Image and Beam Precession of the AST/RO Telescope
AST/RO Technical Memorandum No.25

Xiaolei Zhang
July 12, 1996

The AST/RO telescope is an offset Gregorian telescope. The details of its optical and mechanical arrangement can be found in Stark et al. (1996). It has been found that when the telescope tracks a source around a full circle in azimuth at the South Pole site, the apparent position of the source precesses around a small circle or ellipse, centered not far away from the source's nominal position. This effect is termed beam precession. The parameters of the beam precession pattern also appear to have an elevation dependence.

By analysing the central ray path through the optical system, it can be shown that the AST/RO optics produces no intrinsic beam precession effect, i.e. the focal plane image of the on-axis rays from the sky remains stationary as the telescope is steered in azimuth and elevation. This indicates that the on-axis rays from the receiver also generate only the on-axis beam of the telescope, which should always be pointing at the source as the telescope changes azimuth and elevation.

The observed beam precession effect is therefore due to the alignment errors of the telescope optics, if a good pointing model has been applied to take out the mount error. In the following, we analyse first the focal plane image precession of the on-axis rays from the sky, due to various optics alignment errors. We then turn the problem around and analyse that in such an imperfect optical system, how the radio beam from the receiver precesses on the sky.

Image Precession on the Focal Plane

From Stark et al. (1996), we know that mirrors 1 (the offset paraboloid), 2 (the elliptical subreflector) and 3 (the flat chopper mirror) rotate around the elevation axis (as well as around the azimuth axis) as a rigid structure. Mirror 4 (another flat mirror), however, rotates only in azimuth. Assume first that the mechanical azimuth and elevation axes intersect, and are perpendicular to each other, we now analyse the precession of the image of on-axis rays from the sky at the focal plane, due to various mirror alignment errors.

In Figure 1, assuming that the elevation axis intersects mirror 3 at O_1, and intersects mirror 4 at O_2; assuming also that the mechanical azimuth axis intersects the focal plane at A, and the focal plane image of a fictitious ray which coincides with the elevation axis between mirrors 3 and 4 is B. The non-coincidence of A and B is due to the alignment error (angular or lateral) of mirror 4. If both mirrors 3 and 4 have alignment errors (angular or lateral), the central ray from the sky between mirrors 3 and 4 will not be coinciding with the elevation axis. The focal plane image of the central ray is now designated C, which incidentally is also the image of other on-axis rays, if the alignment errors are chiefly due to mirrors 3 and 4. As the elevation angle is changed from +90 to -90 degrees (imagine AST/RO suspended in space un-obstructed), the image point moves from C_1 through C_2, and finally ends up at C_3, describing a half circle centered on B. Furthermore, when the telescope is steered in azimuth, the points C_1, C_2, C_3 and B precess around A as with the same rate, and with the relative orientation of these points unchanged. We thus obtain that the focal plane image of a star in the lab frame, viewed from origin A, changes with the azimuth and elevation angle of the source as

$$\vec{AC} = (\vec{AB} + \vec{BC}_2 e^{\pm jEl})e^{-jAz}, \tag{1}$$

where the plus or minus sign in the elevation dependence is determined by the nature of the mirror 3 error.

Moreover, if the image of the receiver on the focal plane is D (not shown in Figure 1), which is also misaligned with A, the image precession effect viewed by the receiver is

$$\begin{aligned}
\vec{DC} &= \vec{DA} + \vec{AC} = \vec{DA} + (\vec{AB} + \vec{BC}_2 e^{\pm jEl})e^{-jAz} \\
&= |\vec{DA}|e^{j\theta_a} + (|\vec{AB}|e^{j\theta_b} + |\vec{BC}_2|e^{j(\theta_c \pm El)})e^{-jAz} \\
&= |\vec{DA}|(\cos\theta_a + j\sin\theta_a) + |\vec{AB}|\cos(\theta_b - Az) + j|\vec{AB}|\sin(\theta_b - Az) \\
&\quad + |\vec{BC}|\cos(\theta_c \pm El - Az) + j|\vec{BC}|\sin(\theta_c \pm El - Az),
\end{aligned}$$

