Submillimeter Array Technical Memorandum

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From................................ Andrew Dowd

The Effects of Comparator Hysteresis on Astronomical Spectrometers Part I - Computer Simulations and Telescope Measurements

I. **Introduction**

Recent experiments on the NRAO 12 meter Hybrid Spectrometer have indicated the existence of a problem in the A-to-D converter. Observations taken with the Hybrid Spectrometer will on occasion produce weak features in the baseline of the final spectrum. The features will normally appear as slopes which span a single chunk[1]. In the worst case, the output spectrum has a sawtooth appearance with a period equal to the width of a hybrid chunk. (A greatly exaggerated version of this is shown in figure B1). In actual telescope data, the baseline error is very small. However, I was able to exacerbate this problem in a set of controlled experiments. Some results from these experiments are presented in this memo.

The parameter which seems to have the most control over the baseline error is the change in input power at the sampler. It appears that relatively small changes in noise power alter the frequency response of the instrument. This change in frequency response is non-linear with respect to input power and from empirical observations tends to cause errors in large scale structure of the measured baseline.

The telescope experiments give some clues to this problem. However, complete control over the physical sampler is not possible, therefore, I tried to construct a model in software to gain a better understanding of these effects. The goal of these simulations is to help understand the root cause of this problem. Using this information I hope to avoid these effects in the SMA correlator.

The software simulations presented in this memo were useful tools for examining this problem. They allowed me to get a general feel for some the issues involved. Ideally, some direct mathematical expression of these models and there effects would be useful. At this time, I am looking at this possibility. The results of that work will be presented in Part II of this memo.

II. **Experiments at the NRAO 12 meter**

In January of 1993, I performed a number of experiments on the NRAO 12 meter Hybrid Spectrometer[7] to explore the reported baseline problem. From past experience, the source of the baseline problem was known to be errors in the the calibration scans, specifically the measurement of the hot load. Unlike other astronomical measurements, the hot load causes a significant change in the power level driven into the correlator. Thus, an experiment was devised to recreate this situation. The spectrometer measured the same noise source at two different levels of attentuation. The difference in power from the *Cal*ibrate measurement and the *Ref*erence measurement was +4dB (see Figure 4). These changes in power were applied symmetrically, +2 dB above and -2dB below, the optimum

1. A chunk refers to the portion of the output spectra which is processed by a single sampler

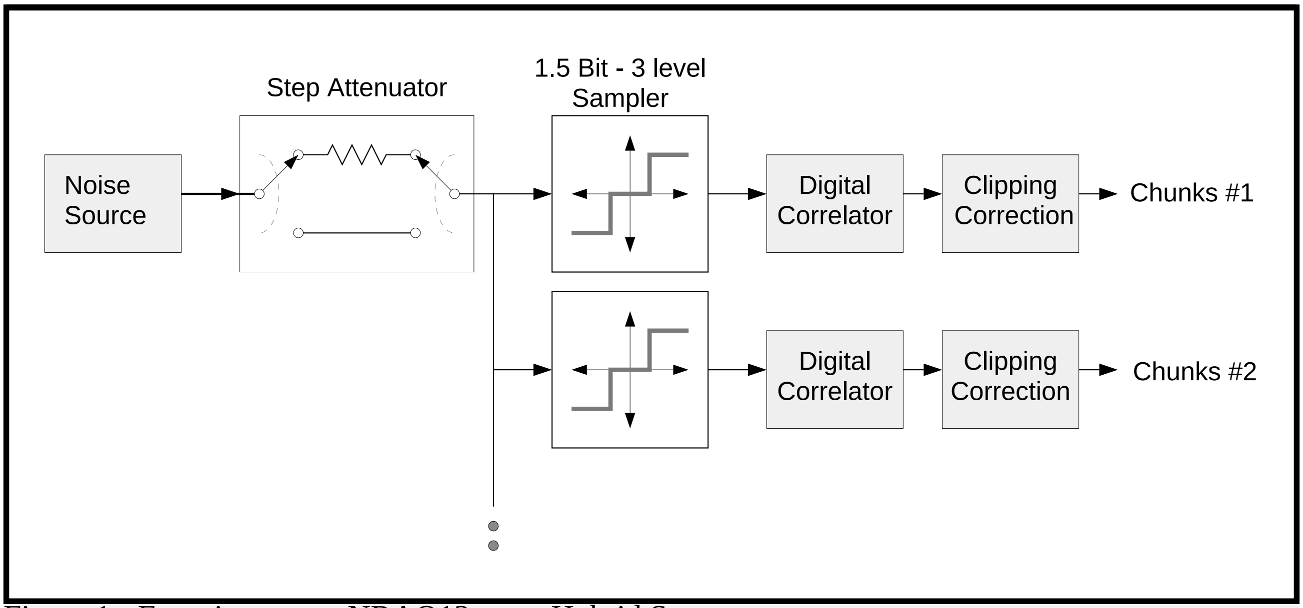

Figure 1 - Experiments on NRAO12meter Hybrid Spectrometer

operating power of the sampler (3 level). The resulting residual baseline after 30 seconds of integration and bandpass correction is sketched in figure B1 (Appendix B). These experiments give an exaggerated example of the errors seen in measuring a hot load. Undoubtedly, the attentuator used in the experiment was imperfect and therefore gives some baseline structure. However, the sawtooth effect on chunk boundaries is certainly an error in the spectrometer. The baseline errors disappear if the power level is unchanged (or nearly constant) between the two measurements. This is demonstrated in Figure B8 (note the change in scale). In this case, the step attenuator was not changed, therefore this experiment resembles the "Signal-Reference" measurement of an astronomical source. The presence of baseline errors is dependent on input power level.

