Accuracy Assessment of GPS Slant-Path Determinations

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Abtract
We have assessed the accuracy of GPS for determining slant-path delays from double-difference postfit residuals [*Alber et al*., 2000] using simulations. We have found that this method has inherent limitations for determining non-homogeneous atmospheres because the method spreads the inhomogeneous signals over all parameters estimated in the least-squares step and, therefore, over all reconstructed GPS slant-path delays, causing significant systematic errors.

1. Introduction

The potential for accurate slant-path determinations from GPS presents us with the possibility of retrieving the vertical distribution of moisture fields in the atmosphere, which could have a significant impact in weather forecasting and climate monitoring. For this reason, several groups have been developing methods for obtaining slant path delays. However, there are potential problems with all these methods because slant delays are determined from estimated parameters and postfit phase residual, generally a statistically non-robust procedure. Therefore, we have performed a rigorous assessment of current methods for GPS slant-path determinations.

In this paper, we present the results from our study. In particular, we use simulations to assess the feasibility of the standard method [*Alber et al*., 2000] for obtaining slant wet delays from GPS double-difference postfit residuals. (Slant path, line-of-sight, one-way phase, and ray are used interchangeably in this paper. Their physical units are in mm of water vapor delay, which can be converted to integrated precipitable water vapor (IPWV) by dividing by ~6.5.)

2. Simulations: Components and Parameters

Figure 1 shows the components of the simulator that we have developed to assess the standard method for obtaining slant wet delays from GPS double-difference postfit residuals. These components reproduce the GPS data flow of the standard method.

In the approach adopted, we simulate one-way phase observations for specific atmospheres, form double-difference phases from the simulated one-way observations, use least squares to estimate atmospheric (and possibly other relevant) parameters, calculate double-difference residuals, unwrap them using *Alber et al.* [2000] "zero-mean" assumption, and reconstruct one-way phase observations.

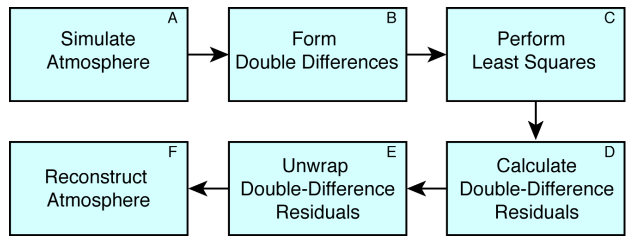

Figure 1. Simulation components.

The direct comparison between the reconstructed and simulated atmospheres then enables us to assess potential errors of the former.

In the simulations, the observing geometry is defined by the actual GPS constellation as observed at a 33-site ground-based network spanning most of the continental US (Figure 2). The shortest and longest baselines are 9 and 3628 km, respectively, with a quasi-continuous distribution between them. This network was used by *Braun et al*. [2003] to compare estimates of integrated slant water vapor from GPS and a microwave radiometer collocated at the central facility of the Atmospheric Radiation Measurement (ARM) Program near Lamont, Oklahoma. As in *Braun et al*. [2003], we assume that satellites signals are observed down to a minimum elevation angle of 10°.

In our simulations, the observing system is perfectly calibrated, i.e., the simulated one-way phase observations are free of multipath, scattering, ionosphere, homogenous atmosphere, satellite orbit and clock, site position and clock, observational or any other GPS error. In other words, the simulated one-way observations are exclusively due to atmospheric inhomogeneities.

