SMA Technical Memorandum 121

Title: Some thoughts on the SGH test data on NYTEX tubes
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Although the cyclic testing and pull-to-failure testing of the NYTEX type-25 tubes show a wide variation in results for the small number of data points available, I believe some useful information can be gleaned from them.

The SGH data available as of Feb. 12, 1998 are tabulated in Tables 1 and 2 (SGH e-mail, attached to Nystrom's Engineering Report, 2/16/98; see also SMA Technical Memo 120). Averaging the **logarithms** of the cyclic data gives the following results (where the lower limits have been treated like regular data):

S (lbs)	log N	n
11,240	2.82 ± 0.62 (0.3)	5
8,992	4.66 ± 0.51 (0.2)	5
6,774	6.11 ± 0.45	5

where N is the number of cycles at stress level S in pounds. These data are plotted in Figure 1. The numbers in parentheses following the rms values are the standard errors of the means ($\sigma/\sqrt{n}$, where n is the number of samples tested). In addition, the value of the pull-to-failure average stress of $15{,}763 \pm 2{,}287$ (808) lbs for the non-cycled tubes is plotted with N = 1. A straight line through the data on a logarithmic scale that gives a "conservative" estimate of the slope (i.e., approximately the smallest slope that is consistent with the data) has the form:

$$\log N = 75.3 - 18.0 \log S \qquad (1)$$

or

$$N = 10^{75.3} S^{-18}$$

The slope of the N/S function seems very high!

The design load (S_D) of tube 25 is 4000 lbs ($10^{3.6}$ lbs) (Davis 1998). This load includes thermal, assembly, gravitational, and hurricane wind loads (56 m/s) combined algebraically. The failure line intercepts the design load of the tube at N = $10^{10.6}$ cycles. This result suggests that if the extrapolation is valid, then: (1) a tube could be stressed to its design limit at the resonant frequency of the BUS for about 130 years before fatigue failure; and (2) because the N/S curve is so steep, low-level stress should have negligible effect on the lifetime of the tube; i.e., tube lifetime is strongly controlled by the number of high-stress events.

The analysis can be cautiously carried a bit farther. There is a very elementary failure-analysis theory, the Palmgren-Minen hypothesis (e.g., Newland 1975), that is essentially based on the simple idea of a linear propagation of damage. The hypothesis is that if a material would fail after N_i cycles at stress level S_i, then it can be expected to fail under the cumulative application of n_i cycles at each stress level when

$$\sum \frac{n_i}{N_i} = 1. \tag{2}$$

If the stress is delivered by a band-limited process with center frequency ν_0 (e.g., resonance frequency) and probability distribution p(S), then the time to failure is given by

$$T = \frac{1}{\nu_0 \int_0^\infty \frac{1}{N(S)} p(S)\, dS} . \tag{3}$$

If $p(S)$ = S(S-So),where δ is a delta function, then equation (3) gives the intuitive result

$$T = \frac{N(S_0)}{\nu_0} . \tag{4}$$

From Figure 1 or equation 1, if S_0 = 4,000 lbs and ν_0 = 10 Hz, then T = $10^{9.6}$ seconds = 133 years.

A commonly assumed form for p(S) is

$$p(S) = \frac{S}{\sigma^2} e^{-\frac{S^2}{2\sigma^2}}. \tag{5}$$

For a general power law N/S function,

$$N = AS^{-B}, \tag{6}$$

substitution of (5) and (6) into (3) gives

$$T = \frac{A}{\nu_0 (2\sigma)^{\frac{B}{2}} \Gamma(\frac{B}{2} + 1)} , \tag{7}$$

where Γ is the Gamma function. If $\frac{B}{2}$ is an integer = m, then

$$T = \frac{A}{\nu_0 2^m \sigma^m m!} (sec). \tag{8}$$

For $\frac{B}{2} = m = 9$, ν_0 = 10 Hz,

$$T_{yrs} = \frac{10^{-6.3}}{\left(\frac{\sigma}{\sigma_0}\right)^{18}} , \tag{9}$$

where σ_0 is the design load of the tube (4000 lbs). T_{yrs} is obviously incredibly sensitive to the parameter σ. The tube would last 30 years if $\sigma/\sigma_0 < 0.37$, or σ = 1500 lbs.

To see if this value is at all reasonable, consider the following. The probability that the stress will exceed S_0, obtained from integration of equation (5), is

$$p(S > S_0) = e^{-\frac{S_0^2}{2\sigma^2}} . \tag{10}$$

Some values are tabulated below.