The experimental data gives some interesting insights. Baseline errors at the 12 meter are qualitatively described as slopes in the chunks. It appears from the correlation differences that this effect is caused by the large deviation in the measured correlation at lag 1. From the correlation residuals (Figures B2-B6), the first lag shows the strongest bias. However, lags 2 and 3 also indicate some errors. A puzzling effect is the ripple at approximately 39 lags. It appears in most of the measured chunks. This ripple is the source of the high frequency oscillation present in the measured baseline (See figure B1). One possible explanation for this effect is the interconnection of correlator cards. After 32 lags, a sample emerges from one correlator board and is driven to another. Between the first correlator board and its sampler are 4 (or 5) stages of delay. From the output of the correlator board to the input of the sampler is a physical separation equivalent to a one or two bits worth of the sample period. All these delays adds up to a round trip travel time perilously close to 39 sample periods. Thus, as far-fetched as it might seem, maybe this ripple is caused by leakage from the digital backplane (which is in a shielded rack!) to the analog data stream. A useful test to verify this hypothesis would be to repeat the experiment with correlations longer than 64 lags. If this explanation is correction, I would expect another ripple pattern to appear at approximately 70 lags deep, i.e. after the signal emerges from the next correlator card.

One final error of interest can be noted in Figure B2. The entire residual correlation is shifted above zero. Most likely, this offset is caused by asymmetry in the threshold voltages. Errors in threshold levels are normally adjusted in the clipping correction, but only if they are symmetric about electrical ground. A skew between the magnitudes of the positive and negative thresholds could create the offset seen in Figure B2.

The discussion of spectrometer errors will now turn to some simulations I have devised.

III. **Hysteresis Model**

When considering the necessary precision of an astronomical sampler circuit, any non-linear behavior must be considered. For example, practical comparators exhibit propagation dispersion [3], which is a variation in the propagation delay with input voltage swing. Another non-ideal aspect is hysteresis. Many comparators have this "feature" built-in to avoid oscillations. However, I demonstrate below that this comparator characteristic reduces the effective dynamic range of the sampler.

A complete model of comparator features is a difficult problem. Therefore, I submit that a useful first order approximation for many comparator shortcomings is hysteresis (in two forms: digital and analog). I will not prove this assertion, but offer some anecdotal support for this assertion in appendix A.

Hysteresis, as defined for electronics, is normally associated with a feedback after one clock period. However, in a practical sampler, leakage can extend beyond one period, or in intervals of less than a full clock period. Thus the term hysteresis is slightly erroneous as applied to my proposed models. More complex representations such are AR and MA models (used in signal processing) have interesting similarities, but are essentially linear operators. The presence of a sampler in the feedback loop for this problem produces a non-linear operator. A quantizing sampler is by nature non-linear, with the clipping correction necessary to persuade the outcome to behave as a linear estimator. In the presence of feedback across the sampler, the assumptions used by the standard clipping correction are incorrect.

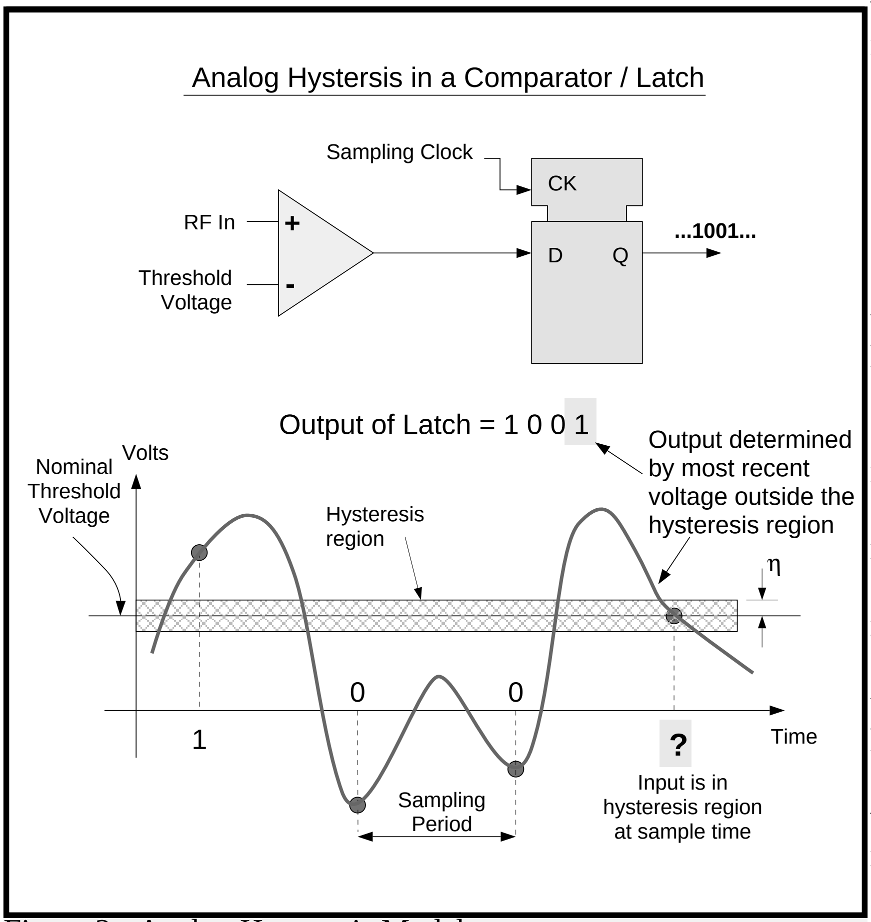

Figure 2 - Analog Hysteresis Model

It is worth noting, that D'Addario[1] and Padin[2] have both considered models of sampler behavior based on *regions-of-uncertainty*. These models have relevance to this discussion and are very useful references on this matter. However, the regions-of-uncertainty model does not incorporate the feedback which seems to exist in the measured data. The proposed hysteresis model produces an error that depends on lag depth, which does not occur in the regions-of-uncertainty model. I demonstrate this difference in Figure B7. If the sloping errors seen at the 12 meter were caused by a region-of-uncertainty mechanism, the correlation error would be proportional to the magnitude of the correlation. In figure B7, I plotted these values from 4 measured chunks. There was only a weak (and non-linear) relationship between these quantities.

