Submillimeter Array Technical Memorandum

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Effect of Pointing Errors on the SMA

Summary

Pointing errors may be divided into two categories, 'fast' errors and 'slow' errors. The errors may be due to different causes, but their effect on observations is primarily dependent on timescale. We refer to the fast errors as jitter and the slow ones as pointing errors. The jitter has the effect of smearing the beam and can be treated similarly to dish surface errors, while the slow pointing errors cause systematic effects in the maps. The natural division between the two is a timescale of about 30 seconds. Our current specifications are 15 μm surface error and 1" rms pointing jitter. For the fast errors, a 1" rms pointing jitter is equivalent to a surface error of 3.6 microns. Our nominal jitter specification is therefore much tighter than the surface specification. This implies that we should accept compromises which improve the surface accuracy at the expense of jitter.

The atmospheric contribution to the jitter is not always negligible. At night the equivalent surface error is about 3 microns, but in the daytime it is typically 20 microns.

The slow errors are more damaging. Even a 1" error causes the strength of a source at the half-power point of the beam to be changed by 20% at 350 microns wavelength. These slow errors are primarily due to thermal effects and to inaccurate mount modelling. A suggested goal for terms in the mount model is that they should be comparable with the beam of the telescope, or with one quarter of the wavelength. Therefore all angular misalignments should be no more than 15 - 60", and translations no more than 0.1-0.5 mm.

1 Introduction

Pointing is one of the most difficult specifications to quantify for the SMA. The goal of this memo is to discuss the effects of pointing errors and to put some of them on the same footing as dish surface errors so that both can be included in a proper way in the error budget. In addition, errors due to the atmospheric irregularities and to the translation of the mount are considered.

It is assumed that pointing for the SM.4 is done in the standard way. At intervals of a few days to weeks, a mount model is calibrated by taking observations of a number of objects distributed across the sky. There are too few bright radio sources to do this quickly so optical measurements are normally used. Even using optical objects, the time required is probably an hour or so. The solution to these observations gives the basic mechanical misalignments of the mount. Next it is necessary to measure the radio beam direction relative to the optical axis which has been defined, and this is done with radio pointing measurements. In most radio telescopes it is found that the mount model is relatively stable and that most pointing variations are due to (probably thermal) distortions of the mount and backup structure. These usually give rise to simple offsets (collimation errors) which are constant in azimuth and zenith angle although they may change slowly with time. It is then sufficient to measure the radio pointing on one reference source and use this for the source under observation. This procedure is not perfectly correct if the reference source is far in the sky from the program source and the offset is due to a cause which depends on position, e.g. the Sun shining on one side of the structure. However, it is not too bad if the reference source can be measured quickly before the structure has time to change temperature. There are relatively few strong radio sources available to the SMA so this step of radio pointing may be done as little as 2-3 times per night and the reference source may be as much as 90” away from the program source.

During measurements of the program source, the telescope will have some error in pointing. This error is due to many causes, but its effect can be analyzed by dividing the error into two categories of timescale, according to whether the variations are fast or slow compared with the basic integration time t. The basic idea is illustrated in Figure 1, and a 1 dimensional sketch of the effective beam shapes is shown in Figure 2.