$\frac{S_0}{\sigma}$	$S_0(\sigma = 1500)$	$p(S > S_0)$
0	0	1
1	1,500	0.6
2	3,000	0.13
3	4,500	0.01

Such a value for σ (1500 lbs) would be consistent with hurricane winds for about 1% of the time. It is clear that a tube is most at risk for failure as a result of rare high-stress events. The figures derived from equation (9) should not be taken too literally, as they depend critically on the form of the probability distribution assumed, and the Rayleigh distribution is probably not accurate for the high-stress regime.

The damage done to the tubes during acceptance or other testing can be estimated for equation (1) in the form

$$\frac{n_T}{N_T} + \frac{1}{N_{f'}} = 1 \; , \tag{11}$$

where n_T is the number of cycles in the testing program at a stress level where N_T cycles produces failure, and $N_{f'}$ is the number of cycles to failure of the "tested tube". Since $N_T = A S_T{}^{-B}$, and $N_{f'} = A S_{f'}{}^{-B}$, where $S_{f'}$ is the failure load of the tested tube,

$$n_T S_T^B + S_{f'}^B = A \tag{12}$$

or

$$\frac{\Delta S_f}{S_f} = 1 - \left[1 - n_T \left(\frac{S_T}{S_f}\right)^B\right]^{\frac{1}{B}} \; , \tag{13}$$

where $\Delta S_f = S_f - S_{f'}$ and S_f is the failure load of the untested tube. This is approximately

$$\frac{\Delta S_f}{S_f} \simeq \frac{n_T}{B}\left(\frac{S_T}{S_f}\right)^B \tag{14}$$

for $n_T\left(\frac{S_T}{S_f}\right)^B \ll 1$. Equation (14) can be converted to a fractional change in number of cycles to failure, $\Delta N_f/N_f$ by use of equation (6),

$$\frac{\Delta N_f}{N_f} \simeq n_T\left(\frac{S_T}{S_f}\right)^B \; . \tag{15}$$

For the acceptance test program, $n_T = 1$; that is, the tube is pulled once to a load $S_T \ll S_f$.

Values of $\Delta S_f/S_f$ and $\Delta N_f/N_f$ are tabulated below.

$\frac{S_T}{S_f}$	$\frac{\Delta S_f}{S_f}$	$\frac{\Delta N_f}{N_f}$
0	0	0
0.2	1.5×10^{-14}	2.6×10^{-13}
0.4	3.8×10^{-9}	6.9×10^{-8}
0.6	5.6×10^{-6}	1.0×10^{-4}
0.8	1.0×10^{-3}	1.8×10^{-2}
0.9	0.9×10^{-2}	0.15
1.0	1	1

Since for our tube sample S_f = 15,763, the decision to test them at 6,000 or 8,000 lbs $(1.5\,S_D$ and 2.0 $S_D)$ corresponds to S_T/S_f < 0.5. Hence this test load should lead to negligible decrease in tube strength $(\Delta S_f/S_f < 10^{-6})$.

Figure 2 shows the failure stress load for tubes that survived the cyclic testing. Any reduction in lifetime is masked by the variation in S_f among tested tubes and by the fact that testing is not expected to harm the tubes unless they are pulled to a stress very close to S_f. The fact that none of the cycled tubes showed any ill effect from cycling confirms that the slope in Figure 1 is indeed a lower limit.

CONCLUSIONS

1. The tubes can be characterized by a fatigue equation of the form

   $$\log N > 75.3 - 18.0 \log S \ .$$

   A type-25 tube could be expected to survive for at least 130 years if it were subjected to a cyclic stress at the design load and the resonance frequency of 10 Hz.

2. Acceptance testing of the tubes to a level of 1.5 – 2.0 S_D should have negligible effect on the strength of the tubes. Note that none of the tubes failed at the 30 kN (6,744 lbs) test level. It is possible that the tubes have a fatigue limit (a stress level below which no damage is done by cycling).

3. The cyclically tested tubes show no decrease in strength at the 10% level. This suggests that the N/S curve is conservative.

REFERENCES

Davis, W. (1998) SMA Backup Structure Tube Loads, SMA Technical Memo 119.

Newland, D. E. (1975) An Introduction to Random Vibrations and Spectral Analysis, Longman: New York, p. 191.