III.A **Analog Hysteresis Model**

III.A.1 **Analog Hysteresis Model - Description**

The first model I will call analog hysteresis. It is based on a region that is centered around the threshold voltages. In this region, the output of the sampler can be either a logical one or zero. In the model described by D'Addario and Padin, the output of the comparator when the input was in this *hysteresis region* was randomly assigned as high or low. Their model was non-linear, but the error in measured correlation was independent from lag-to-lag. This model produces a distortion in the necessary clipping correction, but the error will depend completely on correlation magnitude.

The model I will propose in this memo is slightly more malicious and assumes the output is related to the recent history of the input waveform (See Figure 2). This statistical dependence on past input values will cause the measured correlation errors to be dependent on lag depth. As a result, the proposed model will tend to distort the frequency response as compared to an ideal device.

The proposed analog hysteresis model will react to three possible input conditions. First, when the input waveform is above the hysteresis region at the sample time the output will always be a logic one (high). Second, when the input falls below the hysteresis region the output will always be a logic zero (low). And finally, if the input falls within the hysteresis region (of width $\pm\eta$ volts), the output could be a logic high or low. In this case, the output will be low if the signal entered the hysteresis region from below. Conversely, the output will be a high if it entered the hysteresis region from above.

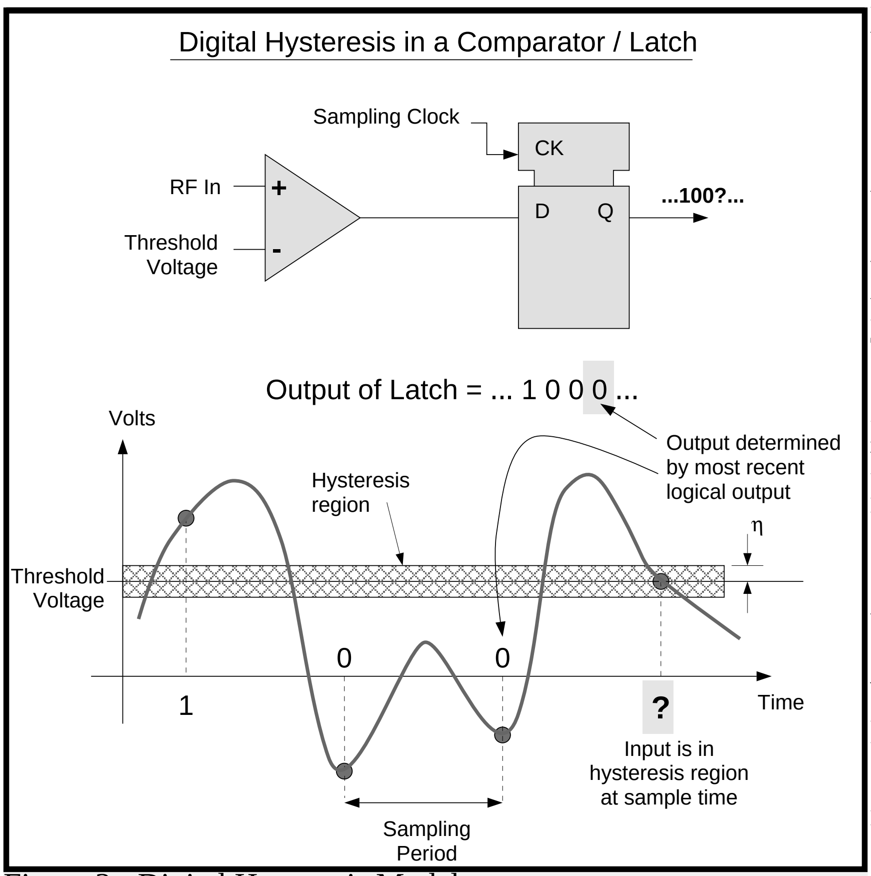

Figure 3 - Digital Hysteresis Model

For waveforms that are encountered in radio astronomy (Gaussian noise with $V_t \approx 0.6\,\sigma$), the time spent in the hysteresis region will be short. The effects of hysteresis will be completely forgotten when the input waveform is outside of the hysteresis region. Therefore, it is reasonable to assume the distortion of the measured correlation caused by analog hysteresis bias will be very weak after a few delays. Most of the bias will occur in the first few lags of the correlation function.

A nonlinear response to changes in input power level is inherent in this model. The hysteresis region will cover different portions of the input distribution function depending on the power level. At a constant power level, the bias will exist, but will remain constant.

III.A.2 **Analog Hysteresis Implementation**

To implement my proposed model for analog hysteresis, it is necessary to reconstruct the waveform between samples. This is possible by using the constructive form of the sampling theorem stated in equation 1:

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT) \frac{Sin\left[\frac{\pi}{T}(t-kT)\right]}{\left[\frac{\pi}{T}(t-kT)\right]}$$

Equation 1 - Sampling Theorem, constructive form for baseband sampling

At the sample time, if the waveform resides in the hysteresis region, the program reconstructs past waveform values (between the samples). This reconstruction works backwards into the past to find the point where the waveform entered the hysteresis region. The last waveform value to reside outside the hysteresis region defines the sampler output state.