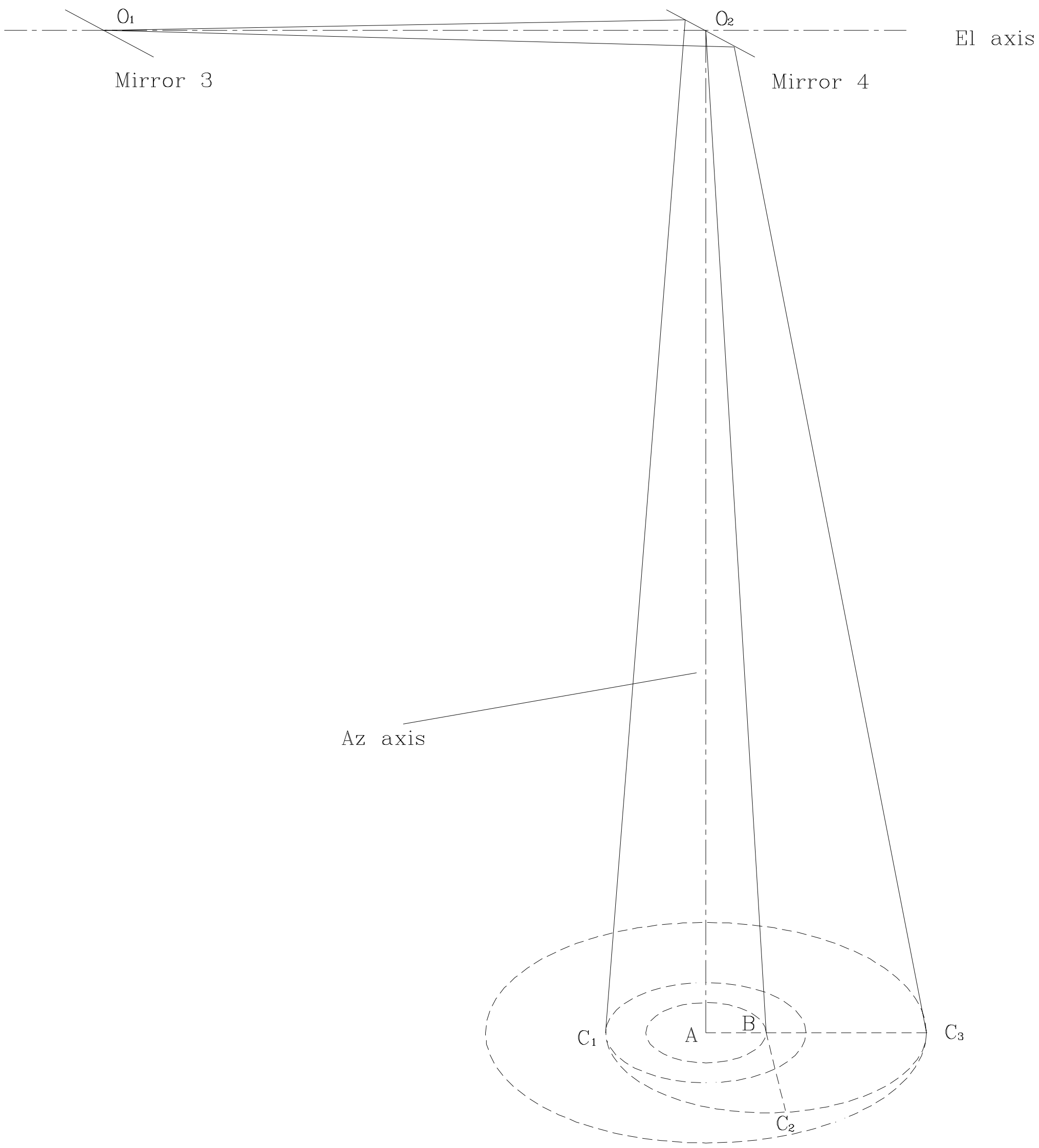

Figure 1: Geometry of the AST/RO optics from mirror 3 to the focal plane, with rays illustrating the various forms of image precession effects

where θ_a, θ_b and θ_c are the phase angles of vectors $\vec{DA}$, $\vec{AB}$, and $\vec{BC}_2$, respectively.

Therefore, the cartesian components x, y of the focal plane image viewed by the receiver changes with the azimuth and elevation of the source as

$$x_{\vec{DC}} = |\vec{DA}|\cos(\theta_a) + |\vec{AB}|\cos(\theta_b - Az) + |\vec{BC}_2|\cos(\theta_c \pm El - Az), \quad (2)$$

and

$$y_{\vec{DC}} = |\vec{DA}|\sin(\theta_a) + |\vec{AB}|\sin(\theta_b - Az) + |\vec{BC}_2|\cos(\theta_c \pm El - Az). \quad (3)$$

Finally, we add that the alignment errors of mirrors 1 and 2 will cause the rays from the sky to come down at a wrong angle, and will smear out the focal plane image somewhat too. The effect of the mirror 1 and mirror 2 alignment error on image precession is similar to that of the mirror 3 alignment error, and will therefore be added on (vectorially) to the mirror 3 error term. A constant radial axis offset also leads to similar beam precession effect. The non-intersection of the azimuth and elevation axes causes image precession effect similar to that of the alignment error of mirror 4 and will be added onto it vectorially.

Radio Beam Precession on the Sky

Since the observed beam precession is actually the precession of the radio beam onto sky, we need to transform the focal plane image precession effect, which we have derived in the last section, into the beam precession effect.

For the AST/RO type of optical and mechanical arrangement, even though the focal plane image of the on-axis rays has no intrinsic precession effect, there is indeed the intrinsic image precession of the off-axis rays on the focal plane, in much the same fashion as the precession of on-axis rays in the case of mirror 3 alignment error.

