SMA TECHNICAL MEMO
TM 150

Title: Elevation Drive Screw bearing and lubrication analysis

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Introduction:

Noise spikes have been observed in servo data files from several antennas. It is believed that these are a result of friction within the elevation drive systems' hardware. These have become evident because of the improved performance of the elevation servo system and its housekeeping data. We can't say whether this is a new problem or has been there all along, but unnoticed. The elevation drive system hardware was reviewed to find the source of these noise spikes. Also to understand possible causes of the performance problems with antenna 8's and problems experienced while re-furbishing antenna 2's lead screws. This review has centered on the: Bearing selection, bearing lubrication, the assembly process and the drive systems alignment to the reflector weldment.

Bearing analysis:

The Elevation drive assembly is described on SMA drawing number 10151760000 with its parts listed on Bill of Material number 10151760001. It is assembled using SMA procedure number 40151760000.

There is a set of FAG bearings number 29422E used to position the elevation drive screw. These bearings are used to carry all the resulting loads for Reflector positioning. The bearing type is a Spherical Roller Thrust Bearing. This type bearing is rated as having excellent axial load carrying capacity with poor radial capacity.

Bearing data:

Dimensions: ID = 110 mm, OD = 230 mm, Ht. = 73mm
Load specifications: C_{dyn} = 212,000 lbs. C_{static} = 630,000 lbs.
Number of rollers = 15, Roller diameter = 33 mm, Roller length = 41.9
Pitch circle = 194.092 mm, Contact angle 50 degrees

Operating speeds:
Slew speed: 2 degrees/sec resulting in a shaft rotation speed ≅ 98 RPM
Sidereal speed: 15 arc-sec/sec resulting in a shaft rotation speed ≅ 1.2 deg/sec (0.205 rpm).

The expected bearing loads (lbs) are:

Load sources:	Top bearing:	Bottom bearing:
Preload	1500	1500
Reflector unbalance	1569 (20^0)	359 (zenith)
Mis-alignment	42	42
Temperature (lbs/F^0)	3268	3268
Totals	6379 lbs.	5169 lbs.

The worst-case load occurs in the top bearing and therefore is the basis for the bearing analysis. The load per Roller is 6379/15 = 425.3 lbs.

The load calculations are as follows:

Mis-alignment calculation:

The alignment specification (40551680000) allows ± 0.25mm when the drive nut is closest and ± 0.50mm when away from the motor. The distances to the nut center from the pivot rotation center for these conditions are 531 and 1536 mm respectively. The 1536mm is for an elevation of 80 degrees, which is the angle usually sited during alignment. The worst-case angle results when the nut is closest to the drive and is 2.69×10^{-2} degrees. This angulation results in a length change between the bearing centers of 4.9×10^{-7} inches. The net spring constant (K) of the bearings, shaft, housing and shaft/nut threads is 8.6×10^{7} lbs/inch (dominated by the threads).

Therefore the expected mis-alignment load is:

$$\text{Load} = K \times \Delta L \text{ or } (8.6 \times 10^{7}) \times (4.9 \times 10^{-7}) = 42.14 \text{ lbs}$$

The expected temperature load per degree difference can be computed by:

$$\Delta L = L_o x\, \Delta T \times CTE_{STEEL}$$
$$\Delta L = 4.5 \times (1) \times 8.4 \times 10^{-6}$$
$$\Delta L = .000038 \text{ inches/deg F}$$

This length change per degree produces an increased load (or reduction) in the bearings of:

$$\text{Load} = (8.6 \times 10^{7}) \times (3.8 \times 10^{-5}) = 3268 \text{ lbs}/^{o}F$$

For analysis, a positive 1-degree difference is used. However, several degrees of temperature difference are possible. This is a fault in the present design because no temperature accommodation is present. A Belleville spring design will be tested on Antennas' 2 and 3 lead screws, which are presently at Haystack. This will guard against the bearings becoming un-loaded should a temperature difference cause expansion of the shaft relative to the housing. The Belleville will be compressed to its solid height, therefore offering no temperature compensation for the reverse case. It will also allow us to apply a known preload.

FAG specifies a minimum bearing preload for proper operation as determined by the formula below.

$$Famin = \left[\left[\frac{C}{1400}\right] + (A \cdot 225)\right] \cdot \left[\frac{Dg \cdot H \cdot n}{1000000}\right]^{2}$$

Where:

C is the static load capacity (630,000 lbs)
A is a FAG specified value for this bearing (0.0021)
Dg is the bearings' Outside Diameter (230mm)
H is the bearings' height (73mm)
n is its maximum operating speed (98 RPM)

This results in a minimum load of 1220 pounds. We have specified a preload of 1500 pounds and therefore meet the minimum recommended pre-load.

FAG formula for bearing life.

$$\text{Life} = \left[\frac{C}{P}\right]^{\left[\frac{10}{3}\right]}$$

Where:

C is the static load capacity (630,000 lbs)
P is the axial load (6379 lbs)

This results in an expected life of 4.6 million cycles.

The Hertzian stresses created at the roller/raceway contact points were also calculated to complete the bearing analysis. Figure 1, shows predicted Hertzian stresses over a range of roller loads. The Hertzian stress for our projected worst-case load (6379) is extremely low and therefore not a concern. All loads shown on Figure 1 are within acceptable Hertzian stress levels.

Bearing Summary:

Analysis indicates that the FAG bearings are well within their static and dynamic operational range and will provide long life, provided that they are properly assembled, lubricated and maintained.