The constructive form of the sampling theorem can be used to fabricate a baseband signal which is Nyquist sampled. However, in some correlators (such as the 12 meter), the input is bandlimited, but is modulated to reside from the folding frequency (fs/2) to the sampling frequency (fs). This form of signal satisfies the Nyquist criterion, but requires a different construction equation. It is conceivable that analog hysteresis will affect the two sampling methods differently. (In this memo, I will term the two sampling methods as *baseband* and *offset*). The constructive form of the sampling theorem for offset sampling is presented in equation 2:

$$x(t) = \sum_{k=-\infty}^{\infty} x(kT) Cos\left[\frac{2\pi}{T}(t-kT)\right] \frac{Sin\left[\frac{\pi}{T}(t-kT)\right]}{\left[\frac{\pi}{T}(t-kT)\right]}$$

Equation 2 - Sampling Theorem, constructive form for offset sampling

III.B. **Digital Hysteresis**

III.B.1. **Digital Hysteresis Description**

The previous model was based on limitations in the differential amplifier. Another source of unwanted feedback is from the digital data bits. Thus a different hysteresis model can be constructed by considering the leakage of digital information to the threshold levels (or analog data pins). This situation is more closely related to "real" hysteresis as it is referred to in industry. This type of hysteresis can be beneficial in circuits that attempt to reject noise from digital data streams. However, in astronomy spectrometers, the last thing we want to do is to reject noise! Hysteresis represents an undesirable feature of the circuitry.

The simplest way to model this aspect of a practical comparator is to change the threshold voltage as a function of the comparator's digital output. For the simulations, I will limit the feedback to one sample period. However, in practice this leakage can occur after longer delay stages. (See Section II). Unlike the analog hysteresis model, the output of the sampler is dependent on the previous values of the *output*. Thus, it is conceivable that the bias caused by digital hysteresis could extend well beyond the depth of the initial corruption.

III.B.2. **Digital Hysteresis Implementation**

The implementation of the digital model is somewhat simpler, because it is not necessary to estimate the input waveform between samples. For this model, the threshold voltage used by the quantizer is given a dependence on the logic level of the previous sampled value. Thus, if the sampled output is a logical one, the threshold applied in the next sample time is increased by a small amount from the nominal operating point. For this experiment, a logical zero makes no change in the input thresholds. As close as possible, the proposed simulation tries to copy the physical arrangement of the NRAO 12 m sampler (See Figure 4). This model make some assumptions about the feedback path in the circuit and the sense of the coupling. The actual coupling may be different and therefore there is some uncertainty in the results. Hopefully, this model will behave like the actual sampler. However, some difference in the details of the response are likely. As presented in Figures #B2-B6, the actual distortion experience at the 12 meter was similar from sampler to sampler, but not identical.

IV. **Software Simulations**

Given these models for possible sampler behavior, I performed a number of software experiments. The simulation experiments were written in "C" and run on a SunSparc. Post-processing

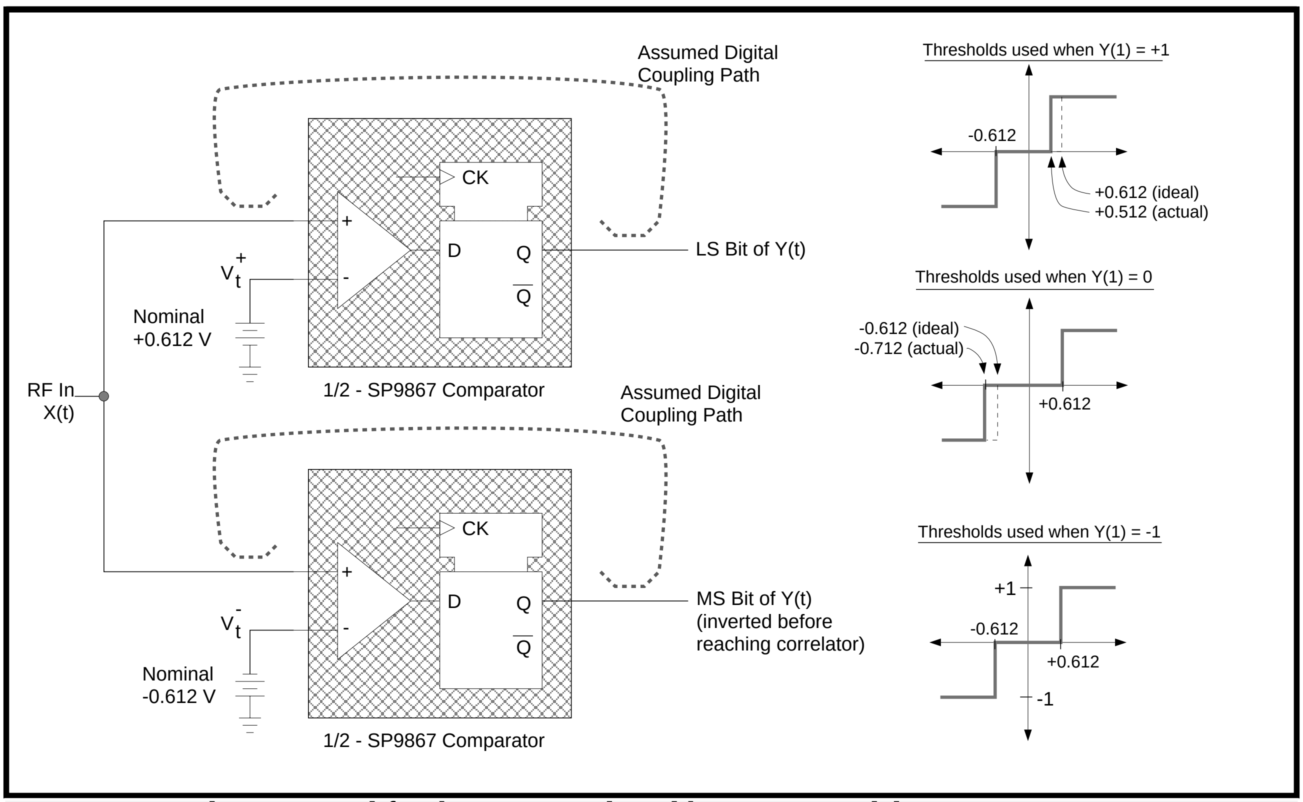

Figure 4 - Coupling assumed for the 12 meter digital hysteresis model

of the data (clipping correction and plotting) was done with Mathematica. Some relevant code fragments are included in Appendix D. All the experiments assumed 3-level quantization, which is the technique applied at the 12 meter. However, a similar results can be expected from a 4 level sampler (such as the SMA correlator).

