# TECHNICAL MEMORANDUM

SMA Technical report number: 114 Date: 9 July 1997

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Subject: SMA Azimuth bearing

**Introduction:**

The Azimuth bearing manufactured by Avon Bearings, Inc., experienced problems when assembled to the base pedestal and upper structure. The problems being:

- Excessive friction, both starting and running.
- Excessive frictional variation during a rotation.
- "Lockup"(required extremely high torque to break free) after a small rotation.
- Non compliance with the moment stiffness specification.

Initially it was felt that the assembly procedure and possibly the machining of the mating parts where at fault. Various attempts where made to measure/ lap the mating surfaces and different assembly techniques where tried. Also discussions between Avon Bearings technical staff, their consultants, and SAO were held. All the above efforts resulted in no definitive conclusions for the bearings non-conformance to specifications and/or the problems experienced. Therefore, this analysis has been undertaken to validate the bearings design and compute various bearing parameters while under full loading . The results and conclusions from that analysis are presented here.

**Approach:**

Several models were developed to compute stresses and resulting deflections in the bearings' components. These models where: inner race (ring), outer race (ring), ball/race indent, bolts (inner and outer races) and the race (or rings) axial deflection between bolts. Also investigated were the industry standards for mating surfaces, assembly techniques and lubrications.

**Modelling and hand calculation results:**

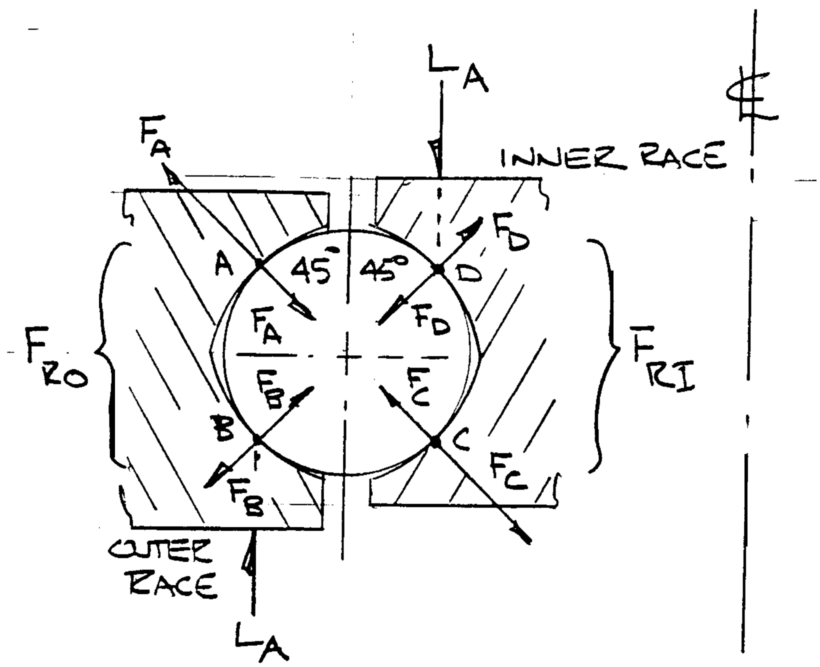
Tom and I met several times to discuss these problems. We independently made calculations to confirm the bearings' design and our results. We now agree and the results are presented below. Reference 1, plots the ball/race indent calculations. References 2 and 3, plot the load versus radial displacement for the inner and outer races. The mounting stiffness created by the bolts at each interface is $2.1\times10^{8}$ lbs/in. The races axial deflection between bolts is $1.78 \times 10^{8}$ lbs/in. These both greatly exceed the ball/race stiffnesses and are not considered to be significant to the overall moment stiffness.

Some observations from above are:

- That the radial spring constants derived for the inner and outer races per references 2 and 3 are essentially equal and deviate from an average of 5.45 x $10^5$ lbs/in by less than 6 %.

- that the total indent spring constant- ie. two identical indents in series, averages about 5.9 x $10^5$ lb/in within the load range from 0 up to 800 lbs. While varying from about 1.9 x 1 $0^5$ up to about 8.7 x $10^5$ lb/in, a factor of about 4.5 to one, over the required load range.