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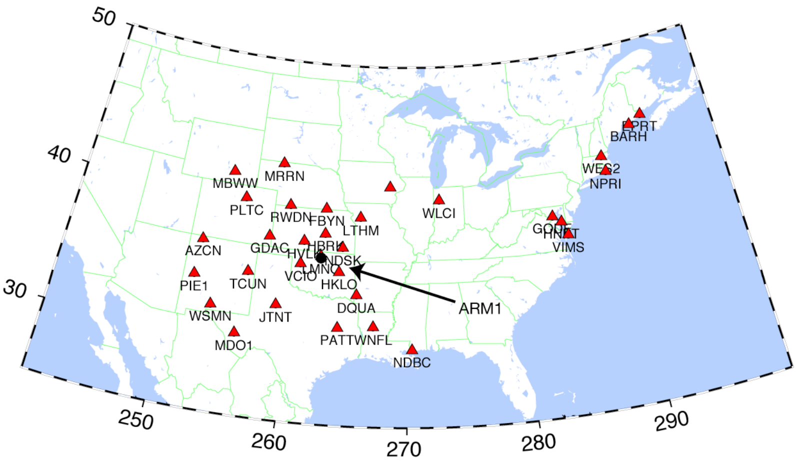

Figure 2. Ground-based GPS network. The location of the ARM GPS site (ARM1) is marked by a black circle.

3. Series of Simulations

We present three variants of a simulated atmosphere. We assume that the atmosphere is perfectly "homogenous" in all three simulations except for a single inhomogeneity that is present along a single slant path. (This model does not violate the "zero-mean" assumption since the zenith delay can be defined to yield such an atmosphere, and since the problem is linear the results are insensitive to the a priori zenith delay value.) For purposes of this study, we define a perfectly homogeneous atmosphere as one that can be characterized (i.e., parameterized) by a zenith delay and gradient parameters. This parameterization is standard in both GPS geodesy and meteorology data analysis [e.g., *Bar-Sever et al*., 1998]. In the first simulation, we follow the sequence of Figure 1. In the second and third simulations, we quantify the effect on the reconstructed atmosphere of the two key algorithm components: adding postfit residuals and least squares.

3.1. Simulation I

We simulate an inhomogeneity in an otherwise homogeneous atmosphere by assigning zero mm to all one-way phases for all 33 sites but a single phase at one site, to which we assign 10 mm of phase (Figure 3A). We have not fully explored the probability of sampling such a simplified atmosphere yet but observing a 10-mm excess phase seems quite plausible [e.g., *Davis et al*., 1993]. We have therefore decided to present results using a simplified atmosphere to assess the effect of the algorithms used by the standard method, and leave the simulation of an atmosphere that emulates nature for follow-up work.

The zenith delay and gradient parameter values of a homogenous atmosphere of zero-mm phases are, of course, zero mm. The outcome of the simulation does not however depend on the "true" values of the parameters because the model used is linear. We use zero mm for improved visualization. The inhomogeneity (Figure 3A) is represented by Ray 1, which happens to be from site ARM1. There are 263 rays (most of the 33 sites observe eight GPS satellites). Ray numbers are sorted by site, i.e., the first eight rays correspond to site 1 (ARM1), the next eight to site 2, and so on. This atmosphere results in zero mm for all double differences but for those that involve Ray 1 (Figure 3B). There are 221 independent double differences, seven of which involve Ray 1. We perform a least-squares solution using these double diffferences (Figure 3C). We estimate four parameters per site (a total of 132 parameters): zenith delay, north and east gradient, and vertical component of site position. In our simulations the last parameter is constrained to zero mm with an uncertainty of 0.001 mm, effectively fixing it. The inhomogeneity is absorbed as an adjustment to all three atmospheric parameters for site ARM1, mainly by the zenith delay parameter (Parameter 1) probably because of the high elevation angle (55°) of Ray 1. Note that the magnitude of this adjustment (~4 mm) is smaller than the accuracy of zenith delay estimates [*Elgered*, 1990]. The adjustment of the zenith delay and gradient parameters for other sites is ≤0.2 mm. The double-difference residuals (Figure 3D) involving Ray 1 are not constant since the least-squares solution modeled the inhomogeneity as (mainly) a zenith-delay adjustment. We use the "zero-mean" assumption for unwrapping the double-difference residuals into one-way residuals (Figure 3E). These residuals reflect the mapping of the inhomogeneity into the estimated parameters. The reconstructed atmosphere (Figure 3F) is the sum of the estimated atmosphere (Figure 3C) and the one-way residuals (Figure 3E). Site ARM1 recovers the 10 mm difference between Ray 1 and the rest of rays, but all its reconstructed phases are biased. Other ~1 and ~2 mm biases in Figure 3E correspond to low elevation-angle satellites (~15º and ~10°, respectively) rising from the east observed by the distant east-coast sites.