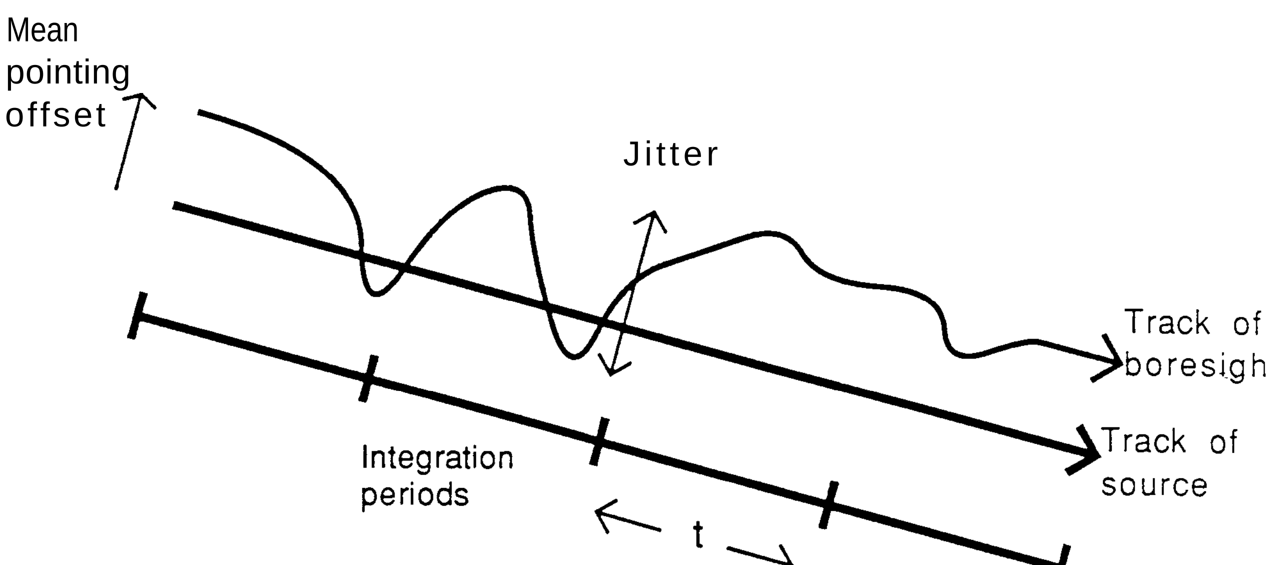


Figure 1. The track of the source on the sky over a few minutes (4 integration periods), compared with the track of the telescope boresight There is a mean pointing offset, plus short-term fluctuations about this mean.

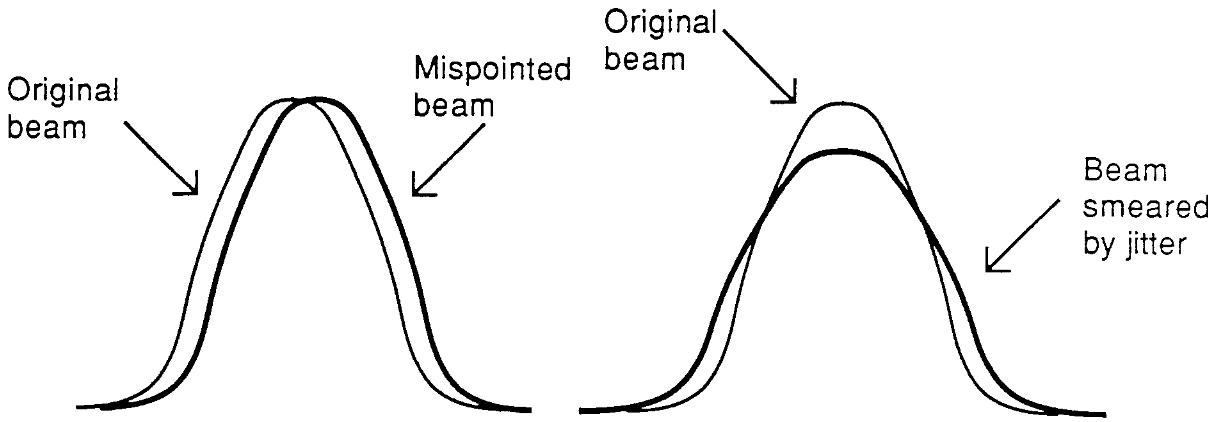
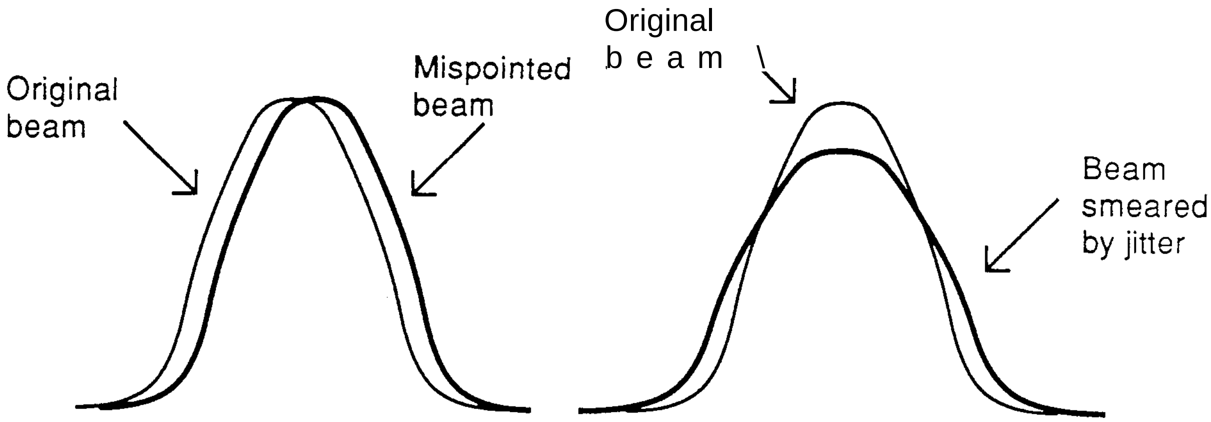