Several definitions need to be made because of the geometry involved, thus:




**For forces:**

$F_A = F_C \cong F_A$ & $F_B = F_D \cong F_B$
$F_{RO} = F_A$ Cos 4.5' + $F_B$ Cos 45”
Therefore $F_{RO} = 1/\sqrt{2}$ (F, + $F_B$)

**Likewise:**

$F_{RI} = F_C$ Cos 45” + $F_D$ Cos 45°
Therefore $F_{RI} = 1/\sqrt{2}$ (F, + $F_B$)

Assuming perfect geometry with no external loading applied (ie. $L_A$ = 0), a preload condition exists due to an interference fit between the ball and two races where the preload forces are identified as prime, then:

$F_A' = F_B' \cong F'$
And $F_{RO} = \sqrt{2}$ x F’ & $F_{RI} = \sqrt{2}$ x F’ therefore $F_{RO} = F_{RI} \cong F_R' = \sqrt{2}$ x F’

When an external axial load , $L_A$ is applied as shown:

$F_A$ = F’ + $L_A$/Cos 45” = F’ + $\sqrt{2}$ x $L_A$

And

$F_{RO} = 1/\sqrt{2}$ x (F’ + $\sqrt{2}$ x $L_A$ + $F_B$)

Also

$F_{RI} = 1/\sqrt{2}$ x (F’ + $\sqrt{2}$ x $L_A$ + $F_B$)

Since the spring constants of the inner and outer races are linear and essentially equal, the expansion of the outer race caused by the $\sqrt{2} \times L_A$ loading increment will reduce $F_B$ by the same amount so that $F_B = F' - \sqrt{2} \times L_A$ , Therefore $F_{RO} = 1/\sqrt{2} \times \left[(F' + \sqrt{2} \times L_A) + (F' - \sqrt{2} \times L_A)\right]$ or h o $=\sqrt{2} \times F'$ . Likewise, $F_{RI} = \sqrt{2} \times F' = F_{RO}$ .

All the above applies as long as $\sqrt{2} \times L_A \leq F'$ ! When $\sqrt{2} \times L_A \geq F'$, the preload contact is lost at points A & C and the bearing no longer has parallel load paths.

The fact that the indent spring constant is non-linear adds a second order effect into the above. However, we can consider it to be constant around a preload value or at other operational values.

The new bearing received recently (see gn- Avon trip report) has a preload value of .0035 thousands of and inch. Then for this preload value (.0035), solve for F' by referring to the plots given in references 1-3.

**For deflections at preload:**

.0035 = double indent + expansion of outer race/ Cos 45" + compression of inner race/ Cos 45°

$.0035 = ( F' / 5.9 \times 10^5) + ( F_{RO} / (5.45 \times 10^5 )(1/\sqrt{2}) + (F, / (5.45 \times 10^5 )(1/\sqrt{2})$

Collecting terms and solving for F'

$F' = 3.50 \text{ X } 10^3 / 9.678 \times 10^{-6} = 387$ lbs.

**Check using model references:**

Reference 1, double indent at 387 pounds $\cong$ 0.0008
References 2 &3 @ $F_{RO} = F_{RI} = 2$ *(F 1J2)* = 540 lbs then;
$\Delta$ R $\cong$ .001 1 in each or 2 x .0011/Cos 45" $\cong$ 0.003 1 for a total preload of 0.0039 which is too high. The models predict a F' = 350. The slight difference results from averaging the races.

With the 0.0035 preload established as F' = 350 lbs. and adding an axial load, $L_A$ = 440 lbs.

Where $L_A$ = total dead weight on bearing / number of balls in bearing

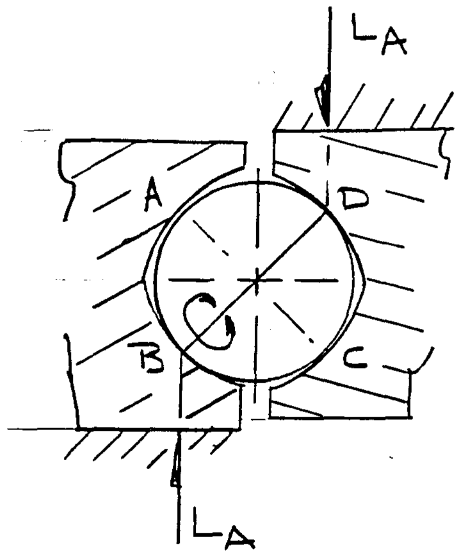
$L_A$ = 56760 lbs / 129 = 440 lbs per ball.

then $F_A = F' + \sqrt{2} \times L_A = 350 + 622 = 972$ lbs.

