# SMA Technical Memorandum # 118

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**Subject:** Clarification on Reynolds Number and the Correlator Thermal Design

**Ref:** Fluid Mechanics, F.M. White, 1979.

Fundamental of Heat and Mass Transfer, F.P. Incropera and D.P. DeWitt, 2nd ed., 1985.

MIL-HDBK-251, Reliability / Design, Thermal Applications, 19 January, 1978.

Viscous Fluid Flow, F.M. White, 1974.

"Design Considerations for Air Cooling Electronic Systems in High Altitude Conditions", Christian L. Belady, 12th IEEE SEMI-THERM Symposium, 1996.

"Thermal Resistance Testing of the Haystack VLSI Correlator Chip", SMA Technical Memo #110, K. McCracken, July 1, 1997.

"Preliminary Digital Correlator Thermal Analysis", SMA Technical Memo #111, K. McCracken, July 28, 1997.

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## Abstract

This memo will discuss the definition of Reynolds number and how it applies to the forced convection cooled PCBs in the digital correlator. It will then compare the pertinent air properties and their changes from sea level to altitude. The basic equations for calculating component surface temperatures will then be examined and reduced to equations with terms that vary with altitude and terms which remain constant with altitude. Finally, the thermal resistance measurement technique of Tech Memo #110 will be compared to the method of component temperature calculations. It will be shown that the dominant terms for the calculation method also apply to the resistance measurements made.

## Introduction

During the December 21, 1997, Control Building A/C Specification Review meeting, the issue was raised about whether or not the Reynolds number was properly accounted for in the thermal design and analysis of the correlator. The following memo provides background on the determination of the applicable fluid properties, heat transfer theory and procedure used to arrive at a thermal design analysis approach for the forced convection cooled digital correlator PCBs.

## Background

When fluids flow over an object or through a duct, its movement can be described as a continual shearing of the fluid. The fluid essentially sticks to the surface of the object or duct (velocity equals zero) and the fluid’s velocity continues to increase as you move away from the surface. It is this "stickiness" or resistance to being sheared that quantifies a fluid’s viscosity.

The Reynolds Number is a nondimensional correlation of the viscosity of a flowing fluid to the average velocity and geometry of the flow. It is defined as

Reynolds Number, Re ≡ (density)(velocity)(characteristic length) / (dynamic viscosity)

= (velocity)(characteristic length) / (kinematic viscosity)

The Reynolds number is used for fluid problems to characterize fluid flows into one of three possible categories. Laminar flow is described as being very smooth and steady with the fluid velocity primarily in one direction at any given point within the flow. Turbulent flow is characterized by flow accompanied by random fluctuations "ranging from one to 20 percent of the average velocity" (p. 305, Fluid Mechanics). Transitional flow is the flow that occurs before the onset of turbulence. This is a very unpredictable situation and will not be examined further by this memo. Heat transfer correlations are also based on determining the correct flow characterization. Table One lists the values of Reynolds number for both internal and external flow and the corresponding flow situation. Note, the internal flow condition corresponds to flow through a duct and the external flow condition corresponds to flow over a flat plate.

Reynolds number

| | Internal Flow | External Flow |
|---|---|---|
| Laminar Flow | $0 < Re < 2300$ | $0 < Re < 500000$ |
| Transitional Flow | $2300 < Re < 4000$ | $Re \cong 500000$ (typical transition Re) |
| Turbulent Flow | $4000 < Re < \infty$ | $1000000 < Re < \infty$ |

Table One - Reynolds number and Corresponding Flow Condition
(p. 321, 369, Incropera, p. 401, Fluid Mechanics)

## Sea Level to Altitude Comparison Approach

Since the system is currently located at sea level and most system and component testing will be done at sea level, a way to compare testing done at sea level to expected results at altitude must be devised. First, the applicable air properties and their dependance on pressure (altitude) and temperature will be determined. Then the relations which describe the heat transfer of the system will be examined to determine the governing parameters. Finally, the relations can then be compared at sea level and at altitude to determine the affect the altitude change will have on the system's heat transfer behavior. This will be done by ignoring the temperature affects (basically stating they would remain as constants) and varying the pressure dependant terms in the heat transfer relations.

Since the physical geometry of the chips, PCBs, and correlator crate will not change from sea level to the Mauna Kea site, the terms describing the geometry at sea level will be identical to the terms at altitude (and can also be considered constants).