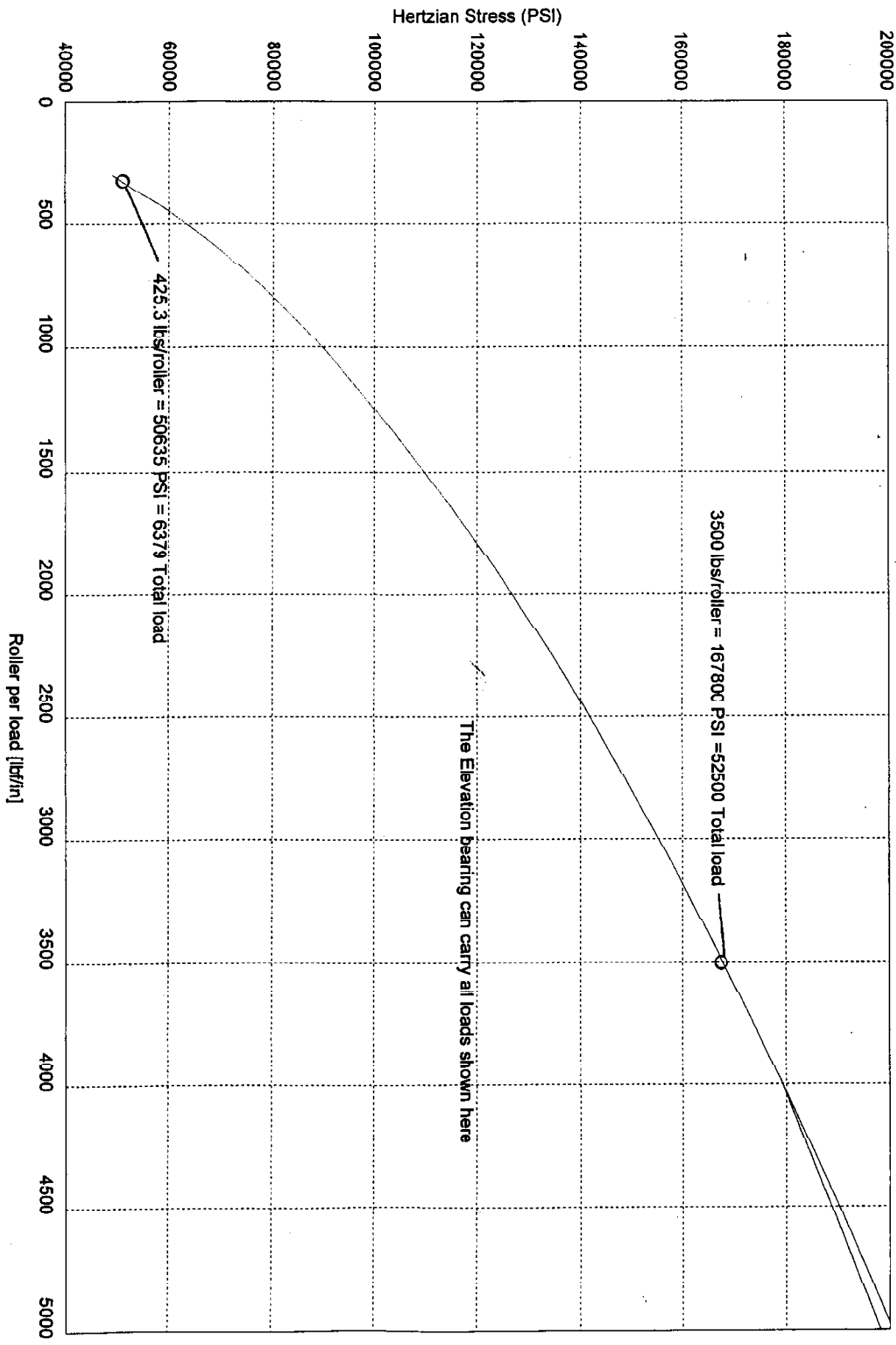

Roller load versus Hertzian stress
SMA Elevation drive bearing
Hertzian Stress (PSI)
40000
60000
80000
100000
120000
140000
160000
180000
200000
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
Roller per load [lbf/in]
3500 lbs/roller = 167800 PSI =52500 Total load
425.3 lbs/roller = 50635 PSI = 6379 Total load
The Elevation bearing can carry all loads shown here

Figure 1

Lubrication analysis:

The present lubrication is a FUCHS hydraulic compressor lubricant PAO HO 32. The question is whether this is a good selection based on the projected loads, operating temperature and rotational velocities for the Elevation drive screw when operating on the summit? The data for this lubricant and a Mobil lubricant are given in the table below.

Lubricant Properties

Property	Fuchs PAO HO 32	Mobil SHC 320	Units
API gravity	32	30.1	
SAE Grade	10	90 Heavy	
ISO Grade	36	320	
Viscosity			
@40 C	32	320	centistokes
@ 100 C		34.6	centistokes
Spec. Gravity			
@ 40 C	.841	.857	
Viscosity Index	135	155	

The analysis follows theory provided by FAG Bearings Inc. in their technical publication number WL 81 115/4 EA. Its relevant pages are provided in Appendix A as a reference. Its formulas and units were placed into MathCAD software for computational purposes. Calculations were made for both the present lubricant (FUCHS PAO HO 32) and Mobil-SHC 320. Calculations are shown in Appendix B for both lubricants. The analysis results are shown in tables 1 and 2 for comparison purposes.

The worst-case film thickness occurs when the lubricant rises in temperature (absolute viscosity thins) and operating at low speeds. The film thickness is relatively insensitive to load because the viscosity rises with increasing load. Therefore, the analysis is based on a load of 6379 pounds, an operating temperature of 20 degrees Celsius and the rotational velocities associated with tracking at sidereal rate. The operating temperature was chosen based on recorded Antenna motor temperature measurements.