However; $F_A = F' \cdot \sqrt{2} \times L_A$ because the added radial force due to $\sqrt{2} \times L_A$ at point B expands the outer race and thereby relieves the contact force at point A by the same amount, ie $\sqrt{2} \times L_A$, because of the races' radial spring constants linearity.

Since $\sqrt{2} \times L_A = 622$ Ibs is greater than F' of 350 lbs. means that there will be no contact at point A! Therefore the geometry becomes:

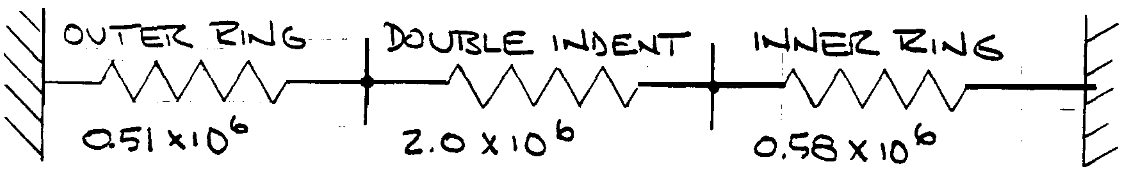
In this configuration the ball can roll freely about its A-C axis with no contact and hence friction at points A-C.




Also, at this total load of $F_B = 972$ lbs., the double indent spring constant increases to about $2 \times 10^6$ lb/in.

The compliance configuration now becomes much simpler. There are three springs in series; the outer race, the double indent and the inner race.

**Thus:**




Then the overall spring constant end to end, $K_{A\text{-}C}$

$$K_{A\text{-}C} = \sum 1/(1/K) = 1/(1/.51 \times 10^6) + (1/\ 2.0 \times 10^6) + (1/.58 \times 10^6)$$

$$K_{A\text{-}C} = 2.38 \times 10^5$$

**Modelling and hand calculation results. continued:**

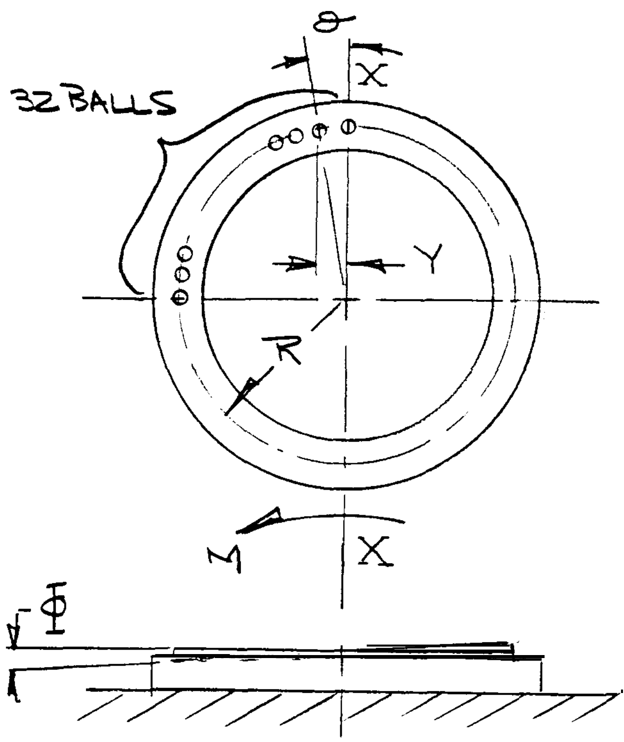
Since $K_{A-C} = \Delta F_A / \Delta d_{A-C}$ and $\Delta F_A \text{Cos } 45” = \Delta L_A$ & $\Delta d_{A-C} \text{Cos } 45” = \Delta V$

Define overall vertical load spring constant thus:

$$K_A = \Delta L_A / \Delta V = \Delta F_A \text{Cos } 45° / \Delta d_{A-C} \text{Cos } 45”$$

Therefore: $K_A = K_{A-C} = 2.38 \times 10^5$ lb/in
This spring constant exists at each and every ball location!

