# Submillimeter Array Technical Memorandum

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**PMAC Servo parameters**

## Summary

This memo summarises the servo equation used in the PMAC and shows how to calculate the parameters for a simple loop, and convert them into PMAC I variables. This should serve to give us a starting point for servo tuning. The calculations are also embodied in a spreadsheet, which is shown at the end with values both for slow and fast loops. This memo will eventually be updated with the final servo parameters.

## Introduction

In the initial setting up of the antenna and other servos, we need a way to get starting values for the PMAC I variables which control the servo loop. Once these are in place, the final values can be found by a combination of the PMAC tuning and more sophisticated servo analysis. The analysis here is based on the choice of a unity gain frequency for the servo, with all parameters derived from that choice. As a consequence of the choice, we can predict several important performance parameters, such as following error. In the final configuration, the unity gain point will be several Hz, slightly below our first resonance at about 10 Hz, but the initial testing should be done with a much lower value of about 0.5 Hz so that the antenna can be treated as infinitely stiff. In the following calculations, I assume that the antennas response is ideal with zero compliance and zero damping.

## Servo Equations

The servo equation for a rotating system is

$$I\, d^2\theta/dt^2 = -\beta\, \theta - \alpha\, d\theta/dt - \gamma \int \theta\, dt, \qquad (1)$$

where $\theta$ is the angular position error in radians, I is the inertia of the antenna in kg $m^2$, $\beta$ is the proportional gain of the servo in N m / radian, $\gamma$ is the damping factor in N m s / radian, and $\varepsilon$ is the integral gain, in N m / radian / s. In the usual way, if we consider a motion of the form $\theta = e^{i\omega t}$, we get the equation

$$-\omega^2 I\, \theta = -\beta\, \theta - \iota\, \omega\, \alpha\, \theta + i\, \gamma\, \theta / \omega. \qquad (2)$$

Considering just the proportional gain at first, we have a loop gain of $\beta$ / I, which falls rapidly with frequency. At frequencies where the loop gain is greater than unity, the servo loop is effective and the servo follows the commanded track, but at higher frequencies it fails. We must pick a frequency initially sufficiently below the antenna resonances that they will not be excited by the loop. With a more sophisticated treatment, we can increase this frequency later. So, if we pick our unity gain frequency of, $\omega'$, and we know the inertia of the antenna, we can calculate $\beta$ as

$$\beta = 1 / (\omega'^2 I) \quad \text{Nm/radian} \tag{3}$$

Given the value of $\beta$, we can calculate the following error at constant velocity if we know the frictional torque, $T_f$. The following error, FE, is given by

$$FE = T_f / \beta \qquad \text{radians} \tag{4}$$

The damping is determined from the roots of the equation

$$- \omega^2 I = -\beta - \iota \omega \gamma, \tag{5}$$

which determine the characteristic frequency of the system

$$\omega = (\iota \alpha \pm \sqrt{[-\alpha^2 + 4 I \beta]}\,) / (2 I) \tag{6}$$

Critical damping occurs when the term under the square root is zero, and there is no oscillatory term. This gives

$$\begin{aligned} \alpha &= \sqrt{[4 I \beta]} \\ \alpha / \beta &= 2 / \omega' \end{aligned} \tag{7}$$

The integral gain is needed to bring the following error to zero. A large integral gain will cause this to happen rapidly, but will affect the loop dynamics at high frequencies. One way to get a starting point is to specify that the magnitude of the integral term should be small, say a factor f ( < 1) times the proportional term at the frequency $\omega'$. Then we get

$$\begin{aligned} \gamma &= f \omega \beta \\ \gamma / \beta &= f \omega \end{aligned} \tag{8}$$

The time constant to reduce a following error is given by $\beta / \gamma = 1 / \ f \omega$, so the settling time to reach a desired error from an initial error is given by

$$\text{time} = \ln(\text{actual error} / \text{desired error}) \ / \ [f \omega] \tag{9}$$

This suggests that we should choose a relatively large value of f, say 0.2, to minimize the settling time. With our initial loop cutoff of 0.5 Hz, the settling time will be about 10 seconds, but with a 10 x faster loop, the settling time will be less than a second, since the following error is also reduced as the loop is speeded up.

The feedforward terms are straightforward. The velocity feedforward should be equal to the feedback term so that there is no damping from the commanded velocity, only from the velocity error. The acceleration feedforward should be equal to I, to compensate from the inertia of the antenna. However, our antennas have such large inertia that the PMAC parameter cannot be made large enough.

**Parameters in PMAC units**

To get PMAC units, we need to know a couple more parameters, and the PMAC feedback equation as given on p 3-62 of the PMAC manual:

$$\begin{aligned} DAC &= 2^{-19}\, Ix30 \,\{\, Ix08\, [\, FE + (\, Ix32\, CV + Ix35\, CA\,) / 128 + Ix33\, IE / 2^{23}\, ] \\ &\qquad - Ix31\, Ix09\, AV / 128 \,\} \end{aligned}$$

| | |
|---|---|
| DAC | = Digital output to D/A converter |
| FE | = Following error in encoder bits |
| AV | = Actual velocity in resolver bits per Servo cycle |
| IE | = Integrated following error in encoder bits |
| CV | = Commanded velocity |
| CV | = Commanded acceleration in resolver bits per (Servo cycle)$^2$ |

| | | |
|---|---|---|
| Encoder scale | radians/bit | used for position feedback |
| Resolver scale | radians/bit | used for velocity feedback |
| Drive scale | N m / bit | |
| Servo cycle | sec | |
| Ix08 scale factor | 96 is the default value | |
| Ix09 scale factor | 96 * Resolver scale / Encoder scale<br>This adjusts Ix31 to be in the right units for the Resolver scale if it is set based on the Encoder scale | |

Proportional gain = **Ix30** $= \beta$ * Encoder scale / Drive scale * $2^{19}$ * Ix08
$= [\, 1 / (\omega'^2\, I)\, ]$ * Encoder scale / Drive scale * $2^{19}$ * Ix08

Velocity feedback = **Ix31** $= \alpha / \beta * 2^5$
$= [2 / \omega']$ * Servo cycle $* 2^5$

Integral = **Ix33** $= f\, \omega' * 2^{23}$ / Servo cycle

Velocity feedforward **Ix32** = Ix31 to damp only velocity errors

Acceleration ff = **Ix35** = $\varepsilon / \beta * 2^7$
= $[ 1 / \omega'^2 ] * 2^7 / [ \text{Servo cycle} ]^2$
(but we can't achieve this since our inertia is so high)

**Comments**

The value of torque/DAC bit should have been 7500 Nm/bit for one motor, I think, and 15000 Nm/bit for two motors, but a value 4 x larger seemed to match reality better in our initial tests.

To start with, we should leave the notch filter alone, until we push the frequency up high enough that it becomes significant.

The integral term can be turned off by setting Ix34 to 1. This should be done before starting a slew and not turned back on again until the dish has had some time to settle after reaching the new position. If we have not moved far, the previous integral will still be pretty good and we will lose little by not updating it, while we stand to lose a lot if we update it with bad values from the settling time.

The PMAC has many limits on acceleration times and rates. In the spreadhseet below, all of these are scaled to the unity gain frequency in a reasonably sensible way so that they do not artificially limit our performance.

Finally, to reiterate, these values are only preliminary ones to get us going. Once we have the servo under control as far as preloading, etc., goes, we can push the speed up and try to optimize the loop. But even a slow loop should be enough to handle the first cut at optical pointing.