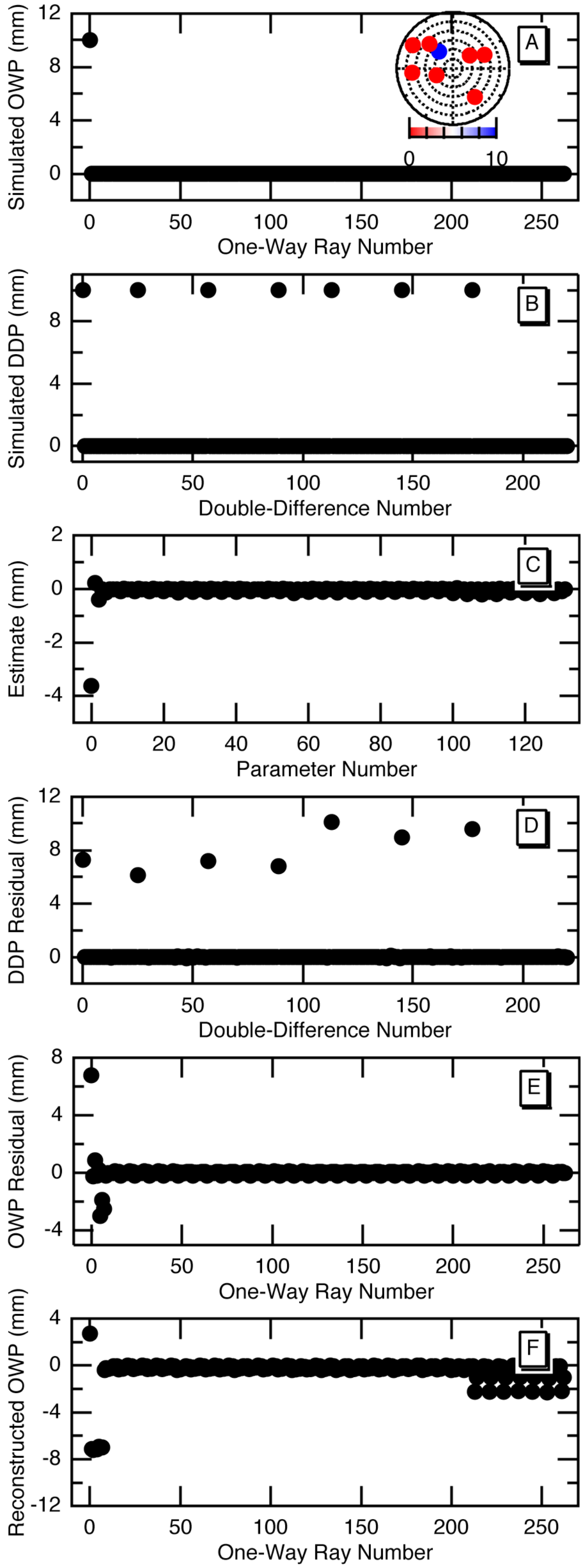

Figure 3. Full sequence of simulation components. There is a one-to-one correspondence between the (A-F) panels in this figure and the (A-F) component labels of Figure 1. Colors in sky plot (inset in A) represent simulated phase values at site ARM1. OWP = One-way phase; DDP = Double-difference phase.

The results of the simulation are summarized in Figure 4, which shows the difference between the simulated and reconstructed atmospheres. All eight reconstructed one-way phases at the site (ARM1) that observed the single inhomogeneity are in error by ~7 mm relative to the simulated phases. For the 10-mm inhomogenous ray, this represents a 70% error. Errors at other sites amount up to ~2 mm.