The bearings moment stiffness can now be determined since the axial stiffness is known at each ball location. Assume bearing races plus their backup structures are infinitely stiff and hence remain planar. The Figure below shows the bearing in plan view:




**For moment. M. about x-x axis:**

$$M = \sum_{n=1\text{-}128} L_{An} \times Y_n$$

$$M = 4\sum_{n=1\text{-}32} L_{An} \times Y_n - 2\, L_{AMAX} \times R$$

**Note: 128** balls was used for ease of analysis versus the 129 actual balls.

**And moment stiffness:**

$$K_{X\text{-}X} = M/\phi$$

For any ball n, $L_{An} = K_A \times \Delta V_n$, where $\Delta V_n = \phi \times Y_n$

collecting terms and simplifying: $K_{X\text{-}X} = 2\, K_A [\, 2 \sum Y^2_n - R^2 \,]$

$K_{X\text{-}X} = 2.07 \times 10^{10}$ see Table 1 for computation values

Evaluate $\sum y_n^2$ where $y_n = R \times \sin \theta_n$ and R=37.50 (ball radius)

| Then for n = | $\theta_n$ | $y_n$ | $(y_n)^2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 1 | 2.8125 | 1.8400377872782 | 3.3857390586116 |
| 2 | 5.625 | 3.6756427623585 | 13.510349716479 |
| 3 | 8.4375 | 5.5023927920761 | 30.276326438291 |
| 4 | 11.25 | 7.3158870756048 | 53.522203703001 |
| 5 | 14.0625 | 9.1117567463724 | 83.024111005063 |
| 6 | 16.875 | 10.885675397042 | 118.49792884977 |
| 7 | 19.6875 | 12.633369502208 | 159.60202497933 |
| 8 | 22.5 | 14.350628713691 | 205.94054447821 |
| 9 | 25.3125 | 16.033316003636 | 257.06722207244 |
| 10 | 28.125 | 17.677377630975 | 312.48967990809 |
| 11 | 30.9375 | 19.278852907246 | 371.67416941922 |
| 12 | 33.75 | 20.833883738235 | 434.0507116183 |
| 13 | 36.5625 | 22.338723918466 | 499.01858630546 |
| 14 | 39.375 | 23.789748156137 | 565.95211733241 |
| 15 | 42.1875 | 25.183460806763 | 634.20669820578 |
| 16 | 45 | 26.516504294496 | 703.125 |
| 17 | 47.8125 | 27.785667200811 | 772.04330179422 |
| 18 | 50.625 | 28.987892001103 | 840.29788266759 |
| 19 | 53.4375 | 30.120282430524 | 907.23141369454 |
| 20 | 56.25 | 31.180110461345 | 972.1992883817 |
| 21 | 59.0625 | 32.16482287501 | 1034.5758305808 |
| 22 | 61.875 | 33.072047413063 | 1093.7603200919 |
| 23 | 64.6875 | 33.899598492129 | 1149.1827779276 |
| 24 | 67.5 | 34.645482469173 | 1200.3094555218 |
| 25 | 70.3125 | 35.307902444363 | 1246.6479750207 |
| 26 | 73.125 | 35.885262589958 | 1287.7520711502 |
| 27 | 75.9375 | 36.376171994795 | 1323.2258889949 |
| 28 | 78.75 | 36.779448015121 | 1352.727796297 |
| 29 | 81.5625 | 37.094119123679 | 1375.9736735617 |
| 30 | 84.375 | 37.319427250207 | 1392.7396502835 |
| 31 | 87.1875 | 37.454829607694 | 1402.8642609414 |
| 32 | 90 | 37.5 | 1406.25 |

$$\sum (y_n)^2 = 23203.125$$

**TABLE**

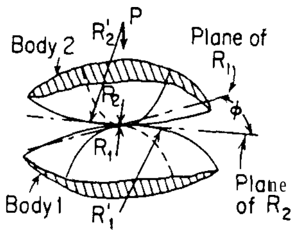
## Models:

### Ball/Race:

The race/ball indent was calculated by constructing a math model using Roarks- Formulas for stress and strain, table 33, Case 4 as a basis. Roarks' formulas are shown below. Load values were varied with the resulting indent calculated and plotted. Reference 1, is a plot of the calculated data.