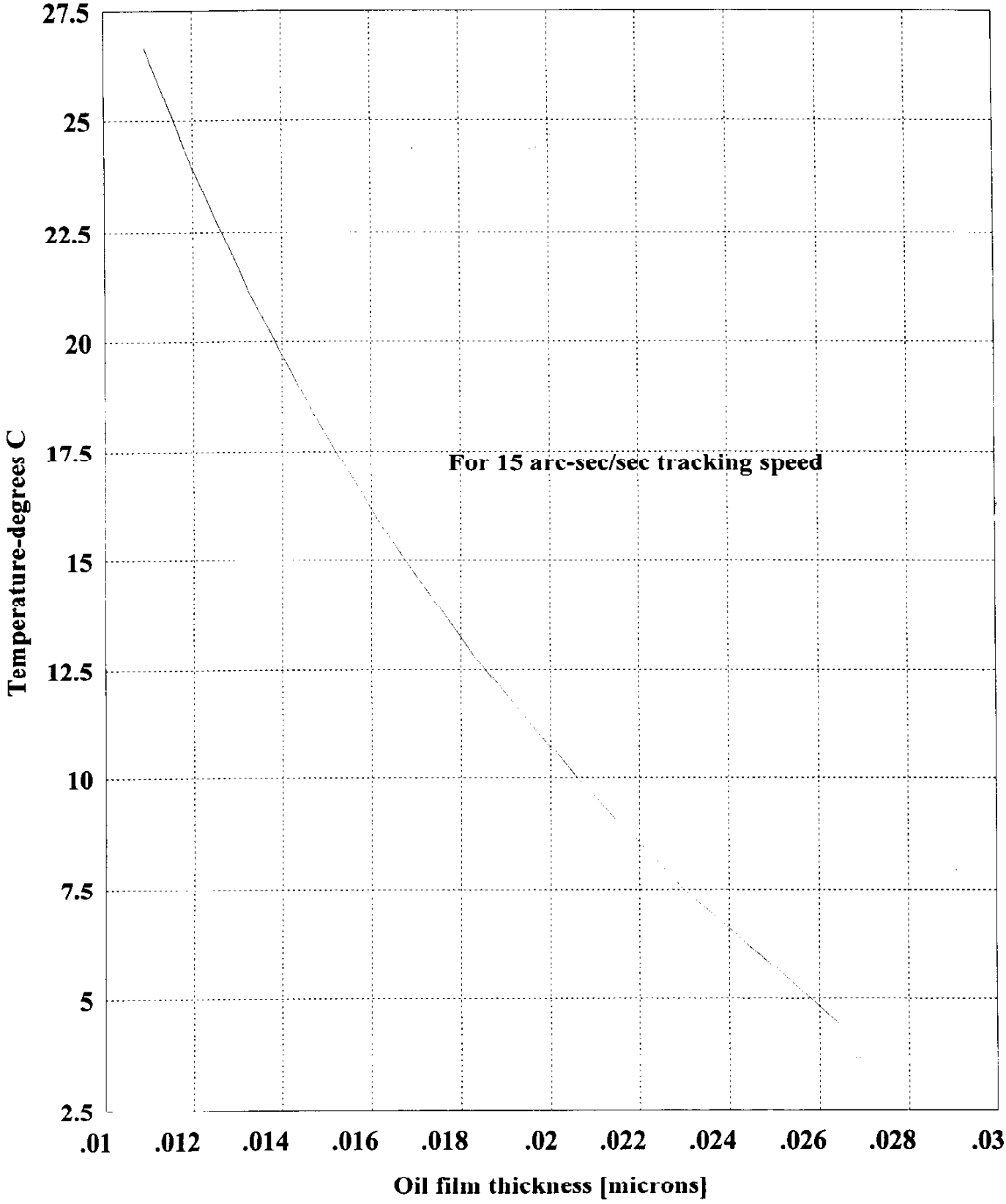

Film thickness versus Temperature-degrees C
FUCHS PAO HO 32
27.5
25
22.5
20
17.5
15
12.5
10
7.5
5
2.5
Temperature-degrees C
For 15 arc-sec/sec tracking speed
.01
.012
.014
.016
.018
.02
.022
.024
.026
.028
.03
Oil film thickness [microns]