TABLES

Table 1. Pull-to-Failure Test for NYTEX Tube Type 25 (design load = 4000 lbs) as a Function of Previous Tube History

Failure Load (lbs)			
No Cycling	30 kN	40 kN	50 kN
11,418	11,677	15,723	16,251
13,698	14,790	16,380*	16,748*
14,894	15,561		
16,287	15,781”		
17,047	15,793”		
17,081			
17,391			
18,290”			

*stud holding tube broke

Table 2. Cyclic Testing Results for NYTEX Tube Type 25 as a Function of Peak Load†

Number of Cycles to Failure		
30 kN	40 kN	50 kN
4,000,000*	9,498	108
4,000,000*	17,777	174
600,000*	103,730”	760
600,000*	107,200*	2,725*
600,000*	107,372	3,372*

† cycling between 4 kN and peak load in tension
* test stopped without failure

Note: 30 kN = 6744 lbs; 40 kN = 8992 lbs; 50 kN = 11240 lbs

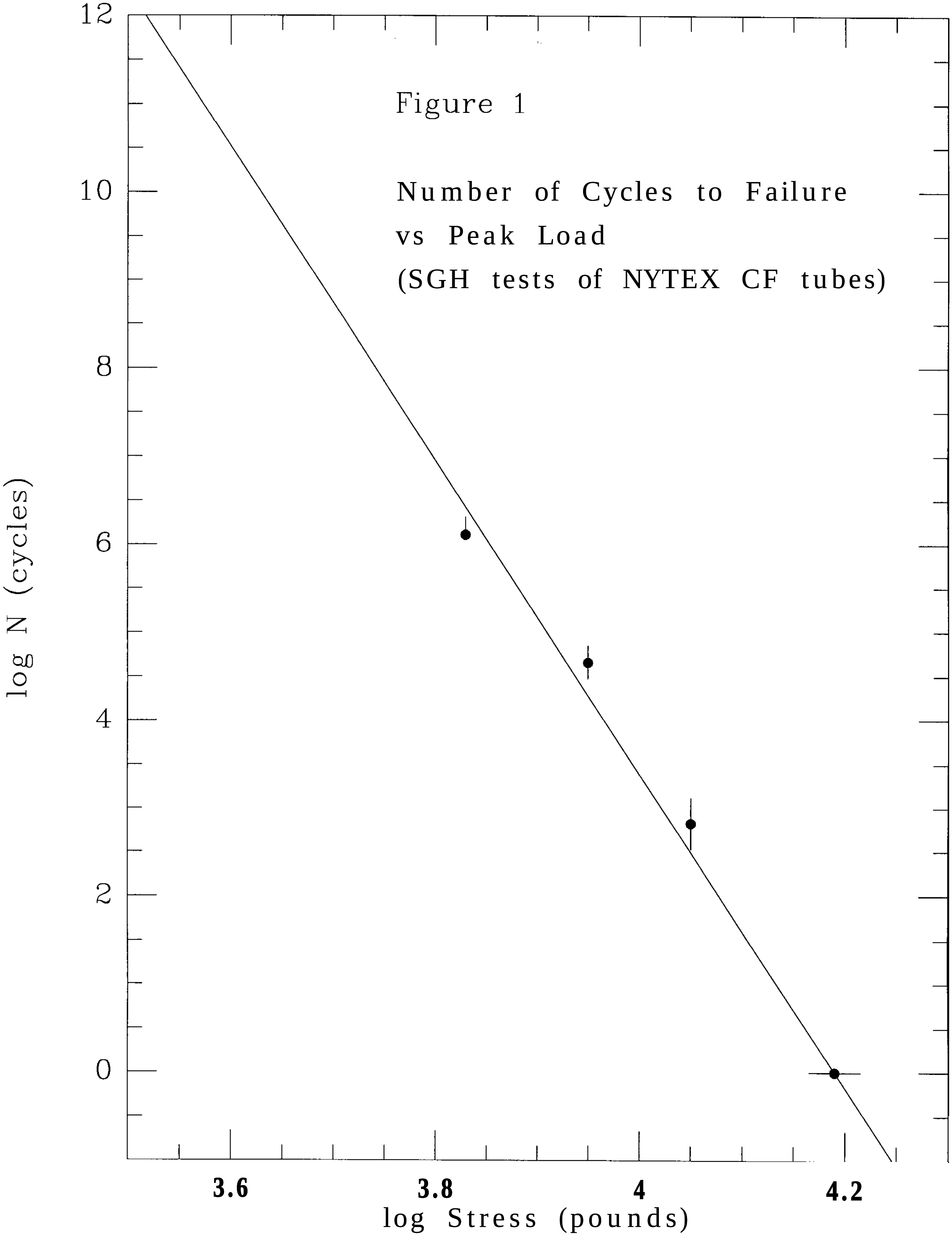
Figure 1
Number of Cycles to Failure
vs Peak Load
(SGH tests of NYTEX CF tubes)
log N (cycles)
12
10
8
6
4
2
0
3.6
3.8
4
4.2
log Stress (pounds)