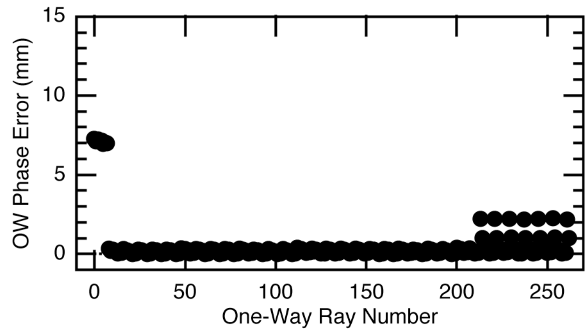

Figure 4. Reconstruction error for Simulation I.

Figure 4 illustrates an inherent limitation of the standard method for determining arbitrary inhomogeneities. The limitation arises from the parameter estimation step. The model describes an atmosphere characterized by zenith delay and gradient parameters. (It has not yet been demonstrated that higher-order parameters can be robustly estimated with GPS.) Inhomogeneities and more complex, and realistic, atmospheres do not fit into this simplified atmospheric model. Therefore, the least-squares solution distributes the atmospheric structure among the estimated model parameters, as any robust estimator is supposed to do. The algorithm of *Alber et al.,* [2000] attempts to overcome this modeling limitation by adding postfit phase residuals to the estimated parameters, a statistically non-robust procedure, as evidenced by the significant systematic errors of Figure 4. This algorithm provides accurate slant-path reconstructions only when the atmosphere behaves exactly as it was modeled. In those circumstances, the use of residuals is superfluous because GPS slant paths can then be accurately reconstructed simply from atmospheric parameter estimates.

To explore the effect of satellite geometry on the reconstructed slant-path delays, we have performed several series of simulations (not presented) whereby the single inhomogeneity is localized along the line-of-sight to, each time, a different satellite. We find that all reconstructed one-way phases are biased, with errors at the site that observed the inhomogeneity between 5–80%, depending on the satellite [*Elosegui and Davis*, 2003]. Other simulated configurations, including (a) a single inhomogeneity that last for not one epoch but say 15 min (and is sampled every 30 s), (b) a single inhomogeneity localized along the line-of-sight to a satellite and common to all sites within a small region, (c) a common inhomogeneity localized along the line-of-sight to pairs of nearby satellites from a single site, produce similar results [*Elosegui and Davis*, 2003].

3.2. Statistical Assessment of Postfit Residual Information

Before proceeding with the next simulation, we address the issue of whether addition of the postfit residual information yields a statistically significant improvement to the atmospheric reconstruction, as has been claimed [*Braun et al*., 2003]. To assess the information that is included in the postfit residuals, we performed a study in which we used the simulated atmosphere of Section 3.1 but we did not use the postfit residual information, i.e., we omitted steps D and E of Figure 1. The resulting one-way phase errors, representing the errors in the reconstructed atmosphere, are shown in Figure 5, which should be compared to Figure 4.

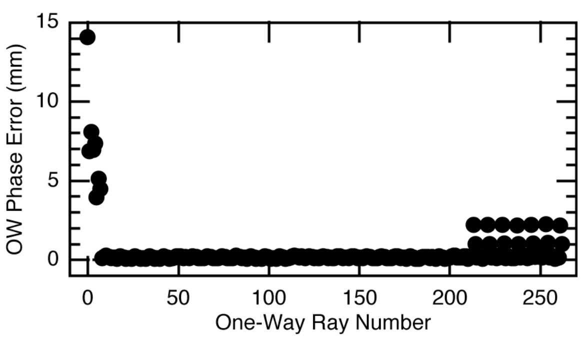

Figure 5. Reconstruction error for the simulated atmosphere of Section 3.1, calculated by omitting steps D and E (see text).