Figure 2. Sketches of cuts through beam profiles on the sky, showing the effects of mispointing and jitter. In the case of mispointing, there is little effect near the center of the beam, but there are large fractional changes near the edge of the beam, with one side increased and the other side decreased. Jitter, in which the beam is rapidly mispointed one way and then the other, has the effect of spreading out the beam, decreasing the forward response and increasing the response near the edge-s.

Fast errors, on timescales shorter than t, produce variations which are indistinguishable from errors in the beamshape. Essentially the beam position jitters around on the sky during the integration producing some average beamshape which is the true beam convolved with the distribution of jitter. Slow errors, which are essentially constant during the time t, produce systematic effects which are most pronounced for sources near the edge of the beam. The sources on one side of the beam appear much brighter than they should be and those on the other side appear fainter. In the usual terminology, the slow errors are simply called pointing errors, while the fast errors are called tracking errors. Since the term tracking error is also used to describe error in following moving targets, we shall use the term 'jitter' to emphasize the short-term nature of these errors.

The basic integration time, t, is set directly by astronomical considerations. At any time, each pair of antennas is collecting radiation from an area in the *uv* plane, which depends on the physical area of the antennas. To be exact, the area is the convolution of the two aperture field distribution functions. Thus it has a radius of d, where d is the diameter of one antenna, but is sharply peaked. As the earth rotates, the center of this area moves in the *uv* plane at a maximum rate of $2\pi D/24$ m/hr, where D is the EW component of the baseline in meters. Independent positions in the *uv* plane are measured every time the center moves by d meters.