The hysteresis region used for these simulations extended (somewhat arbitrarily) from 0.512 to 0.712. In the analog hysteresis experiment this region is symmetric around the nominal threshold voltage of 0.612. The digital hysteresis model I applied is not balanced (See Figure 4). A nominal threshold voltage of 0.612 is convenient because optimum quantization SNR is achieved when the input signal has a variance of 1. This represents a substantial hysteresis region and is much larger than I would expect in an actual comparator circuit. In the analog case, this represents a (dismal) differential voltage gain of 10 given an ECL output swing of 1 volt. However, the simulations were limited by computing power to a modest integration time. Thus, the hysteresis error was exaggerated somewhat to reduce the necessary computing power.

These experiments serve two primary functions. First, I hope to show that the model behavior resembles the NRAO 12 meter experiments. If the simulations using these models show a resemblance to the known correlator problem, I hope to apply the models to alleviate this problem in the SMA correlator. Of course, even if the simulations exhibit identical behavior to the data measured at the 12 meter, this does not constitute a guarantee of model validity. As time and resources permit, I intend to continue this work during the development of the SAO sampler. In particular, I hope to use the models to predict the behavior of the sampler constructed for the SAO, thereby improving my confidence in model validity. The software simulation experiments represent only a first step in this process.

The random number generator (*ran1*) used in these simulations was taken from Numerical Recipes, Second Edition [6]. Several of the experiments were repeated with a different number generator (*ran2*), to monitor any possible problems caused by non-random generator behavior. In fact, I tried to use a third version (*ran3*) supplied by Numerical Recipes, (the least strongly recommended of those offered), and was not pleased with the results. The third generator produced correlations with residual structure. Fortunately, *ran1* and *ran2* of the random generators gave consistent results with reasonable (and different) residual noise patterns. For the sake of brevity, I have only supplied the results from using the *ran1* generator.

It is reasonable to expect the errors to have some relationship to the magnitude of the correlation. Therefore, the random data stream was filtered with a digital AR filter to produce some structure in the spectrum, while still maintaining Gaussian statistics. The nature of hysteresis would suggest there is a connection between the input data correlation function and the bias introduced by the sampling problems. Thus, some interesting effects might be missed with simple white noise.

IV.A. **Software Simulations - Absolute Bias in the Measured Correlation**

The first experiment simulates the bias introduced by the four quantizer models. The experiment is sketched in Figure C1. This configuration measures the correlation coefficient function produced with the various quantizer models and than compares this function to the expected value of the correlation coefficient (derived from the AR coefficients). The result is a plot of the absolute bias caused by the sampling errors. For use in radio astronomy, the important issue is relative bias between two measurements, which is covered in the next two experiments. However, this experiment provides an introduction to this topic.

IV.B. **Software Simulations - Cal-Ref Experiments**

The next simulation experiment is an attempt to simulate the effects of large changes in input power (i.e. a Calibration observation using a hot load). The simulation model is sketched in Figure C8. Figures C9 and C10 sketch the transfer function of the AR filter applied by the data generator. This induced correlation is a linear process applied to the data stream before quantization. In this case, it simulates the bandpass of the telescope instrumentation. In practice, its shape is canceled by the normal baseline correction in an astronomical correlator. In a properly functioning correlator the outcome of this experiment should be a flat, but noisy baseline.

The data samples are processed by the simulated quantizing correlator. The result is an estimate of input correlation function. The experiment is looking for non-linear sensitivity to input power, therefore the data stream is measured at two input power levels relative to the quantizer threshold voltage. This is done by multiplying the same random data by a gain factor (A), then processing with another sampler-correlator model. Thus we get two measurements of same correlation function. The measured correlation coefficient function of these two estimates (after clipping correction) should be identical (expected values) at all lags. The correlation function at lag 0 (*Zerolag*) is used to estimate of the input power relative to the threshold voltage (Vt). This value is needed by the clipping correction to convert the measured correlation into an estimate of the input correlation coefficient.

IV.C. **Software Simulations - Sig/Ref Experiments**

A final set of experiments was performed to investigate another known feature of the NRAO correlator error. Large changes to input power causes significant problems, however, this problem does not appear to adversely effect the correlator's performance when measuring weak lines. Thus, astronomical measurements of *Sig*nal and *Ref*erence (beam switching, position switching, etc.) produce reasonable residuals (*Sig - Ref*). Therefore, if my models are to correctly predict this problem, they should also emulate this behavior.

To simulate a *Sig/Ref* observation, I introduce a change in the AR filter applied by the number generator between ensemble measurements. The change in data statistics represents the presence of an astronomical source. However, the change will be designed to impose very little modification in the total power of the resulting random data stream. A sketch of this experiment is given in Figure C37.

Unlike the Cal/Ref experiment, the difference between the two measurements will not be flat (i.e. zero expected value). The residual is a measurement of the change in the generator's statistics. To examine any bias from the measurement, a further step is needed to remove the induced change in statistics. A "flat" baseline can be produced by subtracting the expected correlation difference from the measured correlation difference. The expected correlation is derived from the two AR filters which are used as *Sig* and *Ref* generators.