Referring back to Figure 1, we see that in order for the receiver to be centered on the image of the source at the different azimuth and elevation, we effectively have to use the image of the off-axis rays, and the amount of this offset in the sky is the beam precession effect we are seeking. The transformation which brings us from vector $\vec{DC}$ on the focal plane to the beam precession on the sky is the inverse of $e^{-jAz \pm jEl + j\theta}$. This can be seen as follows. The on-axis ray precesses on the focal plane as $\vec{DA} + \vec{BC}_2 e^{\pm jEl - jAz} + \vec{AB}e^{-jAz}$, which is made up of three separate terms. A constant off-axis ray of proper orientation could produce a processing focal plane image to cancel the term $\vec{BC}_2 e^{\pm jEl - jAz}$. If the off-axis ray has another component which changes direction with elevation opposite to that of $e^{\pm jEl}$ term in the image precession equation, with a proper magnitude and phase it could cancel the $\vec{AB}e^{-jAz}$ term. Finally, a third component of the precessing beam on the sky of the fashion of $e^{\mp jEl + jAz}$, and of the proper magnitude and phase could cancel the receiver offset term on the focal plane. The separate cancellation of terms is possible because the focal plane is an image of the sky, and the off-axis-ray images precess around the on-axis ones at the focal plane, and the phrase "precess around" is here equivalent to "add vectorially onto". The net effect of this precessing off-axis ray on the sky propagating through the imperfectly aligned optical system is a stationary image of the star on the receiver aperture, as the telescope is steered in azimuth and in elevation.

The beam precession equations can therefore be written as

$$\Delta Az \cos(El) = A_1 \cos(\theta_1 \mp El + Az) + A_2 \cos(\theta_2 \mp El) + A_3 \cos(\theta_3), \quad (4)$$

and

$$\Delta El = A_1 \sin(\theta_1 \mp El + Az) + A_2 \sin(\theta_2 \mp El) + A_3 \sin(\theta_3), \quad (5)$$

where $A_1 e^{j\theta_1}$ corresponds to the error due to the misalignment of the receiver; $A_2 e^{j\theta_2}$ corresponds to the fourth mirror alignment error, as well as that due to the non-intersecting of the axes; $A_3 e^{j\theta_3}$ is the vector sum of the errors produced by the misalignment of the first three mirrors, as well as by the constant radio pointing offset on the sky.

We will obtain the same results as above if we reverse the ray propagation direction and work out the beam path from the receiver end to the sky – for example, the alignment error of mirror 3 should produce only a constant offset on the sky, since mirror 3 rotates rigidly with mirrors 1 and 2 as the telescope changes azimuth and elevation.

We thus see that there are six fitting constants all together (not counting the plus and minus sign uncertainty in the elevation dependence. The sign of this term can always be determined as part of the fitting process, of course, if no *a priori* knowledge of this information is available). A nonlinear algorithm is required to do the fitting. A good sky coverage in both the azimuth and the elevation direction is also required to fit all the constants.

For the beam precession data that were actually gathered by the AST/RO, there was a fair coverage of the azimuth range for each source observed, due to the special condition at the South Pole. The elevation range of the sources observed is however very limited, due to the scarcity of strong point sources in the submillimeter wavelengths. Therefore, compromises need to be made in deriving a workable beam precession model. One way to do this is to fit the azimuth dependence only, and to absorb the elevation dependence in the fitting constants, i.e. we fit rather

$$\Delta Az \cos(El) = B_1 \cos[\theta(El) + Az] + B_2(El), \tag{6}$$

and

$$\Delta El = B_1 \sin[\theta(El) + Az] + B_3(El). \tag{7}$$

For each elevation, there are only four fitting constants now, and one of them is actually elevation independent (i.e. in the case of optics alignment error alone, and no residual mount error terms after the optical pointing correction, we will have a precession circle instead of a precession ellipse). The values of these constants for elevations in between of those actually observed can be obtained by linear interpolation.

Beam Precession Pattern with Residual Mount Error

In the event that the mount error has not been completely taken out by a star pointing model, the residual pointing errors will also contribute to the radio beam precession. This will in general make the beam precession pattern an ellipse rather than a circle, and the enter, size, and the orientation of the ellipse will all be elevation dependent[1].

Therefore, in the most general case, the beam precession model which include both the effect of the residual mount error and the optics misalignment can be written as

$$\Delta Az\cos(El) = B_1(El)\cos[\theta_1(El) + Az] + B_2(El), \tag{8}$$

and

$$\Delta El = B_3(El)\sin[\theta_2(El) + Az] + B_4(El). \tag{9}$$

For each elevation, we have 3 fitting constants for the horizontal and the vertical pointing offsets, respectively. The values of these "constants" for elevations in between of those actually observed can be obtained by linear interpolation.

I thank Matt Rumitz, Maohai Huang, Rich Chamberlin, Alberto Bolatto and Tony Stark for helpful discussions.

Reference

Stark, A. A., Chamberlin, R., Cheng, J., Ingalls, J. & Wright, G. "Optical and Mechanical Design of the Antarctic Sub-mm Telescope and Remate Observatory (AST/RO)," 1996, preprint.

[1]In preserving the ellipitcal shape of the beam precession pattern in the presence of residual mount errors, we have assumed that the azimuth dependence of the residual error terms has mainly the first sinusoidal harmonic component.