

# Meeting #4 of Astronomy 208

On syllabus, we've now covered 1.1, and 1.2 a-g

let me just mention "(h)" Time Scales & Stability

e.g. "dynamical time" "crossing time" "free-fall time" "cooling time" (explain)

Always an important consideration:

what happens to some entity as a function of time

→ SEE also B & D Chapter 22 ←

good 1<sup>st</sup> guess: Virial Analysis for Volume w/in "S"

$$\left( \frac{d}{dt} \right) \frac{1}{2} \frac{d^2 I}{dt^2} = 2T + 3\Pi + \overset{\text{kinetic}}{M} + \overset{\text{internal}}{W} + \frac{1}{4\pi} \int_S (\vec{r} \cdot \vec{B}) \vec{B} \cdot d\vec{S} - \int_S \left( p + \frac{B^2}{8\pi} \right) \vec{r} \cdot d\vec{S}$$

surface pressure term
magnetic stresses
Spitzer 27-218

$I = \int \rho r^2 dV = \text{moment of inertia}$

$T = \frac{1}{2} \int \rho v^2 dV = \text{kinetic energy of the fluid}$  (macroscopic) mass motion

$\Pi = \int p dV = \frac{2}{3} \text{ of random k.E. of thermal ptcls} + \frac{1}{3} \text{ " " " relativistic ptcls}$  (microscopic) molecular motion

$M = \frac{1}{8\pi} \int B^2 dV = \text{magnetic energy w/in } S$

$W = - \int \rho \vec{r} \cdot \vec{\nabla} \phi dV = \text{total grav energy of system, if masses outside } S \text{ don't contribute to pot'l.}$

☑ = terms used most

(2)

often, many terms ignored, e.g. " $2T + W = 0$ "

this kind of simple analysis often used to determine how "bound" a system is & predict its future (e.g. collapse, expansion, evaporation)

(specific examples later in course, including instability analyses)

$$\frac{d^2 I}{dt^2} < 0 \text{ etc.}$$

? time scales for this

Chandrasekhar & Fermi's

1953 virial theorem:

very useful in ISM

$$2 [T_m + T_k] + \Omega + \mathcal{M} = 0$$

$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$T$	$\frac{3}{2} T$	$W$	$\mathcal{M}$	$\frac{d^2 I}{dt^2}$

(much different notation used in the literature, but it's all the same idea)

Note: Virial thm always holds - "inapplicability" is only a problem when important terms are omitted.

# 2. Kinetic Equilibrium & Radiative Processes

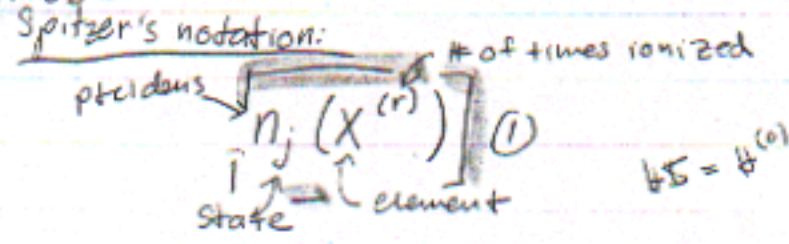
2.1 Thermodynamic Equilibrium  
 often "Local Thermodynamic Equilibrium" or "LTE"  
 is used as "local" assumption

ISM is not usually really in equilibrium, unlike stars  
 & in  $\star$ 's emission, abs., scatt, coll., <sup>(almost)</sup> all take place on very  
 short t-scales, in comparison w/ any dynamical or evolutionary  
 time scales - in ISM,  $n$  is low things are SLOW!

• also <sup>in ISM</sup> hot  $\star$ 's, cosmic rays, X-ray bkgd, shocks mixed  
 in  $\rightarrow$  non LTE

$\Rightarrow$  level populations in atoms & molecules are not always in their  
 "equilibrium distribution" (for a Maxwellian v-distr.)  
 (closest in regions where em & abs of h $\nu$  not too important  
 & collisional ex & de-ex rule)

I'll use



Spitzer's definition:

"ETE" equivalent thermodyk. equil  
 $T = T_{kinetic}$   
 $n_e =$  actual  $e^-$  density

← abstract!  
 but may not  
 tell whole story

for ETE above written  $n_j^*$     absolute value written  $n_j$

and  $n_j^* \neq n_j$  since in real ISM, dist. of atoms in dif stages of ionization will be different

Define:

$$b_j = \frac{n_j}{n_j^*}$$

②

ratio of particle density  
 to ptc dens in ETE

Note:  $b_j$  close to 1 when collisions dominate ionization & recombination

(4)

"LTE"  $\equiv b_j = 1$  for bound levels

Relative popln of levels for the same atom or ion given by the Boltzmann eqn:

$$\frac{n_j^*(X^{(r)})}{n_k^*(X^{(r)})} = \frac{g_{rj}}{g_{rk}} e^{-(E_{rj} - E_{rk})/kT} \quad (3)$$

$E_{rj}$  = energy &  $g_{rj}$  = stat. wt. of level  $j$ , ionization  $r$

So, using (2) & dropping the "r" designation:

$$\frac{n_k}{n_j} = \frac{b_k}{b_j} \frac{g_k}{g_j} e^{-h\nu_{jk}/kT} \quad (4)$$

$\nu_{jk}$  = freq of radiative trans from  $k \rightarrow j$   
convention  $E_k > E_j$   
 $E_{jk} > 0$

To find the fraction of  $X^{(r)}$  atoms which are excited to level  $j$ :

Define  $\sum_k n_k^*(X^{(r)}) = n^*(X^{(r)})$  = particle density of  $r$ -times ionized  $X$  in all states (5)

↓ using Boltzmann ↓

Fraction =  $\frac{n_j^*(X^{(r)})}{n^*(X^{(r)})} = \left( \frac{g_{rj} e^{-E_{rj}/kT}}{\sum_k g_{rk} e^{-E_{rk}/kT}} \right)$  (6) = fraction of  $X^{(r)}$  in level  $j$

Define  $f_r =$  partition function for ion  $X^{(r)} = \sum_k g_{rk} e^{-E_{rk}/kT}$  (7)

So then (6) becomes:  $\frac{n_j^*(X^{(r)})}{n^*(X^{(r)})} = \frac{g_{rj}}{f_r} e^{-E_{rj}/kT}$  (8) gives distribution of atoms over different levels