Conditions and case no.

4. General case of two bodies in contact; $P$ = total load




Formulas

At point of contact minimum and maximum radii of curvature are $R_1$ and $R'_1$ for body 1, and $R_2$ and $R'_2$ for body 2. Then $1/R_1$ and $1/R'_1$ are principal curvatures of body 1, and $1/R_2$ and $1/R'_2$ of body 2; andn each body the principal curvatures are mutually perpendicular. The radii are positive if the center of curvature lies within the given body, i.e., the surface is convex, and negative otherwise. The plane containing curvature $1/R_1$ in body 1 makes with the plane containing curvature $1/R_2$ in body 2 the angle $\phi$. Then:

$$c = \alpha \sqrt[3]{PK_DC_E} \qquad d = \beta \sqrt[3]{PK_DC_E} \qquad \max \sigma_c = \frac{1.5P}{\pi cd} \quad \text{and} \quad y = \lambda \sqrt[3]{\frac{P^2C_E^2}{K_D}} \text{ where } K_D = \frac{1.5}{1/R_1 + 1/R_2 + 1/R'_1 + 1/R'_2}$$

and $\alpha$, $\beta$, and $\lambda$ are given by the following table in which

$$\cos\theta = \frac{K_D}{1.5}\sqrt{\left(\frac{1}{R_1} - \frac{1}{R'_1}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R'_2}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R'_1}\right)\left(\frac{1}{R_2} - \frac{1}{R'_2}\right)\cos 2\phi}$$

| $\cos\theta$ | 0.00 | 0.10 | 0.20 | 0.30 | 0.40 | 0.50 | 0.60 | 0.70 | 0.75 | 0.80 | 0.85 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 0.99 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 1.000 | 1.070 | 1.150 | 1.242 | 1.351 | 1.486 | 1.661 | 1.905 | 2.072 | 2.292 | 2.600 | 3.093 | 3.396 | 3.824 | 4.508 | 5.937 | 7.774 |
| $\beta$ | 1.000 | 0.936 | 0.878 | 0.822 | 0.769 | 0.717 | 0.664 | 0.608 | 0.578 | 0.544 | 0.507 | 0.461 | 0.438 | 0.412 | 0.378 | 0.328 | 0.287 |
| $\lambda$ | 0.750 | 0.748 | 0.743 | 0.734 | 0.721 | 0.703 | 0.678 | 0.644 | 0.622 | 0.594 | 0.559 | 0.510 | 0.484 | 0.452 | 0.410 | 0.345 | 0.288 |

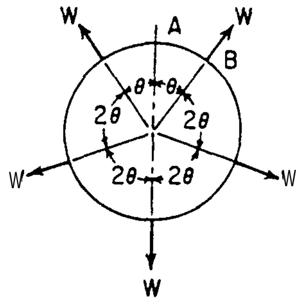
(Ref. 8)

## Races (rings-inner and outer]

The ring deflections where modelled using Roarks-Formulas for stress and strain, table 17, case 7 as a basis. Roarks' formulas are shown below. The model varies the deflection and calculates the required load. References 2 and 3 are plots of the calculated values for each ring. Shear and Hoop Stresses along with bending moments were also calculated and are all nominal and well within the design allowables.

**Reference no., loading, and load terms**

**7. Ring under any number of equal radial forces equally spaced**




**Formulas for moments, loads, and deformations and some selected numerical values**

For $0 < x < \theta$ $\quad M = \dfrac{WR(u/s - k_2/\theta)}{2} \quad N = \dfrac{Wu}{2s} \quad V = \dfrac{-Wz}{2s}$

Max $+M = M_A = \dfrac{WR(1/s - k_2/\theta)}{2}$ $\quad$ Max $-M = \dfrac{-WR}{2}\left(\dfrac{k_2}{\theta} - \dfrac{c}{s}\right)$ at each load position