Figure 2

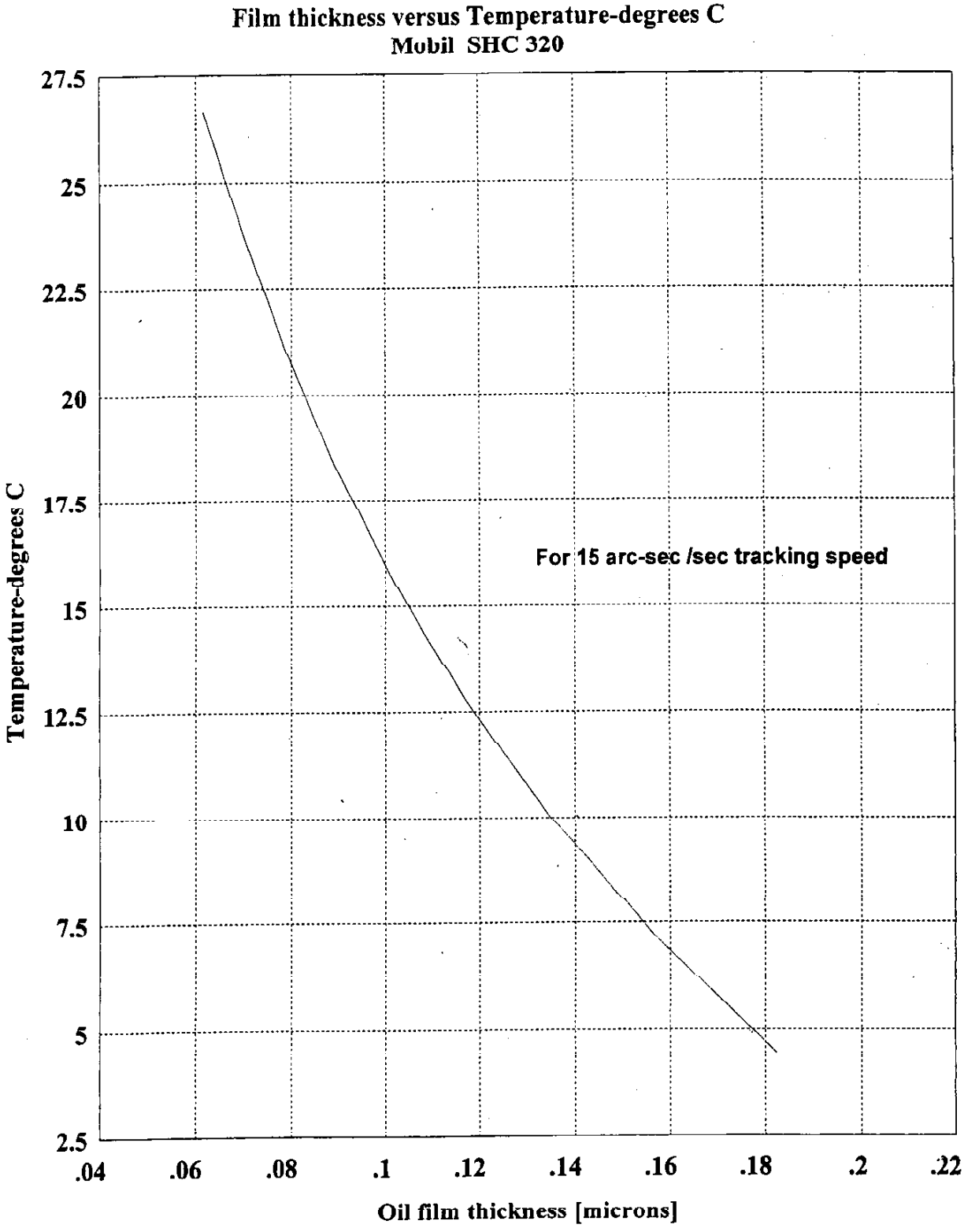

Film thickness versus Temperature-degrees C
Mobil SHC 320
27.5
25
22.5
20
17.5
15
12.5
10
7.5
5
2.5
Temperature-degrees C
For 15 arc-sec /sec tracking speed
.04
.06
.08
.1
.12
.14
.16
.18
.2
.22
Oil film thickness [microns]

Figure 3

Table 1
Comparison of lubricant properties at tracking speed

Item	Fuchs	Mobil	Units
Absolute viscosity	0684	8816	$N*s/M^2$
Kinematic viscosity	80.3	1010.1	mm^2/s
Film thickness	0.014	0.081	Microns
Load Independent moment	0.01	0.06	Ft-lbs
Load dependent moment	1.32	1.32	Ft-lbs
Total moment	1.33	1.38	Ft-lbs

Table 2
Comparison of lubricant properties at slew speed
Note: Viscosities remain as above

Item	Fuchs	Mobil	Units
Film thickness	17.9	107.4	Microns
Load Independent moment	0.7	3.8	Ft-lbs
Load dependent moment	1.32	1.32	Ft-lbs
Total moment	2.02	5.12	Ft-lbs

The Mobil SHC 320 lubricant has EP additives where the present Fuchs lubricant doesn't. EP additives provide extra wear protection (lubricity) by combining with the bearings metallic components. Bearing manufacturers encourage their use in heavy load and/or slow speed applications.

Lubrication summary:

The goal was to achieve FAG's recommended minimum film thickness of 0.1 microns for our worst-case operating load, temperature and speed, while not excessively increasing the (lubricant) viscous torque. Figures 2 & 3 show temperature performance for both lubricants. Clearly, the Mobil lubricant is better suited for our application. It increases the film thickness be a factor of 5.7 with a resulting total torque increase of 2.5. Although, we didn't achieve the 0.1-micron thickness at 20^o C it is achieved at 15.8^o C.

Technical summary:

The Elevation drive system requires precise alignment and assembly to operate properly. The alignment tolerances are achievable; however require that the alignment procedure 40151760000 be followed. Mis-alignment causes stresses to be created in both the drive nut; drive screw and elevation drive bearings resulting in an increased frictional torque. Elevation drive mis-alignment was reviewed in 1997, where a translational over-constraint was revealed. Mis-alignment will also cause the motor and tachometer rotors to tilt within their stators. This tilt is not a problem if alignment tolerances are achieved; however if contact occurs then damage will result. Precise alignment then is extremely important to the proper operation to the Elevation drive system.

Elevation drive assembly is essential to its proper operation. Tolerance studies have revealed that it is possible for the shaft to extend beyond the motor side bearings' face. If this condition exists, then the bearings will not be preloaded by the torquer' housing, as designed. This is a possible cause of antenna 8's elevation "banging noise". Another problem is that the assembly procedure calls for the oil reservoir to be partially filled (≅ 45%). The bearings lubricant coverage therefore varies between 30 to 70% over the elevation range. At Zenith, the top bearing is ≅ 30% covered and the motor bearing is ≅ 70%. These ratios approximately reverse at 20 degrees elevation.

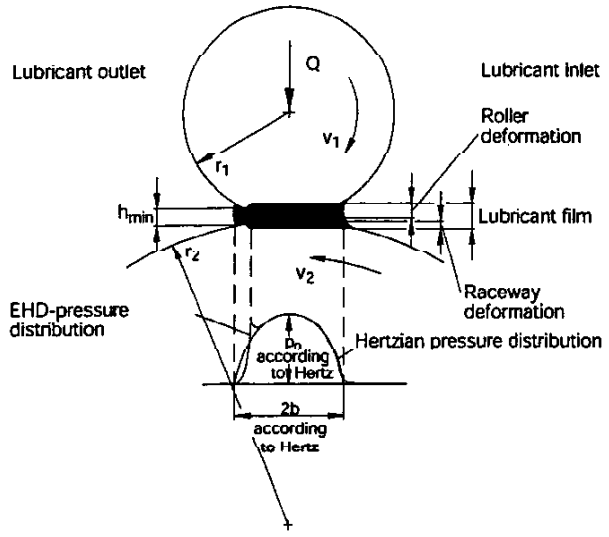
The elevation drive bearings are operating well within their design criteria and should provide excellent performance over the lifetime of the project. However, they require proper lubrication and maintenance to ensure this. Mobil SHC-320 gear lubricant testing is plan for Antenna 1. It will be installed the week of March 23-28, 2003. Antenna 1's servo performance will be monitored thereafter to determine if improvement has been accomplished. Also we will measure the lubricants operating temperature. This will aid in subsequent work should the servo noise spikes persist and/or bearing wear particles be observed in the lubricant. Analysis verifies that the present lubricant is insufficient and this is also supported by a Fuchs analysis of an oil sample taken from Antenna 3's elevation drive, where bearing wear (metallic) particles are present.

1.1.2 Lubricating Film with Oil Lubrication

Main criterion for the analysis of the lubricating condition is the lubricating film thickness between the load transmitting rolling and sliding contact surfaces. The lubricant film between the rolling contact surfaces can be described by means of the theory of elastohydrodynamic (EHD) lubrication. The lubrication under sliding contact conditions which exist, e.g. between the roller faces and lips of tapered roller bearings, is adequately described by the hydrodynamic lubrication theory as the contact pressure in the sliding contact areas is lower than in the rolling contact areas.