The main differences in the reconstruction errors for Section 3.1 and this study are for the site with the inhomogeneous ray, site ARM1. Whereas Figure 4 indicates a relatively constant error for this site of ~7 mm, Figure 5 indicates a variable error with a mean of about the same size. In fact, the root-mean-square (RMS) errors (relative to zero error) are 7.1 mm for Figure 4 and 7.7 mm for Figure 5, a variance ratio of 1.17 that is not very significant for the eight points of Site 1. But if instead we average over all the 263 rays, most of which have ~0 mm error on account of zero heterogeneity, then the RMS errors are 1.3 mm for Figure 4 and 1.4 mm for Figure 5, yielding a variance ratio of 1.15 that appears to be highly statistically significant for 263 points.

The problem with this latter calculation, of course, is that by including all the "zero" information we are not really improving the accuracy of the retrieval of the inhomogeneities, at which this technique is aimed. However, the situation we have posed is probably very likely in that inhomogeneities, like gradients [*Davis et al*., 1993], are nearly zero for most of the time. In other words, the Gaussian distribution is probably a poor descriptor of these phenomena, and comparisons of RMS and other statistics that are based on a Gaussian assumption can lead to erroneous conclusions regarding the significance of these statistics.

3.3. Simulation II

We now quantify the effect on the reconstructed atmosphere of the least-squares step. We thus simulate the same atmosphere (Figure 6A), and perform the same steps as in Simulation I but the atmospheric parameters are tightly constrained to their true value in the least-squares step. This is done by constraining all parameters at all 33 sites to zero mm with an uncertainty of 0.001 mm, which effectively fixes them. The procedure is similar to using external constraints on the atmospheric parameters estimated in the least-squares approach. Therefore, the double-difference residuals are identical to the simulated double-difference phases, that is, to Figure 3B, since all parameter adjusts in the tightly constrained least-squares solution are zero mm.

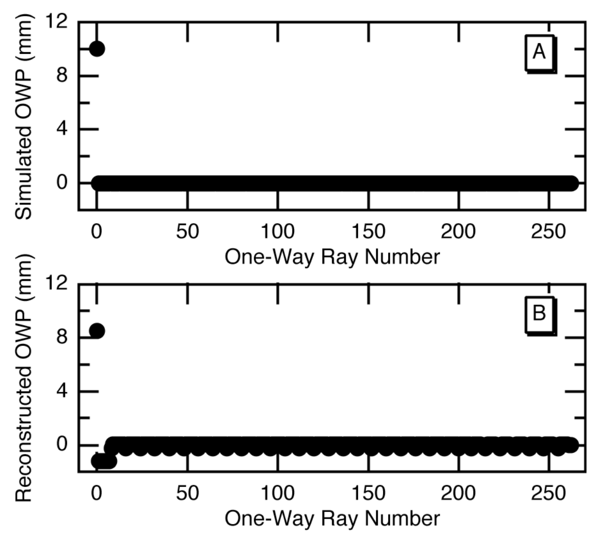

Figure 6. (A) Simulated (as in Figure 3A but without sky plot inset) and (B) reconstructed one-way phases (i.e., label component F in Figure 1).

The reconstructed one-way phases (Figure 6B) are the one-way phase residuals because the estimated atmospheric parameters are zero mm. The reconstruction reflects the spreading, by the “zero-mean” assumption, over all rays of the 10 mm inhomogeneity mapped into the double-difference posfit residuals. Indeed, non-zero rays are those that involve site ARM1 or satellite 1 (i.e., the site and satellite of Ray 1). Site ARM1 reconstructs 9.7 of the 10 mm difference between Ray 1 and its other rays.

Figure 7 shows the reconstruction error. All reconstructed one-way phases at ARM1 are in error by ~1.5 mm. The largest error for other phases is ~0.3 mm. The RMS error of the reconstructed one-way phases for site ARM1 is 1.3 mm (0.2 mm if averaged over all the 263 rays).

