_? \,\sigma_{ni}$$

Instead, we'll leave "n" out & calculate a $\overbrace{\text{rate coefficient}}^{k}$ in $\boxed{cm^3 s^{-1}}$

(8a)

$\boxed{k = \langle \sigma_{ni} v \rangle}$ but (7) $\Rightarrow \sigma_{ni} \propto \frac{1}{v}$ so k indep. of v

(8b) $\boxed{k = \pi z e \left(\frac{2\alpha}{\mu}\right)^{1/2}}$ $\boxed{\approx 2\times10^{-9}\ cm^3 s^{-1}}$ e.g. $C^+ - H$

ion-neutral scattering rate coefficient

for most exotherm ion-neut reaxns

Take "AY209" to find out more! (e.g. calculate "α" from Q.M.)

(9)

So $\boxed{\frac{\text{Rate}}{\text{Volume}} = n_i n_n \langle \sigma_{ni} v \rangle \ cm^{-3} s^{-1}}$

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example say $n_i = n_n = 1$, then $\frac{\text{Rate}}{\text{Vol}} \cong \boxed{2\times10^{-9}\ cm^{-3} s^{-1}}$ ion-neut

(15 yrs btwn transitions!)

For n-n, we found $\tau_{nn} = 500\,yr$ for $n_n = 1$ & $T = 80\,K$

that corresponds to $6.3\times10^{-11}\ s^{-1}$, or $\boxed{6.3\times10^{-11}\ cm^{-3} s^{-1}}$

in a box $1\,cm^3$ w/ 1 ptcl.

In this case, ion-neut interactions are more common by a factor of ~30.

FYI Exothermic (chemical) ion-neutral reactions $\rightarrow k \sim 2\times10^{-9}\ cm^3 s^{-1}$ as above

e.g. $CH^+ + H_2 \rightarrow CH_2^+ + H$

Also Exothermic charge-exchange reaxns often have $k \approx 2\times10^{-9}\ cm^3 s^{-1}$

e.g. $O^+ + H \rightarrow O + H^+$

See Rybicki & Lightman
Ch. 1

Ion-Ion Rates given by Coulomb Interactions, same for Ion-Electron
(see Prob Set 3?)

Radiative Transfer Definitions (more depth next time — this is just so I can use J_ν in discussing Einstein A,B)

dE = energy crossing dA in time dt & freq range $d\nu$
$= I_\nu \, dA \, dt \, d\Omega \, d\nu$ (R1)

(R2) I_ν = specific intensity (distance independent) erg s^{-1} ster^{-1} cm^{-2} Hz^{-1}

(R3) $F_\nu = \int_{4\pi} I_\nu \overset{\cos\theta}{\wedge} d\Omega$ = Flux erg s^{-1} cm^{-2} Hz^{-1} assuming normally incident rays ($\cos\theta = 1$)
(Actually $= \int I_\nu \cos\theta \, d\Omega$)

(R4) $u_\nu = \frac{1}{c} \int_{4\pi} I_\nu \, d\Omega$ = Energy Density erg cm^{-3} Hz^{-1}

(R5) $J_\nu = \frac{1}{4\pi} \int_{4\pi} I_\nu \, d\Omega$ = Mean Intensity erg s^{-1} cm^{-2} Hz^{-1}
(averaged over 4π steradians)

$\left(u_\nu = \frac{4\pi}{c} J_\nu\right)$

next time