The minimum lubricant film thickness h_{min} for EHD lubrication is calculated using the equations for point contact and line contact shown in fig. 2. The equation for point contact takes into account the fact that the oil escapes from the gap on the sides. The equation shows the great influence of the rolling velocity v, the dynamic viscosity η_0 and the pressure-viscosity coefficient α on h_{min}. The load Q has little influence because the viscosity rises with increasing loads and

2: Elastohydrodynamic lubricant film. Lubricant film thicknesses for point contact and line contact



Point contact according to Hamrock and Dowson

$$h_{min} = 3{,}63 \cdot U^{0{,}68} \cdot G^{0{,}49} \cdot W^{-0{,}073} \cdot (1 - e^{-0{,}68 \cdot k}) \cdot R_r \text{ [m]}$$

Line contact according to Dowson

$$h_{min} = 2{,}65 \cdot U^{0{,}7} \cdot G^{0{,}54} \cdot W'^{-0{,}13} \cdot R_r \text{ [m]}$$

with $U = \eta_0 \cdot v/(E' \cdot R_r)$
$G = \alpha \cdot E'$
$W = Q/(E' \cdot R_r^2)$ for point contact
$W' = Q/(E' \cdot R_r \cdot L)$ for line contact

where

h_{min}	[m]	minimum lubricant film thickness in the area of rolling contact
U		speed parameter
G		material parameter
W		load parameter for point contact
W'		load parameter for line contact
e		e = 2,71828..., base of natural logarithms
k		k = a/b, ratio of the semiaxes of the contact areas
α	[m²/N]	pressure viscosity coefficient
η_0	[Pa · s]	dynamic viscosity
v	[m/s]	$v = (v_1 + v_2)/2$, mean rolling velocity v_1 = rolling element velocity v_2 = velocity at inner ring or outer ring contact
E'	[N/m²]	$E' = E/[1 - (1/m)^2]$, effective modulus of elasticity E = modulus of elasticity = $2{,}08 \cdot 10^{11}$ [N/m²] for steel 1/m = Poisson's ratio = 0,3 for steel
R_r	[m]	reduced curvature radius $R_r = r_1 \cdot r_2/(r_1 + r_2)$ at inner ring contact $R_r = r_1 \cdot r_2/(r_1 - r_2)$ at outer ring contact r_1 = rolling element radius [m] r_2 = radius of the inner and outer ring raceways [m]
Q	[N]	roller load
L	[m]	gap length or effective roller length

the contact surfaces are enlarged due to elastic deformation.
The calculation results can be used to check whether a sufficiently strong lubricant film is formed under the given conditions. Generally, the minimum thickness of the lubricant film should be one tenth of a micron to several tenths of a micron. Under favourable conditions the film is several microns thick.
The viscosity of the lubricating oil changes with the pressure in the rolling contact area:

$\eta = \eta_0 \cdot e^{\alpha p}$

η dynamic viscosity at pressure p [Pa s]
η_0 dynamic viscosity at normal pressure [Pa s]
e (= 2,71828) base of natural logarithms
α pressure viscosity coefficient [m^2/N]
p Pressure [N/m^2]

The calculation of the lubricating condition in accordance with the EHD theory for lubricants with a mineral oil base takes into account the great influence of pressure. The pressure-viscosity behaviour of a few lubricants is shown in the diagram in fig. 3. The a_{23} diagram shown in fig. 7 (page 7) is based on the zone a-b for mineral oils. Mineral oils with EP-additives also have α values in this zone.

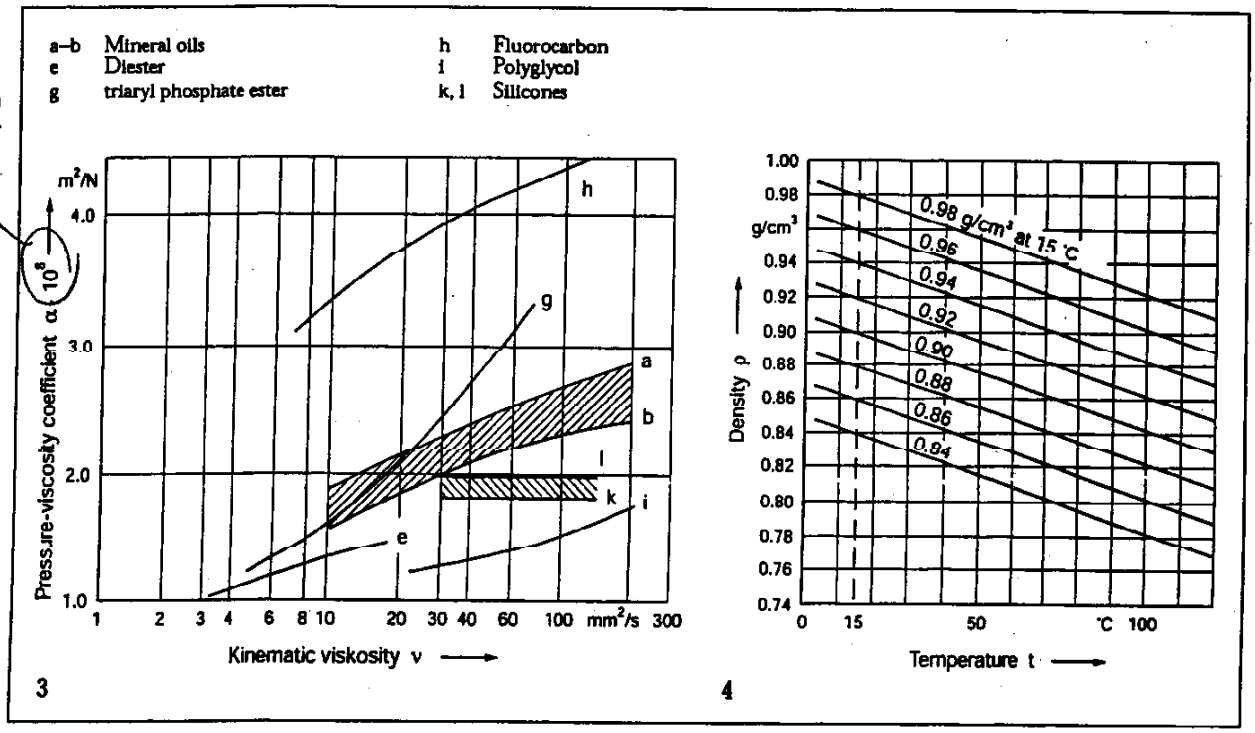
If the pressure-viscosity coefficient has considerable influence on the viscosity ratio, e.g. in the case of diester, fluorocarbon or silicone oil, the correction factors B1 and B2 have to be taken into account in the calculation of the viscosity ratio $\varkappa$.

