

AY208

Handout on Collisional Excitation

to accompany
Meetings 8 & 9

Collisional Excitation

for transitions $u \rightarrow l$ triggered by collision:

$$(C1) \quad C_{ul} \equiv C_{u \rightarrow l} = n \gamma_{ul} = n \langle \sigma_{ul} v \rangle$$

$$(C2) \quad C_{lu} = C_{l \rightarrow u} = C_{ul} \frac{g_u}{g_l} \exp\left(\frac{-h\nu}{T_e}\right)$$

for LTE

$\sigma_{ul} = x \sigma_{ex} n$ for coll $u \rightarrow l$ @ velocity v

γ_{ul} [$\text{cm}^3 \text{s}^{-1}$] = overall rate coeff $u \rightarrow l$

for Maxwellian vel dist @ T_e

$$\gamma_{ul} = \frac{4}{\sqrt{\pi}} \left(\frac{\mu}{2kT_e} \right)^{3/2} \int_0^{\infty} dv \sigma_{ul}(v) v^3 \exp\left(\frac{-\mu v^2}{2kT_e}\right)$$

for neut-neut $\gamma_{ul} \sim 10^{-11}$ to 10^{-10}
neut-ion $\sim 10^{-9}$

(often uncertain by $\sim \times 2$)

(C3)

$\mu = \text{reduced mass}$

rate coeff
↓

Given δ_{ue} , A_{ue} , $\tau_k \neq n$ one can get

$n_u \neq n_e$ from detailed balance ($n = n_{tot} = n_u + n_e$)

$$\left[n_e (\delta_{ue} n) = n_u (\delta_{ue} n + A_{ue}) \right] \textcircled{C4}$$

$$\left[\frac{n_u}{n_{tot}} = \frac{\frac{g_u}{g_e} \exp\left(-\frac{h\nu}{kT_k}\right)}{1 + \frac{g_u}{g_e} \exp\left(-\frac{h\nu}{kT_k}\right) + \frac{n_{crit}}{n}} \right] \textcircled{C5}$$

$$\left[n_{crit} = A_{ue} / \delta_{ue} = \text{critical density} \right] \textcircled{C6}$$

Using (C5), we see that for $n \gg n_{crit}$ u,e thermalized at T_k

$$n \gg n_{crit} \text{ so } \left[\left(\frac{n_u}{n_e} \right)_{\text{thermal}} = \left(\frac{g_u}{g_e} \right) \exp\left(-\frac{h\nu}{kT_k}\right) \right] \textcircled{C7}$$

but for $n \ll n_{crit}$ spontaneous > collisions
 \Rightarrow each collision $e \rightarrow u$ gives emission

$$\left[\frac{n_u}{n_e} = \left(\frac{n}{n_{crit}} \right) \left(\frac{n_u}{n_e} \right)_{\text{thermal}} \right] \textcircled{C8}$$

$$\left[\begin{array}{l} \text{Flux} \propto A_{ue} \int n_u dz \\ \int_{n_{tot}} n dz \propto n^2 \quad n < n_{crit} \\ \int_{n_{tot}} dz \propto N \quad n > n_{crit} \end{array} \right] \textcircled{C9}$$

Astronomy 208 Meeting # 9

Cold ISM, cont'd

Last time a) Atomic Gas

$$\text{Recall: } \tau_\nu = 5.49 \times 10^{-19} \frac{N_{\text{Total}} \phi(\nu)}{T_{\text{ex}}}$$

See handout for

$$\tau(\nu) = \frac{N(\nu)}{C \times T_s} \quad C = 1.82 \times 10^{18}$$

(units onboard may have been confusing last time)

e.g. for $\tau = 1$ @ $T_s = 1000 \text{ K}$

$$1 = \frac{N(\nu)}{1.82 \times 10^{18} \cdot 1000} \Rightarrow N \approx 2 \times 10^{21}$$

(1 mag A_ν)

& for $N \gg 2 \times 10^{21}$ $\tau \gg 1$
 $N \ll 2 \times 10^{21}$ $\tau \ll 1$

Three more points about HI

(i) Line shapes

(ii) Radial & Vertical Dist'n.