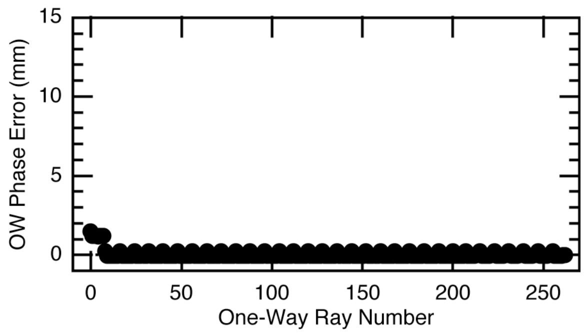

Figure 7. Reconstruction error for Simulation II.

Comparing Simulations I and II, we find that the reconstruction errors of the standard method (Figure 4) are significantly larger than those that result when external constraints on the atmospheric parameters estimated in the least-squares are used (Figure 7). The RMS error of the reconstruction has also improved, from 1.3 mm to 0.2 mm. This simulation demonstrates that the large errors in Section 3.1 were not due to the "zero-mean" assumption being applied to a "non-zero-mean" atmosphere.

The reconstruction error still represents ~15% of the signal that we are trying to measure. This 15% error is, for this particular case, entirely due to the "zero-mean" unwrapping algorithm. Perhaps most importantly, the availability of external constraints on atmospheric parameters at the 0.001 mm level would render this GPS approach to slant-path determinations superfluous.

4. Discussion and Conclusions

The scientific goal of this project is to determine whether it is possible to measure, and with what accuracy, a single line-of-sight GPS signal delay. In the present study, we have used simulations to assess the feasibility of the standard method [*Alber et al*., 2000] for obtaining slant wet delays from GPS double-difference postfit residuals.

Our studies have confirmed our intuition regarding linear least-squares solutions: that the standard method has significant problems. The origin of the problems lies in the least-squares step because the simple (relative to the complexity of nature) atmospheric model spreads the inhomogenous signals over all estimated parameters, as any robust estimator would do. Adding postfit residuals to a model calculated from "corrupted" parameter estimates does not "uncorrupt" them. Therefore, the standard method results in reconstructed GPS slant wet delays that present significant systematic errors.

The standard method provides accurate slant-path reconstructions only when the atmosphere can be described exactly by the parametric model. In these conditions, the use of residuals becomes superfluous because GPS slant-paths can then be accurately reconstructed using only atmospheric parameter estimates. Otherwise, we find that with the standard model the magnitude of the error in reconstructing slant wet delays is commensurate with the size of the inhomogenous signal that we are trying to measure.

We have found that the standard method leads to improvements in the RMS error of reconstructed slant wet delays of ~10%. This improvement is not a real measure of algorithm performance since the algorithm leads to significant errors in the reconstructed slant wet delays. Also, the RMS statistic is probably not a valid measure of statistical variation because atmospheric inhomogeneities are probably not Gaussian distributed.

We have also found that the accuracy of the standard method would be improved if there were external constraints on the atmospheric parameters estimated in the least-squares step. However, the GPS approach would be unnecessary if such independent constraints were available.

Heretofore, methods for determining GPS slant paths have been based on applying "ad hoc" conditions as extensions to estimation techniques and software packages developed with geodetic applications in mind. We have demonstrated that methods based on these conditions have significant problems. Obtaining accurate slant-path delays, if feasible, will require innovative atmospheric methods that make better use of the strengths of GPS than current methods. For example, though some methods incorporate statistical information regarding temporal correlations in the atmosphere, information regarding spatial correlations are still entirely ignored.

Acknowledgments
This research was supported by a contract from the Forecast Systems Laboratory (FSL) of the National Oceanic and Atmospheric Administration (NOAA). We thank S. Gutman of FSL for ideas on new method(s). J. Braun of UCAR provided us with the site coordinates used in these simulations

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