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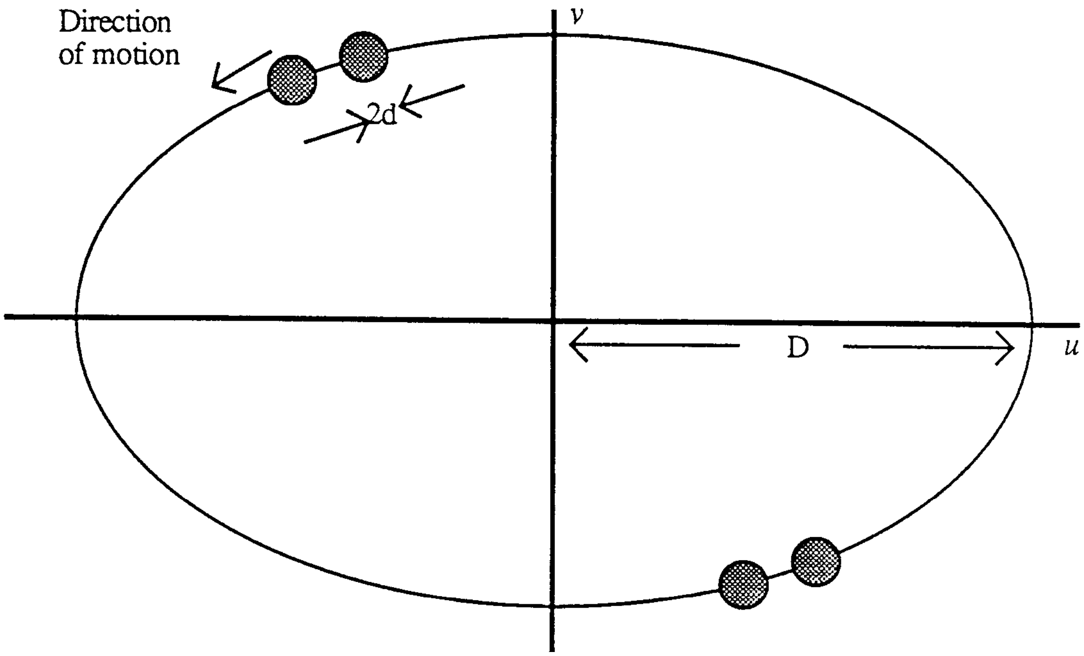

Figure 3. Track in the *uv* plane for one baseline, of EW extent D. As time moves on, the baseline moves around the ellipse. The grey circles show the areas sampled at two different times which are far enough apart that the samples are independent. (The extra pair of circles in the lower right are equivalent to the other two, but with the antennas swapped.)

To be conservative, we divide this time by a factor of 2, giving

$$t = (6 / \pi)\ (d/D)\ \text{hours.}$$

Substituting d = 6 m, and D = 1000 m, we find

$$t = 41 \text{ seconds.}$$

This result can also be derived by considering the Nyquist sampling rate for fringes from a source at the edge of the field. In principle, the relevant time could be shorter if the antenna is moved quickly from one field to another in mosaicing observations, but it is unlikely that times will be much shorter than this because of the extra overhead due to slewing. In single dish operation, although the chopping cycle may be much faster than this, many cycles are always added together so that the basic integration time is typically 30 seconds or more. For weak sources, longer times are forced by the need to get enough signal/noise, while even for strong sources, operational overheads impose large penalties at short times. For almost all purposes then, t will be 30 seconds or more.

We then choose $t = 30$ seconds, and define any pointing fluctuations on shorter timescales as jitter.

II The Effect of Jitter

The effect of pointing jitter is rather easily estimated. It spreads out the beam over an area wider than the ideal and is therefore equivalent to surface error, which also reduces the forward gain and spreads the beam. Quite generally, the forward gain, relative to the ideal case, is given by

$$G = \left\langle e^{-f^2} \right\rangle$$

where f is the *wavefront* error in radians and the average is taken over the surface of the antenna. This does not depend on the form of the errors, provided that they are on a scale large compared with the wavelength. In particular, the reduction in forward gain is the same for rms wavefront errors due to pointing as it is for wavefront errors due to dish inaccuracy. In terms of the antenna rms surface error, s,

$$f = 4\pi s/l,$$

where the extra factor of 2 is due to the reflection. For small f, this can be approximated by

$$G = 1 - \left\langle f^2 \right\rangle$$

$$G = 1 - 0.29 \left(\frac{s}{15\ mm}\right)^2 \left(\frac{350\ mm}{l}\right)^2$$

In this limit, the reduction in forward gain is just equal to the square of the rms wavefront error, in radians.