IV.C. **Software Simulations - Number of Iterations**

A final note on the number of iterations. All simulations were performed with 100 million iterations. In general this would represent about 1 second of integration time at the 12 meter. However, to help minimize computer time, I performed the *Cal/Ref* experiments using the same noise pattern for the Calibrate and Reference measurements (not ensembles as would be performed at the telescope). Thus, the simulated baselines contain only quantization noise, which has only 1/3rd the variance as compared to 1 second of telescope measurements. Thus the baseline noise of these simulations is equivalent to approximately 3 seconds of observing time at the 12 meter. Even so, the 3 seconds worth of observations time took approximately 12 hours of computing time.

For the *Sig/Ref* experiment, I was blessed with more computing power, so the measurements were performed with different ensembles. The number of iterations is the same (100 million). Therefore, these baseline have somewhat more measurement noise than the first simulations. The use of ensembles was not necessary, but I though this was prudent to look for any inconsistencies (sanity check). Fortunately, no problems appeared, however, the large computing times dissuade me from re-doing these simulations using the more efficient approach I applied in the *Cal/Ref* experiments.

V. **Software Simulations - Results**

V.A. **Software Simulations - Results from the Absolute Bias Experiment**

As an introduction to this matter, the first group of figures in Appendix C (figures C1-C8) presents a simulation of the absolute bias generated by the quantizer models. The generator used for these experiments was AR filter #4, described in Figures C2 and C3. (Forgive the ordering of these, but there is actually a reason for starting at #4: it will be used again in Section V.C). The correlation coefficient of the generated data stream is measured using all 4 models: ideal quantizer(C4), plus a quantizer affected by digital hysteresis (C5), analog-baseband hysteresis(C6) and analog-offset hysteresis(C7). The expected correlation of the AR generator is calculated by inverting the Yule-Walker equation[8]. Subtracting the expected value from the measured correlation coefficient gives a simulation of the bias imposed by the quantizer models. This is done in Figures C4-C7. These plots have identical vertical scaling to indicate the relative bias of each method. (In latter figures, the scaling is modified to maximize information content).

As expected, the ideal quantizer shows no bias (Figure C4). The worst error is found in the sampler plagued with analog-baseband hysteresis, which shows an especially strong effect at lag τ=1. The difference in the bias strength between various hysteresis models portends a similar relationship witnessed throughout these experiments.

V.B. **Software Simulation - Results from the Cal/Ref Experiment**

The next set of figures (C8- C36) presents the effects created by the introduction of a large power step in a hysteresis-affected sampler. The simulation model is sketched in Figure C8, while a description of the AR generator (#1) used for this experiment is presented in figures C9 and C10. In the previous experiment, the absolute bias (relative to the predicted outcome) was found. In this setup, the relative bias between measurements is the goal.

Figures C11-C14 are plots of the residual between the measured correlation function taken at two power levels (about 6 dB difference). Figure C11 was performed with an ideal sampler. As expected there is no obvious bias. The next figure on this page (C12) shows the correlation residual when digital hysteresis is present. The correlation residual is quite strong at lag τ=1. Otherwise the main effect is a shift in the entire baseline.

Some thought on the quantizer model suggests the bias should be related to the AR filter in some messy fashion. Therefore, a second experiment was performed using a different AR process. The AR generator for this second set of Cal/Ref experiments is presented in figures C27 and C28. Of concern at the moment is Figure C30, which presents the correlation residual (like C12) when using AR generator #2. This residual indicates similar problems: large bias at lag τ=1 and an offset. However, there is also a major distortion in the short lags which does not appear in the first experiment.

The offset of the entire residual is easily explained. This signifies a change in the effective mean of the input signal between *Cal* and *Ref* scans. It is caused by an imbalance in the feedback between the upper and lower thresholds. The bias is unevenly applied to different quantization regions, thus causing an apparent shift in the mean of the signal. Its occurrence in the simulations was engendered by my attempt to tailor the hysteresis to the structure of the 12 meter sampler, which is not physically symmetric (see Figure #4). The same asymmetry could be the cause of the offset in the 12 meter experiment data (Figure B2). However, I suspect the primary cause of offset in the 12 meter data is skew in the positive and negative threshold voltages.

The next two figures C13 and C14 present the residual correlation when analog hysteresis is present in a Cal/Ref simulation. Figure C13 models standard baseband sampling, while figure C14 show the error induced by offset sampling in the presence of analog hysteresis. The offset sampling case shows little or no bias, while the baseband sampling case shows a dramatic distortion. The second AR generator residuals given in Figures C31 and C32 are similar. However, the offset sampling experiment now indicates a weak distortion at lag τ=1. The dramatic distortion of low lags witnessed in the digital hysteresis model are likewise present in the analog-baseband hysteresis model.

Next, figures C15-C18 present the spectral version of the previously plotted correlation residuals. They represent a simulation of the spectral baselines that a hysteresis plagued correlator would produce. For these plots, the measured correlations are Fourier transformed, then the bandpass is removed: Cal-Ref /Ref. Figure C6 produces the expected flat baseline for the ideal quantizer. (In reality, these plots should be translated by the input power difference:A, but this offset is normalized for clarity). The hysteresis affected correlators have baseline "features" caused by their non-linear frequency response to input power changes. Figure C16 shows a sloping feature that is similar to those documented by the 12 meter. (The two-sided truncation of chunk spectrum to remove overlap will convert the "feature" plotted by Figure C8 into a slope). Figure C17, the simulated baseline with analog-baseband hysteresis shows an even stronger sloping term. Of course, the 12 meter uses offset sampling, which is modeled in Figures C18. The sloping term is much weaker for offset sampling (However, more integration could easily exacerbate the weak bias)

Although I have not tested this fact, I would expect the baseline features produced by digital hysteresis to depend on the coupling model (Figure 4). Also, as seem by differences in Figures C12 and C30, the bias is, in some complex way, dependent on the underling correlation function. At this time, I won't examine this relationship in great deal (non-linear, god save us). Hopefully. in time I can improve this sketchy relationship.