## Mauna Kea Site and Air Properties

The SMA site is at 4100 meters (13,451 feet) altitude. The atmospheric pressure at that altitude can be found by interpolating the U.S. Standard Atmosphere, (Table 5, p. 680, Fluid Mechanics). The atmospheric pressure is 8.829 psia. And from p.120, MIL-HDBK-251, the density of dry air is given by the following equation

density of dry air = 0.0807 (273/(T + 273))(p / 14.7)

where p = atmospheric pressure (psia)
T = air temperature (°C)
density = lb/cu. ft

From this equation, it can be shown that for any given air temperature, the dry air density at our site is 0.601(or, 8.829/14.7) times the density at sea level and is a function of altitude.

The additional relevant air properties for this analysis are the conductivity and the dynamic viscosity. The specific heat and Prandlt No. (Pr) are both calculated from the three previous properties. Both the conductivity and the dynamic viscosity are function of temperature only for the pressures and temperatures that concerns this analysis. The fluid conductivity, k, can be described by a power law relation based on temperature only that is accurate within ±3% over the temperature range of -85°F to 1340°F (p. 32, Viscous Fluid Flow). The dynamic viscosity, $\mu$, can also be calculated from a power law relation that is accurate within ±4% over the temperature range of -82°F to 2960°F (p. 29, Viscous Fluid Flow).

The kinematic viscosity, $\nu$, is equal to the dynamic viscosity divided by the density. Since the density is a function of both temperature and pressure, the kinematic viscosity is also a function of the altitude. The specific heat, Cp, is defined as the amount of heat required to raise a unit mass of the material one degree at constant pressure and is a function of temperature only. The Prandlt No. (Pr) reduces to the dynamic viscosity times the specific heat divided by the conductivity. All are function of temperature only and so is the Pr.

## Heat Transfer Analysis

There are two major parts of the calculations which determine the component surface temperature in a forced air cooled design. First, there is the temperature rise of the air as it passes through the chassis or rack. This air temperature rise is calculated by determining the amount of heat put into the airstream before reaching the component or location in question. Knowing the mass flow of the fluid and its heat capacity a calorimetry equation is then used to determine the air temperature rise. Second, the convection coefficient must be determined. By knowing the system geometry and the fluid properties a temperature rise from the air passing over a component to the surface can be calculated.

Tcomponent surface = Tair entering unit + Trise of air through chassis + Trise from air to comp. surface

Both parts of the component surface temperature calculation will be examined for their dependance on altitude.

### Air Temperature Rise through chassis

The temperature rise for a given amount of heated air is described by a simple calorimetry equation.

Air Temperature Rise = Heat / ((mass flow) (specific heat))

However, most manufacturers list their fan data as volume flow rather than mass flow. Volume flow is simply mass flow divided by the density. The calorimetry equation becomes

Air Temperature Rise = Heat / ((Vol. Flow )(density)(specific heat))

To compare the sea level and altitude differences on the calorimetry equation, assume the power dissipation is constant. Also, as was shown in the Air Properties section, the specific heat is a function of temperature only. So, to obtain the equivalent cooling at altitude as would occur at sea level, the air temperature rise at sea level will be set equal to the air temperature rise at altitude. In equation form this becomes

$$\text{Heat}_{\text{sea level}} / (\text{Vol. Flow}_{\text{sea level}}\ \text{Dens.}_{\text{sea level}}\ \text{Cp}_{\text{sea level}}) = \text{Heat}_{\text{altitude}} / (\text{Vol. Flow}_{\text{altitude}}\ \text{Dens.}_{\text{altitude}}\ \text{Cp}_{\text{altitude}})$$

Eliminating constant terms yields

$$\text{Volume Flow}_{\text{sea level}}\ \text{Density}_{\text{sea level}} = \text{Volume Flow}_{\text{altitude}}\ \text{Density}_{\text{altitude}}$$

The convection heat transfer relations most often use the air velocity over the components or PCB to calculate a heat transfer coefficient. The average velocity of the air flowing through the duct created between two PCBs can be calculated by the following equation.

$$\text{Air Velocity} = \text{Vol. Flow} / \text{Cross-sectional Area available for Flow}$$

Since the Cross-sectional area is a geometry term and is constant from sea level to altitude, substitution of the equation for air velocity into the calorimetry equation comparing sea level to altitude yields (neglecting constants)

$$\text{Air Velocity}_{\text{sea level}}\ \text{Density}_{\text{sea level}} = \text{Air Velocity}_{\text{altitude}}\ \text{Density}_{\text{altitude}}$$

Solving for the sea level velocity yields

$$\text{Air Velocity}_{\text{sea level}} = \text{Air Velocity}_{\text{altitude}}\ (\text{Density}_{\text{altitude}} / (\text{Density})_{\text{sea level}})$$

From the Site and Air Properties section, the ratio of density at 4100m altitude to sea level is 0.601. Substituting this value produces an equation that relates sea level air velocity to an equivalent air velocity at 4100m altitude based on air temperature rise (calorimetry).