(iii) Using Absorption & Emission Together
to get τ , T_s

(i) Line Shapes

keep in mind.

for a photon emitted by an atom @ rest

$$\Delta E \Delta t \geq \hbar \quad (\text{Heisenberg})$$

$$2\pi \Delta \nu \Delta t \geq 1$$

$\Delta t = 1/A_{ul}$ \approx occupation time in upper state

e.g. for HI $A_{ul} \approx 3 \times 10^{-15} \text{ s}^{-1} \Rightarrow \Delta \nu_{\text{nat}} = \frac{A}{2\pi} = 5 \times 10^{-16} \text{ Hz}$

In velocity, @ $\nu = 1420 \text{ MHz}$ (21cm) this gives:


$$\Delta v = \Delta \nu \frac{c}{\nu} \approx 10^{-19} \text{ km/s} \ll \ll \text{any obs'd } \Delta v$$


In 21-cm (not all molec. line obs.) "natural" linewidth inconsequential


Actual lineshape determined by:

- ① distribution of material as function of velocity (Doppler profile)
- ② temperature distribution along l.o.s.
 \rightarrow (radiative xfer gives overall profile)

Gross example of ②

If $T_{\text{bg}} > T_s \Rightarrow$ absorption 

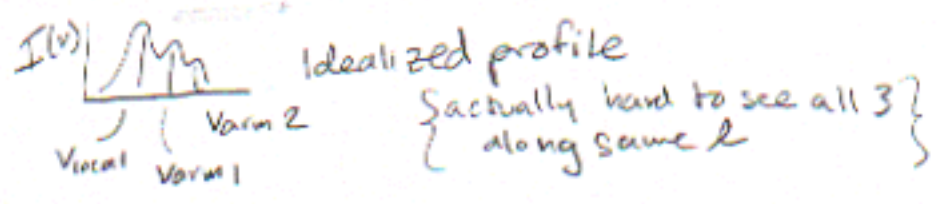
$T_{\text{bg}} < T_s \Rightarrow$ emission 

$T_{\text{bg}} = T_s$ NOTHING 

e.g. what if
extragal HI
is @ 3k?

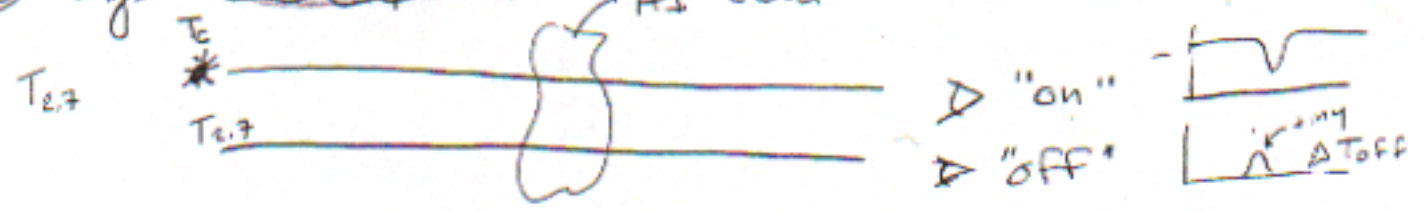
(ii) Radial & Vertical Distribution of HI

radial: spiral arms can be identified from radial velocities of peaks



vertical: can model as a plane-parallel atmosphere, but HI not isothermal w/ z, so this doesn't work perfectly
 see handout for empirical determination

(iii) Using HI in Absorption & Emission Together to get τ , T_s



$$T_{on} = T_c e^{-\tau} + T_{2.7} e^{-\tau} + T_{HI} (1 - e^{-\tau}) \quad (R-J)$$

$$- T_{off} = T_{2.7} e^{-\tau} + T_{HI} (1 - e^{-\tau})$$

$$T_{on} - T_{off} = \Delta T = T_c e^{-\tau} \Rightarrow \text{gives } \tau \text{ directly} \quad (1)$$

$$\Delta T_{off} = (T_{HI} - T_{2.7}) (1 - e^{-\tau}) = T_{off} - T_{2.7} \quad (2)$$



observation of off position above.

So (1) gives you τ , sub into (2) gives T_{HI}

spin temp of HI in absorbing region

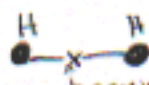
Note: We'll talk more about abs lines later in course. e.g. hot LSM

§ 3.2 b. Molecular Gas

Why can't we just observe H₂?