The coupling model for analog hysteresis has much less potential variation. For example, I would not expect an analog input which rapidly traverses the hysteresis region to be *less likely* to change digital state than a slow moving input. Consequently, I suspect the negative sign of the error at Lag τ=1 produced in these experiments is characteristic of analog hysteresis, (when performing R{High Power} - R{Low Power}). This may be helpful in separating the two effects in a laboratory experiment.

Figures C19-C22 appear similar to the previous plots of simulated spectral baseline (C15-C18). Actually, they take the same measured correlation function, but use a different power level in the clipping correction. In the previous examples (and as is done at the 12 meter), the input power level relative to the threshold voltage is derived from the zero lag correlation. From table C2, the value of Vt/σ taken from the zerolag is different when hysteresis is present (both digital and analog). Thus, I wanted to examine the baseline that is produced if the clipping correction were based on a different measure of input power; namely, the value estimated by the ideal sampler. Thus, figures C19-C22 present the simulated baseline when the clipping correction is taken from a non-biased measurement of input power. (the Vt/σ used by the clipping correction is given in Table C1). The digital hysteresis baseline was not significantly altered. This result could be predicted from Figure C24, where I plotted the correlation residual versus the correlation magnitude when digital hysteresis is present.

Conversely, the analog-offset hysteresis baseline (C21) has gotten, if anything, worse when the new clipping correction is applied. The offset sampling baseline (C22) is a complete mess (notice its relationship to the input AR spectrum, figure C9). A power level derived from the zerolag seems to be a much better estimate of the "effective" power level than a direct measurement of power and threshold voltage. Where "effective" would mean the power level applied by the clipping correction which gives the smallest RMS in the baseline. However, from the plots of residual versus correlation magnitude, (figures C25 and C35), the effective power level taken from the zerolag may also be less than perfect. The presence of some relationship between correlation bias and magnitude would indicate an erroneous clipping correction. The optimum power estimate when analog hysteresis is present could be found by iterating to the minimum mean squared error (excluding the correlation at lag τ=1). However, whether this is prudent is unclear at this time.

The offset sampling experiments demonstrates a very dramatic effect. The zerolag measured with analog-offset hysteresis is quite different from the ideal quantizer measurement (See Tables C1 and C2). However, looking at figures C22 and C18, its clear an input power level calculated from the measured zerolag is a better estimator of the "effective" input power level for use in the clipping correction.

Attempts to make accurate predictions about the nature of correlation errors with hysteresis are difficult. The loss of a six-pack of beer by this author after a brash and incorrect prediction gives some indication as to the difficulty in making generalization about these effects. More likely, the lost beer may have greater relevance to the questionable judgement of this investigator when betting on the outcome of non-linear equations. That said, there are still a few characteristic features of hysteresis that can to be culled from these simulations.

V.C. **Software Simulations - Results from the Sig/Ref Experiment**

For the first Sig/Ref experiment, the change applied to the AR filter between ensembles was quite small. Therefore, I called the two filters used for this experiment #3 and #3+ε. Thus "ε" is the residual correlation that should remain after the correlations are subtracted. The power levels used in both measurements are virtually identical.

Figure C38 shows the #3 AR filter response and Figure C39 gives the #3+ε filter response. The difference can be noted around the 8th lag. Figures C41-C44 shows the correlation residuals measured with the various quantizer models. The "ε" correlation appears in all cases, and with no discernible difference between the various quantizer models. Figures C45-C48 show the residual spectra , i.e a simulated baseline for a Sig/Ref experiment with a line present. (Not a particularly realistic line, I'm afraid). To search for any small distortions, the measurement of "ε" is subtracted from the expected outcome in Figures C49-C52. There are some differences in the noise, particularly in the offset sampling case, but nothing of obvious concern.

This experiment emulates the experience at the telescope. When the power level remains fixed, the correlator seems to operate quite well. The presence of hysteresis does not degrade its performance in Sig/Ref type experiments.

To test this hypothesis, I invented a pathological case. A follow-up experiment was performed using two very different AR filters. Like the previous experiment, the power levels are corrected to ensure they are virtually identical. The two AR generators used were #4 (shown in figures C1-C2) and #5 (shown in figures C53-C54). The measurement of residual difference is presented in figures C55-C58. Like the previous case, the residual correlation measured by all quantizer models shows no discernible distortion. However, when the expected value of this simulated "line" is subtracted, a distortion does appears (See figures C59-C62). Thus, keeping a fixed power level does will not necessarily remove all bias from a difference measurement. A final note, unlike previous distortions created by hysteresis, this distortion does not seem to favor lag τ=1.

VI. **Conclusions**

The software simulations show similar behavior to the problems encountered at the NRAO 12 meter telescope. Although as yet unproven, it seems reasonable to speculate that the ripples seen in the Hybrid Spectrometer residual correlations at lag 39 are digital leakage. However, at this point it would be difficult to ascribe the errors in low lags as either digital or analog hysteresis. The problems at the 12 meter could easily be caused by some combination of digital or analog hysteresis.

The effects of hysteresis on samplers can be visualized as a non-linear change in the instrumental response. Sampler errors cause an absolute bias in instrumental response. This distortion is dependent on input power level. If the input power is not changed, the bias introduced by the sampler hysteresis can be removed like any normal instrumental bandpass shape. However, the dependence on input power level of this distortion greatly reduces the useful dynamic range of a sampler circuit. The effective dynamic range will depend on the magnitude of the feedback and the integration time. Consequently, the specified dynamic range of a comparator chip is probably not a useful indicator of the dynamic range in a practical sampler circuit. The usable dynamic range will depend on the hysteresis and coupling issues.