$$\mathbf{Air\ Velocity_{\ sea\ level} = 0.601\ (Air\ Velocity_{\ 4100\ meters})}$$

### Convection Heat Transfer or Temperature rise from Component Surface to Air

Since the internal air flow around PCBs and chips is extremely variable, comparisons of laminar and turbulent air flow will be made. As was done in the "Design Considerations ... Memo" by Belady, laminar and turbulent internal heat transfer will be compared to determine the dominant parameters and their dependance on altitude. However, PCB and component convection coefficients are also calculated based on external flow relations, such as flow over a flat plate. This memo will examine both types of calculations, however, transitional flow calculations will be ignored.

Convection heat transfer is described by the following equation

$$Q = h\, A\, (T_{surface} - T_{air})$$

where $h \equiv$ average convection coefficient
$A \equiv$ surface area of device exposed to the air
$Q \equiv$ heat transfer between the surface and the air

Also, the convection coefficient can be defined in terms of another dimensionless parameter, the Nusselt number, Nu.

$$h = Nu\ k / L$$

where $Nu \equiv$ dimensionless temperature gradient at the surface
$k \equiv$ fluid conductivity
$L \equiv$ characteristic length of the geometry

### Laminar Internal Flow

From p. 389, Incropera, the Nu is shown to be constant for laminar, fully developed internal flow. Substituting a constant into the equation for the convection coefficient yields

$$h = \text{Constant}\ (k / L)$$

And since the fluid conductivity is dependant only on temperature, **the result is that for laminar internal flow the heat transfer coefficient is only dependent on geometry and temperature, not altitude.**

### Laminar External Flow

By examining a Nu for external flow, a similar relationship describing the convection coefficient in terms of constants and properties dependant on pressure and temperature can be found. From p. 318, Incropera, the average Nu for laminar convection off a flat plate is given by

$$Nu = .664 (Re^{0.5})(Pr^{0.33})$$

Substituting this equation into the convection coefficient equation yields

$$h = .664 (Re^{0.5})(Pr^{0.33}) k / L$$

Since the Prandlt No. (Pr) and the fluid conductivity (k) are both functions of temperature only for air and the length (L) is a constant, the equation for laminar external flow becomes

$$h = \text{Constant} (Re^{0.5})$$

Substituting the definition of the Reynolds No. yields

$$h = \text{Constant } [\ ((\text{density})(\text{velocity})(\text{characteristic length})) / \text{dynamic viscosity}\ ]^{0.5}$$

Since the characteristic length is a function of geometry (constant for both s.l. and altitude), and the dynamic viscosity is a function of temperature only, the equation can be rewritten as

$$h = \text{Constant} (\text{density}^{0.5})(\text{velocity}^{0.5})$$

**Therefore, for laminar external flow, the heat transfer coefficient is dependant on altitude and air velocity** , in addition to geometry and temperature.

## Turbulent Internal Flow

From p. 394, Incropera, the local Nusselt No. for turbulent, fully developed flow can be described from the Dittus-Boelter equation

$$Nu = .023 (Re^{0.8}) (Pr^{0.4})$$

Substituting this Nusselt No. equation into the convection coefficient equation yields

$$h = .023 (Re^{0.8}) (Pr^{0.4}) k / D \qquad \text{where } D \equiv \text{diameter}$$

Again, since the Prandlt No. (Pr) and the fluid conductivity (k) are both functions of temperature only for air and the diameter can be assumed constant, the equation of convection coefficient for turbulent, internal flow becomes

$$h = \text{Constant} (Re^{0.8})$$

Similar to the previous case, substituting the definition of Re and rewriting yields

$$h = \text{Constant} (\text{density}^{0.8})(\text{velocity}^{0.8})$$

**Therefore, for turbulent internal flow, the heat transfer coefficient is dependant on altitude and air velocity** , in addition to geometry and temperature.

## Turbulent External Flow

From p. 319, Incropera, the local Nusselt No. for turbulent flow is described by

$$Nu = .0296\,(Re^{0.8})\,(Pr^{0.33})$$

Again, as was shown for the turbulent internal flow case, the convection coefficient becomes a function of a constant times ($Re^{0.8}$).

$$h = \text{Constant}\,(Re^{0.8})$$

Similar to the previous case, substituting the definition of Re and rewriting yields

$$h = \text{Constant}\,(\text{density}^{0.8})(\text{velocity}^{0.8})$$

**Therefore, for turbulent external flow, the heat transfer coefficient is dependant on altitude and air velocity** , in addition to geometry and temperature.

| Flow Condition | | Convection Coefficient Dependance |
|---|---|---|
| Laminar | Internal Flow | h = constant |
| | External Flow | $h \propto (\text{density}^{0.5})(\text{velocity}^{0.5})$ |
| Turbulent | Internal Flow | $h \propto (\text{density}^{0.8})(\text{velocity}^{0.8})$ |
| | External Flow | $h \propto (\text{density}^{0.8})(\text{velocity}^{0.8})$ |

Table Two - Summary of Convection Coefficient Dependance

The next step is to take the convection dependance results and determine the conditions at sea level that will result in an equivalent convection coefficient at altitude. First, the temperature is assumed constant (20°C) and the density is calculated for both sea level and 4100m altitude (p. 120, MIL-HDBK-251).