(We can: ^{UV} Interstellar Absorption Lines (of H₂) is Big Business - but you need a background source; see Spitzer p. 96-98 & for ISM §)

In emission:



(most radio transitions come from ro-vib dipole radiation. - see CO, below) \Rightarrow no dipole radiation possible

Observe "Trace" Species Instead

Some H₂ vib emission from shock - later

• choice of tracer depends on \bar{n} for R.O.I & " n_{crit} " for tracer

e.g.	type	n in cm^{-3}	tracer
In PRACTICE for cold gas $0 \lesssim T \lesssim 100K$	low	$10 \lesssim n \lesssim 500$	¹² CO
	"dark"	$300 \lesssim n \lesssim 5 \times 10^3$	¹³ CO, OH
	dark/dense	$10^3 \lesssim n \lesssim 10^4$	C ¹⁸ O, CS
	dense	$5 \times 10^3 \lesssim n \lesssim 10^6$	NH ₃ , CS
	very dense	$n \sim 10^8$	OH masers
very, very, dense	$n \sim 10^{10}$	H ₂ O masers	

recall

$$n_{crit} = \frac{A_{ul}}{\gamma_{ul}}$$

trouble is - some tracers emit line @ $n < n_{crit}$ due to radiative trapping, etc.

- density @ which collisional excitation beats spontaneous emission rate

- leads to continuing emission from the tracer (otherwise, all just go to lower state & stay there)

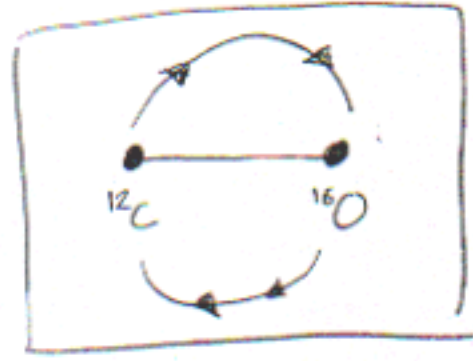
see handout

Energy ↓
vibrational rotational

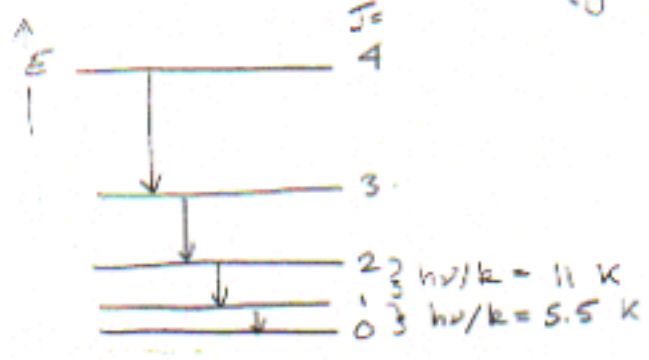
(5)

CO as Tracer of H₂

(next most abundant after H₂, but down by $\approx 10^{-5}$)



~ Rotational Ladder ~ (Vibrational ground state v=0)



Energy levels for linear molecules are those allowed by Q.M. for a "rigid rotator"

spacing of lines is equidistant in frequency

$$\nu_{J \rightarrow J-1} = 2BJ \quad (3a)$$

since $E = hBJ(J+1)$, $J=0, 1, 2$ and $\Delta J = \pm 1$ (3b) (selection rule)

$$B = h / 8\pi^2 I \quad (3d)$$

- has dipole moment (center of mass \neq center of charge)
- rotational states are quantized \rightarrow
- important coolant of molecular gas
- key tracer of molecular gas

Note: T_{rot} for these rotational trans. = "T_{rot}"

(for CO: $B = 5.75 \times 10^{10}$)

¹²C¹⁶O

$\nu(J=1-0) = 115.271 \text{ GHz}$ 285 (J=1)

$\lambda(J=1-0) = 2.7 \text{ mm}$

$A_{10} = 6 \times 10^{-8} \text{ s}^{-1}$

Easily excited @ low temperatures... 1st excited state only 5.5 K above ground ($kT_{rot} \approx 8.6 \times 10^{-5} \text{ eV} \cdot 5.5 \text{ deg} = 4.7 \times 10^{-9} \text{ eV}$)

Maximum emission in a rotational ladder @ T_{rot} occurs @

$E_J = hBJ(J+1) \approx kT_{rot}$, so $J_{max} \approx \sqrt{T_{rot} / hBk}$

(4)

From 3b

Which ^{12}CO transition is strongest where?

<u>T_{rot}</u>	<u>J</u>	<u>λ</u>
10 K	$2 \rightarrow 1$	1.3 mm
50 to 100 K	4...7	sub-mm
> 1000 K	20...40	far-IR

← SMT !!

→ relative strengths of these ^{12}CO lines can give T_{rot}
(but absolute level of emission depends on collision rates, too
thus various combinations of n & T_{rot} can give
same flux ... see Conzel Fig. 8 (on web site) AG