$\varkappa_{B1,2} = \varkappa \cdot B_1 \cdot B_2$

$\varkappa$ viscosity ratio for mineral oil (see section 1.1.3)
B_1 correction factor for pressure-viscosity behaviour
$= \alpha_{synthetic\ oil} / \alpha_{mineral\ oil}$
(α values, see fig. 3)
B_2 correction factor for varying density
$= \rho_{synthetic\ oil} / \rho_{mineral\ oil}$

The diagram, fig. 4, shows the curve for density ρ as a function of temperature for mineral oils. The curve for a synthetic oil can be assessed if the density ρ at 15°C is known.

3: Pressure-viscosity coefficient α as a function of kinematic viscosity ν, for pressures from 0 to 2000 bar
4: Density ρ of mineral oils as a function of temperature t



Appendix A-2

1.2 Calculation of the Frictional Moment

The frictional moment M of a rolling bearing, i.e. the sum total of rolling friction, sliding friction and lubricant friction, is the bearing's resistance to motion. The magnitude of M depends on the loads, the speed and the lubricant viscosity (fig. 15). The frictional moment comprises a load-independent component M_0 and a load-dependent component M_1. The black triangle to the left of the dot-dash line shows that with low speeds and high loads a considerable mixed friction share R_M can be added to M_0 and M_1 as in this area the surfaces in rolling contact are not yet separated by a lubricant film. The zone to the right of the dot-dash line shows that with a separating lubricating film which develops under normal operating conditions the entire frictional moment consists only of M_0 and M_1.

$$M = M_0 + M_1 \quad [N\ mm]$$

M	[N mm]	total frictional moment of the bearing
M_0	[N mm]	load-independent component of the frictional moment
M_1	[N mm]	load-dependent component of the frictional moment

Mixed friction can occur in the raceway, at the lips and at the cage of a bearing; under unfavourable operating conditions it can be very pronounced but hard to quantify.

In deep groove ball bearings and purely radially loaded cylindrical roller bearings with a cage the mixed friction share according to fig. 15 is negligible. The frictional moment of axially loaded cylindrical roller bearings is determined by means of the equations given at the end of section 1.2.

Bearings with a high sliding motion rate (full-complement cylindrical roller bearings, tapered roller bearings, spherical roller bearings, thrust bearings) run, after the run-in period, outside the mixed friction range if the following condition is fulfilled:

$$n \cdot \nu / (P/C)^{0,5} \geq 9000$$

n	[min^{-1}]	speed
ν	[mm^2/s]	operating viscosity of the oil or grease base oil
P	[kN]	equivalent dynamic load
C	[kN]	dynamic load rating

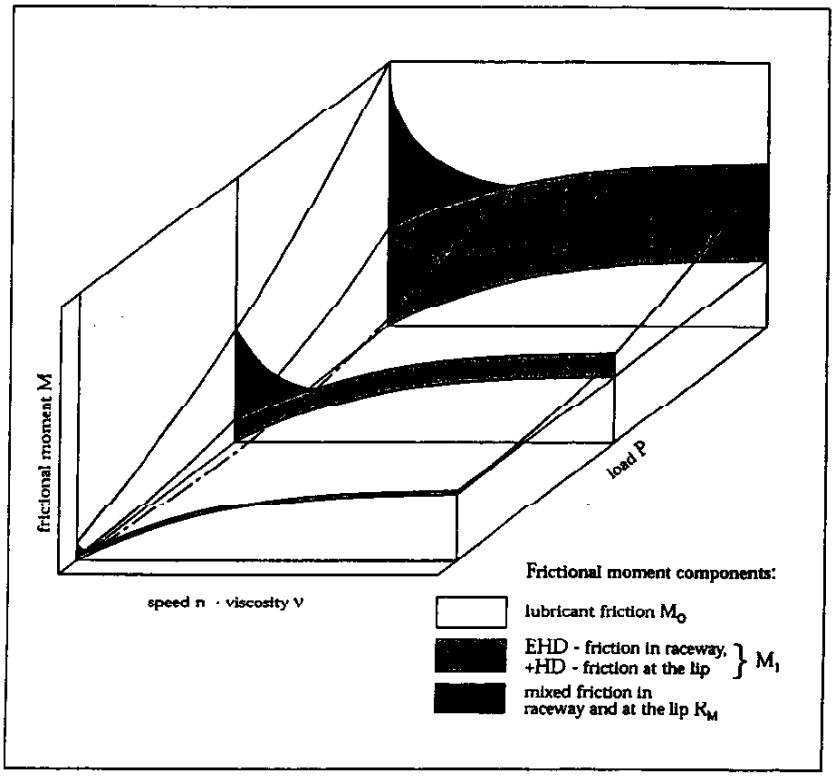
The load-independent component of the frictional moment, M_0, depends on the operating viscosity ν of the lubricant and on the speed n. The operating viscosity, in turn, is influenced by the bearing friction through the bearing temperature. In addition, the mean bearing diameter d_m and especially the width of the rolling contact areas – which considerably varies from type to type – have an effect on M_0. The load-independent component M_0 of the frictional moment is determined, in accordance with the experimental results, from

$$M_0 = f_0 \cdot 10^{-7} \cdot (\nu \cdot n)^{2/3} \cdot d_m^{\ 3} \quad [N\ mm]$$

where

M_0	[N mm]	load-independent component of the frictional moment
f_0		index for bearing type and lubrication type (table, fig. 16).

