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EQUATIONS OF THE SECONDARY MIRRORS FOR AST/RO

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This memorandum describes the design and fabrication of secondary mirrors for the Antarctic Submillimeter Telescope and Remote Observatory. The optics as a whole and their interface to receivers is described in another memorandum, *AST/RO Optics*, by John Bally and Antony A. Stark.

Figure 1 is a schematic representation of the optics. Although AST/RO itself is offset in both azimuth and elevation, the primary and secondary mirrors are symmetric about a vertical plane, and this figure shows the intersection between the mirror surfaces and the plane of symmetry. The central ray is a short dashed line. In this figure the telescope is looking toward the zenith, along the y axis. The coordinate system, (x, y, z) has its origin a the vertex of the primary mirror paraboloid, and the focus of the paraboloid is at $(0, 1200, 0)$. These dimesions are in millimeters; we will switch to inches later for the machine shop. In this coordinate system, the surface of the primary mirror satisfies the equation

$$y = \frac{x^2 + z^2}{4f}, \tag{1}$$

where $f = 1200$mm. Seen from above, the primary mirror is bounded by a cylinder 1700mm in diameter, with its axis at $x = 1150, z = 0$. The central ray follows this axis from $y = \infty$ until it intercepts the center of the primary mirror at (1150, 275.521, 0), where it reflects at an angle of incidence $\theta_p = 25^\circ\!.60219$, through Focus 1 at $(0, 1200, 0)$, to the center of the secondary. The focal ratio at the prime focus is 0.779. For a 15 dB edge taper at 492 GHz, the prime focus beam waist radius is $w_0 = 0.405$mm. Following the secondary mirror are a flat chopper which directs the beam along the elevation axis, and a flat "fourth mirror" which directs the beam down the azimuth axis. From the secondary to the chopper is 127.8mm, from the chopper to the "fourth mirror" is 389.6mm, and from the "fourth mirror" to the optics table is 3942.6mm. The geometrical coudé focus is 800 mm above the optics table; the 492 GHz beam waist is 155mm closer than this to the secondary mirror because of finite-wavelength effects.

There are two secondary mirrors, called "long" and "short", intended to be used with instruments at the coudé focus and Nasmyth focus, respectively. As of March 1994, only the "long" mirror has been fabricated. First consider the long focal length mirror. Its surface is determined by four criteria: (1) the mirror has two focii, one coincident with the primary mirror; (2) the line segment connecting the two focii is aligned with the y axis (minimum abberation condition); (3) the vertex-to-focus distance is (arbitrarily choosen to be) exactly $1/12^{\text{th}}$ that of the primary, i.e. 100mm; (4) the distance from the mirror center to the second focus is 3660mm, in order to place the geometric focus of the secondary 800mm above the optics table in the coudé room. The surface of this table is 4460mm from the center of the secondary along the optical path. These conditions are satisfied by an ellipsoid whose equation is

$$\left(\frac{y-y_c}{a}\right)^2 + \left(\frac{x}{b}\right)^2 + \left(\frac{z}{b}\right)^2 = 1, \tag{2}$$

where

$$y_c = -591.099\text{mm} = -23.272'',$$

$$a = 1891.099\text{mm} = 74.45272'',$$

$$b = 606.811\text{mm} = 23.8902''.$$

The input phase front radius of the "long" mirror is therefore $R_1 = 122.20$mm, the output phase front radius is $R_2 = 3660$mm, and half the angle through which the beam is turned is $\theta_i = 24\overset{\circ}{.}8594$, according to the gaussian beam formalism of Chu (1983 *IEEE Trans. Ant. and Prop.* AP-31, 614).

The "short" mirror is also determined by four criteria: (1) it has two focii, one coincident with the primary mirror; (2) the second focus lies on the central ray determined by the "long" mirror, so that third and fourth mirrors do not have to be realigned when the secondary mirror is changed; (3) the second focus is $40.00'' = 1016$mm from the focus of the long mirror—this is the difference in path lengths between the coudé focus and the Nasmyth focus; (4) one point on the short mirror's surface is coincident with a point on the long mirror's surface—this point is chosen to be point of intersection of the surface with the ray closest to the y axis, at $(-24.98, 1298.38, 0)$. The choice of this point of intersection results in the two surfaces lying close together over most of the area of the secondary mirror. These conditions are satisfied by an ellipsoid with semi-major axis $a = 570.01$mm $= 22.441''$, semi-minor axis $b = 321.25$mm $= 12.648''$, and tilted by $4\overset{\circ}{.}1742$ with respect to the y axis.

These ellipsoids must be transformed into the local coordinate system of the secondary mirror, in order to describe how the surface is machined. The local coordinate sytem is determined by four accurately-made "construction balls" attached to the surface of the mirror. These are Reid Tool Supply Company (2265 Black Creek Road; Muskegon, MI 49444; tel:800-253-0421) part number TCB-26875. The balls are $0.500''$ diameter, and the ball center is $0.3125''$ from the plane of the front of the mirror. The center of the local coordinate system lies in the same plane as the centers of these balls, at the point centered between them. Call the local coordinates (x', y', z'). As shown in Figure 2, the z' axis is positive going into the mirror, the front surface of the mirror is parallel to the plane $z' = 0$, and the plane $y' = 0$ is the plane of symmetry for the mirror. For both the long and short mirrors, consider the coordinate system (x'', y'', z'') in which the equation of the secondary surface is as simple as can be:

$$\left(\frac{y''}{a}\right)^2 + \left(\frac{x''}{b}\right)^2 + \left(\frac{z''}{b}\right)^2 = 1. \tag{3}$$

(The double prime coordinate system will be different for the two mirrors.) These coordinates are related to the (x', y', z') coordinate system by a translation in x'' and y'', followed by a rotation through angle θ about the z'' axis and a rotation of $-90°$ about the new x axis:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x'' - x_0 \\ y'' - y_0 \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ 0 & 0 & -1 \\ -\sin\theta & \cos\theta & 0 \end{pmatrix} \begin{pmatrix} x'' - x_0 \\ y'' - y_0 \\ z'' \end{pmatrix}. \tag{4}$$

The inverse transform is

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ \sin\theta & 0 & \cos\theta \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \\ 0 \end{pmatrix}. \tag{5}$$

Using this relation, Equation 3 can be re-written in the form

$$(\alpha z' + \beta)^2 + (\gamma z' + \delta)^2 + \epsilon^2 = 1, \tag{6}$$

where

$$\alpha = \frac{\cos\theta}{a}, \qquad \gamma = \frac{-\sin\theta}{b}, \qquad \epsilon = \frac{-y'}{b},$$
$$\beta = \frac{x'\sin\theta + y_0}{a}, \qquad \delta = \frac{x'\cos\theta + x_0}{b}.$$

Equation 6 can be solved for z':

$$z' = \frac{-(\alpha\beta + \gamma\delta) \pm \sqrt{(\alpha\beta + \gamma\delta)^2 - (\alpha^2 + \gamma^2)(\beta^2 + \delta^2 + \epsilon^2 - 1)}}{\alpha^2 + \gamma^2}; \tag{7}$$

this is the desired expression for the depth of the mirror's surface in the local coordinate system.

For the long secondary, $x_0 = -79.375$, $y_0 = 1820.939$ and $\theta = 27\overset{\circ}{.}2208$. For the short secondary, $x_0 = -81.33$, $y_0 = 494.85$ and $\theta = 31\overset{\circ}{.}395$. These values result from a particular placement of the aluminum mirror part in the structure of the telescope.