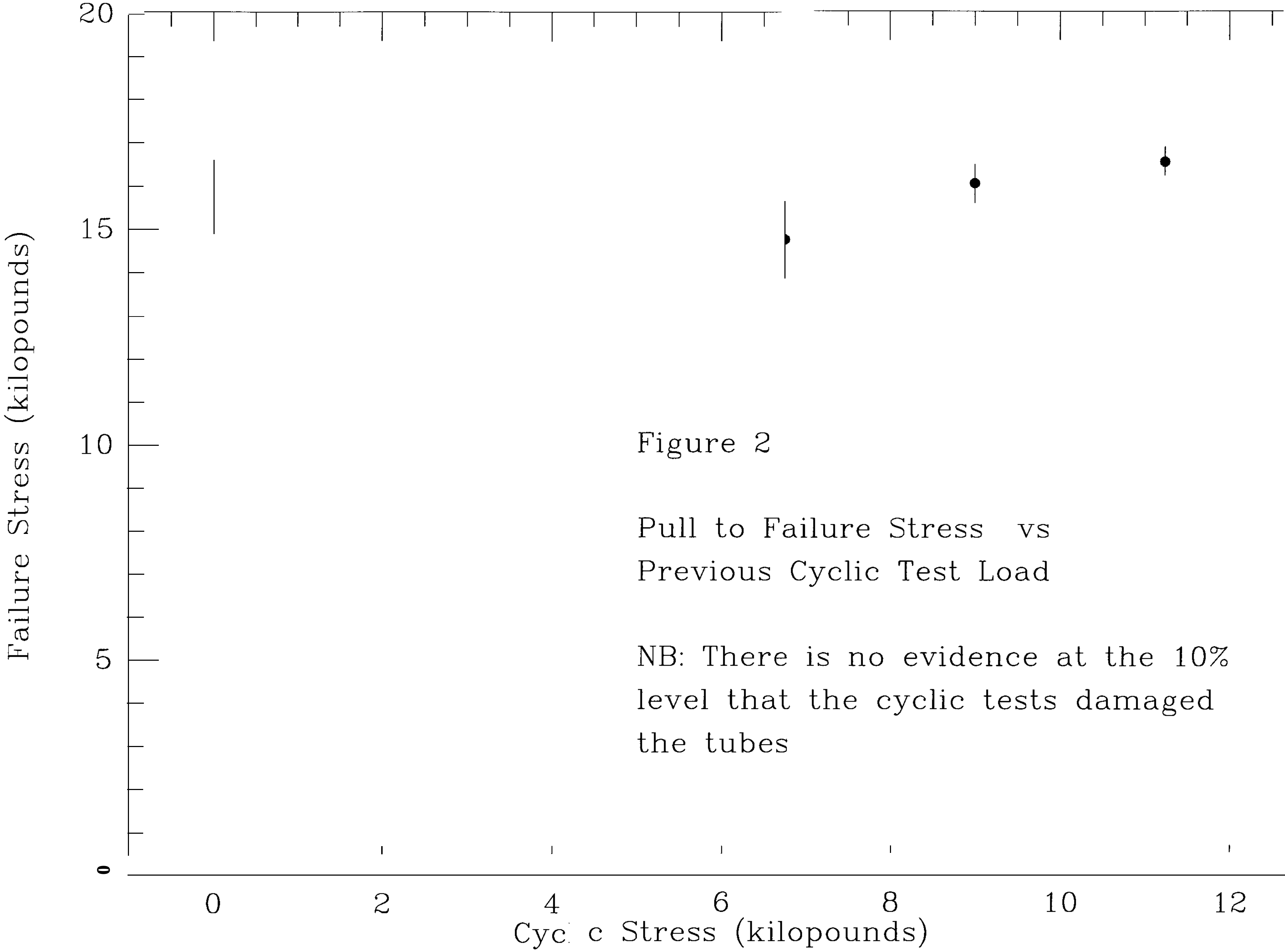

Figure 2

Pull to Failure Stress vs
Previous Cyclic Test Load

NB: There is no evidence at the 10% level that the cyclic tests damaged the tubes