15: Frictional moment in rolling bearings as a function of speed, lubricant viscosity and loads.
In ball bearings (except thrust ball bearings) and purely radially loaded cylindrical roller bearings the mixed friction triangle (left) is negligible, i.e. $R_M \approx 0$.



ν	[mm²/s]	operating viscosity of the oil or grease base oil fig. 5, page 6)
n	[min⁻¹]	bearing speed
d_m	[mm]	(D + d)/2 mean bearing diameter

The index f_0 is indicated in the table, fig. 16, for oil bath lubrication where the oil level in the stationary bearing reaches the centre of the bottommost rolling element. F_0 increases – for an identical d_m – with the size of the balls or with the length of the rollers, i.e. it also increases, indirectly, with the size of the bearing cross section. Therefore, the table indicates higher f_0 values for wide bearing series than for narrow ones. If radial bearings run on a vertical shaft under radial load, twice the value given in the table (fig. 16) has to be assumed; the same applies to a large cooling-oil flow rate or an excessive amount of grease (i.e. more grease than can displaced laterally).

The f_0 values of freshly greased bearings resemble, in the starting phase, those of bearings with oil bath lubrication. After the grease is distributed within the bearing, half the f_0 value from the table (fig. 16) has to be assumed. Then it is as low as that obtained with oil throwaway lubrication. If the bearing is lubricated with a grease which is appropriate for the application, the frictional moment M_0 is obtained mainly from the internal frictional resistance of the base oil.

Exact M_0 values for the most diverse greases can be determined in field trials. On request FAG will conduct such tests using the friction moment measurement instrument R 27 which was developed especially for this purpose.

16: Index f_0 for the calculation of M_0, depending on bearing type and series, for oil bath lubrication; for grease lubrication after grease distribution and with oil throwaway lubrication these values have to be reduced by 50 %.

Bearing type Series	Index f_0 for oil bath lubrication
deep groove ball bearings	1,5...2
self-aligning ball bearings	
12	1,5
13	2
22	2,5
23	3
angular contact ball bearings, single row	
72	2
73	3
angular contact ball bearings, double row	
32	3,5
33	6
four point bearings	4
cylindrical roller bearings with cage:	
2, 3, 4, 10	2
22	3
23	4
30	2,5
full complement	
NCF29V	6
NCF30V	7
NNC49V	11
NJ23VH	12
NNF50V	13
needle roller bearings	
NA48, NA49	5...5,5
tapered roller bearings	
302, 303, 313	3
329, 320, 322, 323	4,5
330, 331, 332	6
spherical roller bearings	
213, 222	3,5...4
223, 230, 239	4,5
231, 232	5,5...6
240, 241	6,5...7
thrust ball bearings	
511, 512, 513, 514	1,5
522, 523, 524	2
cylindrical roller thrust bearings	
811	3
812	4
spherical roller thrust bearings	
292E	2,5
293E	3
294E	3,3

Appendix B
Fuchs PAO HO 32 Lubricant
Tracking speed Analysis

Analysis is base on present FUCHS lubricant PA0 HO 32 operating at 20 degrees C, 15 arc-sec/sec speed operating in FAG bearing number 29422E.

Determine the reduced curvature radius, R_r

R_b= roller radius. 16.mm

R_s = bearing pitch radius,194 mm

$R_b := 0.0165m$ $\quad R_s := 0.097m$

$$R_r := \frac{R_b \cdot R_s}{R_b + R_s}$$

$$R_r = 0.014\,m$$

Determine the effective Youngs modulus, E_e

μ is possion's ratio

E is Young modulus

Values from FAG catalog

$$E_s := 2.08 \cdot (10)^{11} \frac{N}{m^2} \qquad \mu_s := .3$$

$$E_e := \frac{E_s}{\left[1 - \left(\mu_s^2\right)\right]}$$

$$E_e = 2.286 \times 10^{11} \frac{N}{m^2}$$

A tracking speed of 15 arc-sec per sec. is used as the basis for calculation. The change in lead screw length per degree of elevation change (averaged over the range from Zenith to 20 degrees elevation) is 16.4059 mm/degree. Drive screw lead is 20 mm/revolution, therefore 15 arc sec of elevation change equals 1.2304 degrees of screw rotation. Also, the bearing race is assumed lock to the shaft.

Determine the speed parameter, U

η_0 is the oil's absolute viscosity

v are the component velocities

$$v_s := \pi \cdot 2 \cdot R_s \cdot \left(\frac{1.2304}{360}\right) \cdot \frac{1}{s} \qquad v_s = 2.083 \times 10^{-3} \frac{m}{s}$$

$$v_r := \left(v_s \cdot \frac{R_s}{R_b}\right) \qquad v_r = 0.012 \frac{m}{s}$$

$$v := \frac{v_r + v_s}{2} \qquad v = 7.164 \times 10^{-3} \frac{m}{s}$$

η_0 is the oil's absolute viscosity at 20 degrees C calculated from Mobil data

$$\eta_0 := .0684 \cdot \frac{N \cdot s}{m^2}$$

$$U := \frac{(\eta_0 \cdot v)}{(E_e \cdot R_r)}$$

$$U = 1.52 \times 10^{-13}$$

$$\beta := \left(\frac{1.9}{2.2}\right) \cdot \left(\frac{.86}{.87}\right) \qquad \beta = 0.854$$

Determine the material parameter; G
α is the pressure-viscosity coeff.
E is the effective modulus
Note: FAG catalog and their lubrication publication WL 81 115/4 EA lists the exponent as +8, I notified FAG and they agree it is in error, should be -8 as shown

$$\alpha := 1.9 \cdot (10^{-8}) \cdot \beta \cdot \frac{m^2}{N}$$

$$\alpha = 1.622 \times 10^{-8} \frac{m^2}{N}$$

$$G := (\alpha \cdot E_e) \qquad G = 3.708 \times 10^3$$

Determine the load parameter, W
6379 is the estimated axial load in lbs
Q is the load normal to surface
L is the spherical length of roller

$$Q := \frac{\left(\frac{6379 \cdot 4.4482}{15}\right)}{\cos(50 \cdot deg)} \cdot N \qquad Q = 2.943 \times 10^3 N$$

$$L := .0421 \cdot m$$

$$W := \frac{Q}{(E_e \cdot R_r \cdot L)} \qquad W = 2.169 \times 10^{-5}$$

Determine the minimum oil film thickness: h_{min}

$$h_{min} := 2.65 \cdot (U^{0.7}) \cdot (G^{0.54}) \cdot (W^{-.13}) \cdot R_r$$

$$microns := \frac{m}{(10^6)}$$

$$h_{min} = 0.014 \, microns$$

Calculate the frictional moment (torque) values and M values are N mm

Determine the load independant moment, M_0
where: f_0 taken from table 16
ν is oil kinemetic viscosity
n is rpm
d_m is pitch diameter