To calculate the loss in forward gain due to pointing jitter, we can use the fact that a pointing error is equivalent to a phase gradient across the aperture. Then we can find the reduction in forward gain due to jitter by calculating the equivalent rms wavefront error for that gradient. If we consider a tilt of the whole dish due to a pointing error, with a displacement at the edge of the dish of e, relative to the center, then the rms displacement averaged over the whole dish is e/2 and the rms contribution, f,, to the wavefront error is

$$f_p = 2\pi(e/2)/l.$$

In this case there is no extra factor of 2, since the subreflector and the rest of the optical train moves with the reflector. The displacement at the edge of the reflector is simply proportional to the pointing offset, q,

$$e = 14.5\ q \text{ microns/arcsec}\ .$$

Comparing the pointing and surface errors, we can see that 1” of pointing error is equivalent to a surface error of 3.6 microns. Note that this is 2-dimensional pointing error. It is immediately obvious that our pointing jitter specification of 1” rms is much tighter than the surface error specification of 15 microns rms. Therefore tradeoffs which improve the surface at the expense of the jitter may be worthwhile.

III The effect of slow pointing errors

Slow variations in pointing can be much more damaging than pointing jitter, since they may be coherent over a significant part of any observation. In a single-dish map, such errors obviously produce shifts and distortions in the map. In an interferometric map, the effect can be more complicated. For a single field, a pointing shift has a small effect on the intensity of a source at the center of the beam, but a much larger effect on sources away from the center. For a mosaic observation, the effect of such errors on the final map depends on the processing algorithm. Recent work by Emerson suggests that a kind of self-calibration can be used to fix up pointing errors in mosaic images in most cases of interest.

One way to estimate the effect of pointing errors is to calculate the effect on the intensity of a source at the half-power point of the beam. For a uniformly illuminated circular 6 m aperture, the fractional change in intensity of a source at the half-power point is

$$\frac{DI}{I} = 0.22\left(\frac{Dq}{1''}\right)\left(\frac{l}{350\ mm}\right)^{-1}$$

where Dq is the pointing error and I is the observing wavelength. In this case, Dq is a 1-dimensional error since the sources affected are those at the half-power point in the direction of the pointing error. This equation is a slight overestimate of the error in a real antenna, since the taper of the illumination will broaden the beam, lessening the effect of pointing offsets by a few percent.

From the above expression, it is clear that even a small pointing error can have a large effect at the shortest wavelengths and we should strive to keep our slow pointing errors down to 1” (1-D). These errors are largely due to two types of effect. The first is thermal distortions in the mount and backup structure. The magnitude of these can be estimated by modelling. The second cause is errors in the mount model, and any effects, such as hysteresis in the encoders, which are not included in it. It is hard to anticipate all such errors. For the obvious misalignments in the mount, I suggest that these should be kept to values comparable with the beam (15 - 60”) so that a mount model accurate to 1% could reduce the residuals to acceptable levels. Most slow effects of wind should be removed by tiltmeters, while fast effects will be counted as jitter.

IV Other sources of loss

Two other sources of loss can be put on the same scale as jitter and surface errors. The first is atmospheric phase and the second is interferometer path error due to motions of the structure.

Atmospheric phase. The atmospheric phase has a power-law distribution of scales, and is dominated by large-scale effects. Its effect can be modelled well as a variable tilt of the incoming wavefront. In this approximation, the excess path length at the edge of the antenna, relative to the center, is half of the path difference between the edges of the antenna. We can write the excess path length at the edge of the 6 m dish relative to the center in terms of p_{100}, the measured path difference at 100m baseline, as

$$e_a = 0.5\ p_{100}\,(6/100)^{0.65}$$
$$= 0.08\ p_{100}\ .$$

The slope of 0.65 for the wavefront path difference as a function of baseline length is taken from the preliminary phase monitor data on Mauna Kea. As for pointing jitter, this can be converted into an equivalent rms antenna surface error, s_a, by dividing by 2 to convert to rms, and a further factor of two to get the equivalent surface error.

$$s_a = 0.02\ p_{100}\ .$$

Based on the preliminary analysis of the first month of atmospheric data, the median value of p_{100} at night is 160 microns, and 950 microns during the day. At night then, the atmospheric contribution to the losses is very small, equivalent to 3.2 microns of surface error, but during the day, the median atmospheric loss is equivalent to 19 microns of surface error, which dominates other sources.

