Center for Astrophysics

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SUBMILLIMETER ARRAY TECHNICAL MEMORANDUM

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Point Source Sensitivity Improvement Achieved by Adding a Larger Antenna to an Array of Smaller Elements

A common situation may be the one in which a large antenna is added to an existing array (eg CSO or JCMT to the SMA; 45m to the array at NRO; GBT to the VLBA). If the data are weighted optimally, the point source sensitivity scales approximately as the total collecting area. The total collecting area is not achieved because the electric field autocorrelations are not included in the interferometric data processing (the reason for this is that the autocorrelation terms do not have any fringe rate associated with them and hence it is more difficult for them to achieve the theoretical sensitivity because of systematic effects). Optimum weighting means that baselines to the large antenna are counted more heavily, which may lead to point spread functions with high sidelobes. If uniform weighting is used, the sensitivity effectiveness of the large antenna is greatly diminished. However, this weighting scheme is the norm for conventional image processing.

The sensitivity to a point source is just the rms variance of the sum of the baseline visibilities normalized to Janskys. These can be calculated by the usual statistical formulas.

Optimum Weighting

$$m = \frac{\sum \frac{V_i}{\sigma_i^2}}{\sum \frac{1}{\sigma_i^2}} , \tag{1}$$

$$\sigma = \left[\sum \frac{1}{\sigma_i{}^2}\right]^{-1/2} , \tag{2}$$

or for Uniform weighting

$$m = \frac{1}{N} \sum V_i , \tag{3}$$

$$\sigma = \frac{1}{N} \left[\sum \sigma_i{}^2\right]^{1/2} , \tag{4}$$

where V_i are the N individual measurements with rms noise errors of σ_i and m and σ are the estimate of the mean and the standard deviation of the estimate of the mean.

The rms error on each baseline measurement is

$$\sigma_i = 2k \left[\frac{T_{R_0} T_{R_1}}{A_0 A_1 \quad (2B\tau)} \right]^{1/2} , \tag{5}$$

where k is Boltzmann's constant, T_{R_0} and T_{R_1} are the receiver temperatures, A_0 and A_1 are the collecting areas, B is the bandwidth and τ is the integration time. We assume that all receiver temperatures are the same and that the collecting areas equal the geometric areas of the antennas. Now, assume we have an N antenna array with element area A_0 and a large antenna of area A_1. For optimally weighted data equation (2) yields

$$\sigma^2 = \frac{4k^2 T_R^2}{B\tau} \left[N (N-1) A_0^2 + 2N A_0 A_1 \right]^{1/2} , \tag{6}$$

We can identify the term in brackets as an equivalent area, A,, so that

$$A, = \left[N (N\text{-}1) A_0^2 + 2N A_0 A_1 \right]^{-1/2} . \tag{7}$$

Note that if $A_1 = A_0$, then the area is equal to

$$\sqrt{N (N+1)}\, A_0 ,$$

as expected since the total number of antennas is N+1. The result in eq (7) can be thought of as adding the terms

$$\sqrt{N (N-1)}\, A_0 \text{ and } \sqrt{2N A_0 A_1} ,$$

in quadrature. Hence, the large antenna of diameter d_1 will have major impact on the sensitivity if

$$d_1 > d_0 \sqrt{\frac{N-1}{2}} , \tag{8}$$

when d_0 is the diameter of the array antennas. The results for various arrays are listed in Table 1.

If, on the other hand, the baseline visibilities are equally weighted, then the effective area is given by

$$A, = \frac{N (N+1) A_0}{\left[N (N\text{-}l) + 2N \frac{A_0}{A_1} \right]^{1/2}} , \tag{9}$$

Note that if $A_1 = A_0$ then this weighing becomes equal to the optimal weighting,

$$A_e = \sqrt{N (N+1)}\, A_0 , \tag{10}$$

as expected. However as A_1 becomes large with respect to A_0 little of the effectiveness of a large dish is realized and the performance is about the same as if the extra antenna had the same size as the array antennas. The results for equal weighting are listed in Table 2.

Table 1

Sensitivity Improvement from Adding a Large Antenna to an Array

	A_e m (1)	A_T m (2)	η (3)	baseline sensitivity improvement (4)	total sensitivity improvement (5)
SMA(Ae_0, A_{T_0})	155	170	0.91		
SMA+CSO	225	248	0.91	1.7	1.5
SMA+ JCMT	290	346	0.84	2.5	1.9
SMA+JCMT+CSO*	372	410	0.91		2.4
NRO (Ae_0, A_{T_0})	'351	392	0.89		
NRO+45m	1171	1982	0.59	4.5	3.3
VLBA(A_{e_0}, A_{T_0})	4658	4910	0.95		
VLBA+GBT	9939	12760	0.78	4.0	2.1

* obvious generalizations of equations below

N = number of array antennas
d_0 = diameter of array antenna
d_1 = diameter of extra antenna

$A_0 = \frac{\pi}{4} d_0^2$

$A_1 = \frac{\pi}{4} d_1^2$

1. A , $= \sqrt{N(N-1)}\ A_0$
   $A_e = \left[N(N-1)A_0^2 + 2NA_0A_1\right]^{1/2}$

2. $A_{T_0} = NA_0$
   $A_T = NA_0 + A_1$

3. $\eta = A_e/A_T$ or A_{e_0}/A_{T_0}

4. $R_B = d_1/d_0$

5. $R_s = A_e/A_{e_0}$

Table 2

Sensitivity Improvement with Equal Weights

	$A_e(opt)$ (m) (1)	$A_e(equal)$ (m) (2)	η_{equal} (3)	η_{opt} (4)
SMA+CSO	225	202	0.81	0.91
SMA+ JCMT	290	210	0.61	0.84
SMA+JCMT+CSO*	372	263	0.64	0.91
NRO+45m	1171	520	0.26	0.59
VLBA+GBT	9939	5654	0.44	0.78

$*$ obvious generalizations of equations below

(1) $A_e(\mathrm{opt}) = \left[N(N-1)\,A_0^2 + 2NA_0A_1\right]^{1/2}$

(2) $A_e(\mathrm{equal}) = \dfrac{N\,N+1\,A_0}{\left[N(N-1) + 2NA_0/A_1\right]^{1/2}}$

$A_T = NA_0 + A_1$

(3) $\eta_{equal} = A_e(equal)/A_T$

(4) $\eta_{opt} = A_e(opt)/A_T$