$\text{density}_{\text{sea level}}$, 20°C = .0752 lb/cu ft

$\text{density}_{\text{4100m}}$, 20°C = .0452 lb/cu ft

Second, set the sea level convection coefficient equal to the convection coefficient at 4100m. This will not be done for the laminar internal flow case because it has already been determined that the convection coefficient is constant (not dependant on altitude). For the laminar external flow case,

$$h_{\text{sea level}} = h_{\text{4100m}}$$

This can also be written as (neglecting the constant terms)

$$(\text{density}_{\text{sea level}})^{0.5}(\text{velocity}_{\text{sea level}})^{0.5} = (\text{density}_{\text{4100m}})^{0.5}(\text{velocity}_{\text{4100m}})^{0.5}$$

Solving for velocity at sea level yields

$$\text{velocity}_{\text{sea level}} = ((\text{density}_{\text{4100m}})^{0.5}(\text{velocity}_{\text{4100m}})^{0.5} / (\text{density}_{\text{sea level}})^{0.5})^2$$

Substituting in the density values above reduces the equation to

**Air Velocity $_{\text{sea level}}$ = .601 (Air Velocity $_{\text{4100m}}$)**

So to obtain the identical convection coefficient at sea level as at 4100m for laminar internal flow, use a sea level test velocity that is 60.1% of the anticipated velocity at 4100m altitude. This ratio of the densities is the same result found in the Air Temperature Rise section.

Since both turbulent convection coefficient relations are dependant on density and velocity to the same power, examine their situation simultaneously. As was done above for laminar flow

$h_{\text{sea level}} = h_{\text{4100m}}$

$\text{velocity}_{\text{sea level}} = ((\text{density}_{\text{4100m}})^{0.8}(\text{velocity}_{\text{4100m}})^{0.8} / (\text{density}_{\text{sea level}})^{0.8})^{1.25}$

Substituting in the density values reduce the equation to

**Air Velocity $_{\text{sea level}}$ = .601 (Air Velocity $_{\text{4100m}}$)**

Both turbulent convection coefficient relationships agree with the findings in the Air Temperature Rise section.

## Final Comparison of Thermal Resistance to Convection Calculations

Finally, the thermal resistances and the convection calculations must be compared. From the Convection Heat Transfer section the heat flowing out of a surface by convection was given by

$$Q = h A (T_{\text{surface}} - T_{\text{air}})$$

The thermal resistance measurements of SMA Technical Memo #110 were based on measurements of the junction temperature, the top and bottom case temperatures and knowing the input power. Writing this in equation form for the thermal resistance top case to ambient yields

$$Q = (T_{\text{case top}} - T_{\text{air}}) / R_{\text{case top to air}}$$

By examining these two equations it is evident that the resistance can be written as

$$R_{\text{case top to air}} = 1 / h A$$

Since the component area is a constant, a theoretical prediction that is made for the convection coefficient should also be applicable to a thermal resistance measurement that is dominated by convection.

## Conclusions

**This analysis has shown that the convection coefficient for laminar flow is either constant or dependant on the ratio of the density at altitude to the density at sea level. Likewise, the convection coefficients for turbulent flow are also shown to be dependent on the ratio of the density at altitude to the density at sea level.**

In SMA Technical Memo #111, the junction temperature prediction for the Haystack correlator chips are based on interpreting the thermal resistance test data (from SMA Technical Memo #110) made at sea level. While the thermal resistance measurements (Rjunction to ambient) include the conduction through the package to the die, they primarily account for all the convection and area affects. The resistance junction to ambient used for SMA Tech Memo #111 altitude temperature predictions were based on the sea level resistance value determined at 59.4% of the altitude air velocity. The difference in the ratios (59.4% to 60.1%) comes from the differences in the altitude (14,000 ft to 4100m) assumed. These previous calculations are consistent with the findings of this memo.

In addition, if the convection coefficient truly is constant (not dependent on altitude) as in the laminar internal flow case, then the temperature predictions are very conservative.