Path error. Changes in the dish position or internal optical paths due to thermal and wind effects can produce path errors which decorrelate the interferometer signal and produce further loss in amplitude. These path errors are naturally expressed in terms of displacements, d, at each antenna. Fixed and slowly varying displacements, such as those in the receiver cab, should be removed by the regular phase calibration, but those which vary rapidly with time or dish orientation cannot easily be calibrated. Once again, these can be treated in the same way as pointing jitter.

The net path error on any baseline is $\sqrt{2}$ times greater than the path error in a single antenna. To put this on the same scale as dish surface error it must be divided by 2, since the surface errors are doubled to find their effect on wavefront phase. Thus each 1 micron of path error is equivalent to 0.71 microns of dish surface error.

V Discussion and Conclusions

I have shown that several sources of gain loss can be expressed on comparable scales, and have calculated conversion factors so that pointing jitter and path errors can be compared directly with dish surface errors. Since these may all be traded off against each other in the dish design, this comparison permits an optimization for the best overall gain of the system. In the limit where the losses due to these errors are small, the expression for the combined loss has a particularly simple form. The different sources of error are independent and combine in a root-sum-square fashion, while the loss depends on the sum of the squares. Therefore, by combining all the losses discussed, we can write the relative gain as

$$G = e^{-\left[0.29\left(\frac{350\ \mu m}{\lambda}\right)^2\left\{\left(\frac{\sigma}{15\ \mu m}\right)^2+\left(\frac{\theta}{4.2''}\right)^2+\left(\frac{\delta}{21\ \mu m}\right)^2+\left(\frac{\rho_{100}}{750\ \mu m}\right)^2\right\}\right]}$$

$$G \approx 1 - 0.29\left(\frac{350\ \mu m}{\lambda}\right)^2\left\{\left(\frac{\sigma}{15\ \mu m}\right)^2+\left(\frac{\theta}{4.2''}\right)^2+\left(\frac{\delta}{21\ \mu m}\right)^2+\left(\frac{\rho_{100}}{750\ \mu m}\right)^2\right\}$$

where λ is the operating wavelength, σ is the dish surface error, θ is the rms pointing jitter, δ is the jitter in the position of each antenna, and ρ_{100} is the path length fluctuation on a 100 m baseline. The approximation in the second equation is valid when the loss is small compared with 1. Under most conditions, the dominant loss is due to the antenna surface error, but under typical daytime conditions the atmospheric irregularities may be worse than the antenna errors.

There is a significant caveat to be added here about the variability of errors. If the surface error, for example, varied over the range of O-15 μm, then the gain at 350 μm would vary from 100-75 %. Such unpredictable gains would distort our maps because of the inconsistency between measurements at different times. Therefore we should be more conservative with losses of gain which might vary substantially from one time to another (e.g. pointing jitter due to wind), than with stable losses.

There is a similar question about large scale surface errors, such as astigmatism, versus small-scale ones, such as panel misalignments. The large scale ones scatter power close to the main lobe and therefore can produce larger relative changes in gain for sources near the boresight of the antenna, while the small-scale errors scatter power far from the main beam, where it cannot affect the observations significantly. For interferomeny, the small-scale errors are slightly less serious than the large-scale ones, assuming that the rms is equal. For single dish work, mapping sources larger than the main beam, the situation is less clear, since large-scale errors do not degrade the beam efficiency as much as small-scale errors do.

Slow pointing errors must be handled differently and can be more damaging. Our goal should be 1” rms for slow pointing errors, although it is difficult to predict all possible causes of error. The mount model may be limited by a number of effects, but we should try to limit the magnitude of any misalignments. As a rule of thumb, we should keep all angular misalignments less than or equal to the beam size (15-60”), and relevant mechanical offsets (which affect the path errors) to a quarter of the wavelength (0.1-0.4 mm), so that they can be accurately calibrated out in the mount model.