$f_0 := .00000033$ $\quad \nu := 80.3$

$n := .205$ $\quad d_m := 194$

$M_0 := f_0 \cdot (\nu \cdot n)^{.667} \cdot (d_m)^3$

$M_0 = 15.606$ $\quad M_0 = 0.01$ ft-lbs

Determine the load dependant moment, M_1
where: f_1 taken from table 17
P is axial load (N)

$f_1 := .00033$

$P := 6379 \cdot 4.4482$

$M_1 := f_1 \cdot P \cdot d_m$

$M_1 = 1.817 \times 10^3$ $\quad M_1 = 1.32$ ft lbs

Calculate the total frictional Moment, M

$M := M_0 + M_1$

$M = 1.832 \times 10^3$ $\quad M = 1.33$ ft lbs

Appendix B
Mobil SHC 320 lubricant
Analysis

Analysis is base on Mobil oil SHC 320 operating at 20 degrees C, 15 arc-sec/sec velocity and FAG bearing number 29422E.

Determine the reduced curvature radius, R_r

R_b= roller radius, 16.mm

R_s = bearing pitch radius,194 mm

$R_b := 0.0165m$ $\quad R_s := 0.097m$

$$R_r := \frac{R_b \cdot R_s}{R_b + R_s}$$

$R_r = 0.014\,m$

Determine the effective Youngs modulus, E_e

μ is poesion's ratio

E is Young modulus

Values from FAG catalog

$E_s := 2.08 \cdot (10)^{11} \frac{N}{m^2}$ $\quad \mu_s := .3$

$$E_e := \frac{E_s}{\left[1 - \left(\mu_s^2\right)\right]}$$

$E_e = 2.286 \times 10^{11} \frac{N}{m^2}$

A tracking speed of 15 arc-sec per sec. is used as the basis for calculation. The change in lead screw length per degree of elevation change (averaged over the range from Zenith to 20 degrees elevation) is 16.4059 mm/degree. Drive screw lead is 20 mm/revolution, therefore 15 arc sec of elevation change equals 1.2304 degrees of screw rotation. Also, the bearing race is assumed lock to the shaft.

Determine the speed parameter, U

η_0 is the oil's absolute viscosity

v are the component velocities

$v_s := \pi \cdot 2 \cdot R_s \cdot \left(\frac{1.2304}{360}\right) \cdot \frac{1}{s}$ $\quad v_s = 2.083 \times 10^{-3} \frac{m}{s}$

$v_r := \left(v_s \cdot \frac{R_s}{R_b}\right)$ $\quad v_r = 0.012 \frac{m}{s}$

$v := \frac{v_r + v_s}{2}$ $\quad v = 7.164 \times 10^{-3} \frac{m}{s}$

η_0 is the oil's absolute viscosity at 20 degrees C calculated from Mobil data

$$\eta_0 := .8816 \cdot \frac{N \cdot s}{m^2}$$

$$U := \frac{(\eta_0 \cdot v)}{(E_e \cdot R_r)}$$

$$U = 1.96 \times 10^{-12}$$

$$\beta := \left(\frac{1.9}{2.2}\right) \cdot \left(\frac{.86}{.87}\right) \qquad \beta = 0.854$$

Determine the material parameter; G
α is the pressure-viscosity coeff.
E is the effective modulus
Note: FAG catalog and their lubrication publication WL 81 115/4 EA lists the exponent as +8, I notified FAG and they agree it is in error, should be -8 as shown

$$\alpha := 1.9 \cdot \left(10^{-8}\right) \cdot \beta \cdot \frac{m^2}{N}$$

$$\alpha = 1.622 \times 10^{-8} \frac{m^2}{N}$$

$$G := (\alpha \cdot E_e) \qquad G = 3.708 \times 10^3$$

Determine the load parameter, W
6379 is the estimated axial load in lbs
Q is the load normal to surface
L is the spherical length of roller

$$Q := \frac{\left(\frac{6379 \cdot 4.4482}{15}\right)}{\cos(50 \cdot deg)} \cdot N \qquad Q = 2.943 \times 10^3 N$$

$$L := .0421 \cdot m$$

$$W := \frac{Q}{(E_e \cdot R_r \cdot L)} \qquad W = 2.169 \times 10^{-5}$$

Determine the minimum oil film thickness: h_{min}

$$h_{min} := 2.65 \cdot (U^{0.7}) \cdot (G^{0.54}) \cdot (W^{-.13}) \cdot R_r$$

$$microns := \frac{m}{(10^6)}$$

$$h_{min} = 0.081 \text{ microns}$$

Calculate the frictional moment (torque) values (N mm)

Determine the load independant moment, M_0
where: f_0 taken from table 16
ν is oil kinemetic viscosity
n is rpm
d_m is pitch diameter

$f_0 := .00000033$ $\quad \nu := 1010.1$

$n := .205$ $\quad d_m := 194$

$M_0 := f_0 \cdot (\nu \cdot n)^{.667} \cdot (d_m)^3$

$M_0 = 84.483$ $\quad M_0 = 0.06$ ft-lbs

Determine the load dependant moment. M_1
where: f_1 taken from table 17
P is axial load (N)

$f_1 := .00033$

$P := 6379 \cdot 4.4482$

$M_1 := f_1 \cdot P \cdot d_m$

$M_1 = 1.817 \times 10^3$ $\quad M_1 = 1.32$ ft-lbs

Calculate the total frictional Moment, M

$M := M_0 + M_1$

$M = 1.901 \times 10^3$ $\quad M = 1.38$ ft-lbs