Now, given that these problems exist, is it possible to avoid them in future designs? Steps can be taken to help alleviate these problems. The brute force method to avoid these problems is to limit the dynamic range of the signal which enters the sampler. The JCMT uses an AGC circuit to maintain a fixed power at the input to the correlator. This is particularly important if a broadband calibrator is used, because it typically will introduce a large step in power compared to the sky measurements. However, as show in section V.C., while an AGC is very hopeful, it is by no means a cure-all. Bias in difference measurements can occur even when power is constant. Thus, some direct effort at reducing the sampler hysteresis in conjunction with the AGC would seem prudent.

Other improvements can be made by optimizing the sampler design to reduce feedback. From the telescope experiments it appears digital feedback is particularly problematic. Thus improvements in shielding and ground planes are advised. Every effort must be made to remove digital noise from the analog signal.

Other techniques can be used to mitigate this problem. Dr. Alan Rogers at Haystack observatory, mentioned a method to reduce the effects of digital hysteresis. Feedback from the internal latch of a comparator chip to the input of the analog stage is particularly insidious. By resetting the internal latch before every acquisition (sampler) clock, the comparison will always be performed under the same input conditions. Feedback from other latches in the system also create hysteresis, but proximity tends to make the first stage especially important. (Note, Dr. Rogers indicated this trick was originally used by Dr. Weinreb at NRAO)

Another question is whether it is better to use the internal latch of the comparator chip or drive the logic signals to an independent latch for time sampling. This issues is a quandary between digital hysteresis and analog hysteresis. By using the internal latch, the effects of analog hysteresis are mitigated by improving the frequency response and gain of the quantizing stage. Conversely, the proximity of this latch to the data represents a potential feedback path. Ideally, the solution would be to use the internal latch with some form of reset to remove the digital feedback path.

If a design is being contemplated from scratch, what characteristics of a commercially available comparator are desirable? A fast comparator which can operates at much a higher frequency than the input waveform would not be wasted complexity. The better frequency response will tend to produce less analog hysteresis. I would also note that one of the most commonly used comparator for astronomical correlators, the Plessey SP9380x is specifically design with reduced gain[4]. This feature is undesirable for astronomical sampler applications. In evaluating comparators, I would also consider the propagation dispersion of the comparator. This specification is very important to designers of ATE equipment (a large user of comparator chips) and therefore most commercially available comparators specify this value. Ideally, an astronomical sampler should have zero propagation dispersion. A smaller propagation dispersion will translate into less analog hysteresis and more dynamic range.

Although I don't believe the simulations are conclusive on this point, it seems offset sampling is less susceptible to analog hysteresis as compared to baseband sampling. However, I suspect this is only true if the frequency response of the comparator is much higher than the sample frequency. If the comparator is operating on the edge of its range, the advantage of offset sampling will be reduced and possibly be negated completely.

A final note, the data from the 12 meter suggests the primary mechanism for problems is digital feedback. Conversely (or perversely, I should say), I have spent a large effort on analog hysteresis. I'll defend this by stating that the effects of analog hysteresis will become more critical as sampler speeds are increased.

Appendix A - **Relating Comparator Limitations to Analog Hysteresis**

When I first approached this subject, one of the most difficult issues I faced was relating known comparator "features" to the errors seen at the telescope. It seems reasonable that comparator specifications such as propagation dispersion and differential gain have an affect on the performance a sampler circuit. But how to convert these known limitations into a useful model was not obvious. The hysteresis model (especially analog) was the result. I believe it is a reasonable approximation, but by no means a perfect model of the situation. In this appendix, I hope to show, in a fashion, how I arrived at this model.

Consider the response of an ideal sampler to an input which crosses one of its threshold voltages. At this point, an ideal comparator would need infinite differential gain to drive the logical output to the new state. In an ideal sampler, this means infinite gain and zero propagation delay. In practice, some propagation delay is not necessarily a problem, if it means a simple shifting of the time axis of a stationary signal. However, even this is not perfect because one component of the propagation delay is rise time of the digital output. Therefore, extremely short forays above a threshold (cross and then return) will be lost. These short pulses are statistically significant because the clipping correction assumes they are measured.

As a consequence, limited gain in the quantizer will cause bias in the measurements of correlation. This bias will change with power level because the proportion of time spent in the area of the threshold will decrease with increasing input signal power. This relates to the comparator feature of propagation dispersion. Propagation dispersion is defined as the difference in propagation delay which depends on input overdrive[3]. A comparator will have less propagation delay if the input signal swings rapidly past the threshold and moves well beyond it. To the first order this can be seen as a limited gain in the comparator differential gain stage. Thus, for the comparator to react the input must actually pass the threshold by a fixed amount (the voltage gain of the device). This occurs sooner for fast moving inputs. (This fact may explain why my simulations of analog hysteresis in the case of offset sampling showed little bias)

We can even make a rough calculation of the voltage gain associated with a given propagation dispersion. Consider a typical comparator which has 100 psec of dispersion over a 10 nsec sampler cycle. Given an input at half the sample clock and a one volt logic swing (ECL), this gives the following voltage gain:

$$G_v \approx \frac{1}{Sin\left(\pi \frac{100psec}{10nsec}\right)}$$

$$G_v \approx 32$$

This is naturally a very crude estimate, but gives an idea of the problem. The simulations described in this memo are based on a voltage gain of 10, which produces substantial errors after only 3 seconds of integration. Therefore, modest propagation dispersion could be a significant source of errors.

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