Radial displacement at each load point $= \Delta R_B = \dfrac{WR^3}{EI}\left[\dfrac{k_1(\theta - sc)}{4s^2} + \dfrac{k_2 c}{2s} - \dfrac{k_2^2}{2\theta}\right]$

Radial displacement at $x = 0, 2\theta, \ldots = \Delta R_A = \dfrac{-WR^3}{EI}\left[\dfrac{k_1(s - \theta c)}{4s^2} - \dfrac{k_2}{2s} + \dfrac{k_2^2}{2\theta}\right]$

If $a = \beta = 0$, $M = K_M WR$, $AR = K_{\Delta R} WR^3/EI$

| e | 15" | 30" | 45" | 60° | 90" |
|---|---|---|---|---|---|
| $K_{MA}$ | 0.02 199 | 0.04507 | 0.07049 | 0.09989 | 0.18169 |
| $K_{MB}$ | -0.04383 | − 0.08890 | -0.13662 | -0.18879 | -0.31831 |
| $K_{\Delta RB}$ | 0.00020 | 0.00168 | 0.00608 | 0.0 1594 | 0.07439 |
| $K_{\Delta RA}$ | −0.00018 | −0.00148 | -0.00539 | -0.01426 | -0.0683 1 |

**Moment Stiffness:**

A model was constructed to compute the moment stiffness by using a stored energy approach. It uses equations similar to those given earlier. An angular deflection is applied to the bearing and its indent at the maximum radius computed. The indent then is computed for each ball based on its distance from axis X-X (see figure on page 5). The force at each ball location is then calculated. Having the ball location and force then the moment is compute using the equations given on page 5. The model predicts a moment stiffness of 2.4 x $10^{10}$. Other preloads were modelled and demonstrated that the bearings moment stiffness doesn't change significantly with preload.

**Summarv:**

The bearings design has been evaluated against its specification for:

- axial and radial stiffness.
- moment stiffness.
- friction: starting and running.

The evaluations main purpose was to understand how the bearing acts under-loading to see if the **"lockup"** and high friction could be explained. The evaluation didn't disclose any reason for the **"lockup".** However, we did find that the bearing by itself cannot attain the required moment stiffness. The specifications and the calculated values are listed below:

| Attribute | Specification | Calculated |
|---|---|---|
| Axial stiffness | 1.5 x $10^7$ lbs/in | 2.9 x $10^7$ lbs/in |
| Radial stiffness | 1.5 x $10^7$ lbs/in | 2.9 x $10^7$ lbs/in |
| Moment stiffness | 6.84 x $10^{10}$ in-lbs/Radian | ≈ 2.0 x $10^{10}$ in-lbs/ Rad |

The bearings radial and axial stiffness are equal and exceed the specification. Therefore, they are not a concern. The bearings moment stiffness is a factor of 3.4 to low and is a concern needing resolution. Recommendations are made later. The re-machined bearings starting and running friction was measured to be 683 ft-lbs for the condition of .0035 preload, no load applied and with the seals installed. For this condition we can compute a coefficient of friction ($\mu$) using the standard formula $\mu$= T/D(F).

Where: T= applied torque (ft-lbs), D=diameter of ball centerline (ft) and F=Force applied (lbs)

Thus: $\mu$=683/6.25(350x129) = 0.0024 This $\mu$ value is excellent for a bearing this size.

Also: We can compute the $\mu$ for the bearing itself, without the seals install. This is an excellent measure of machining quality. Thus:

$\mu$ = 450/6.25(350x129) = .0016 This is excellent, the industry standard would be- .003.

The other areas address where:

- Flatness of surfaces: The industry recommendation for a bearing this size is .OO 15 inches per radial inch, **out of flatness,** which can be combined with mount deflections of .0045 in per radial inch. This is called allowable- Radial Dish. The Radial Dish needs to be summed across the bearing (apportioned to all four surfaces). This yields a per surface flatness requirement of .OO 15 in per radial inch. Any deviation from flatness must be gradual (slope angle $\leq$ 10 arc-min.).

- Assembly techniques: There has been several recommendations on assembly techniques with several adopted into the present Azimuth bearing assembly procedure. All bearings assembled to their mounts indicate that the bearings performance is sensitive to the method of assembly. Avon factory and SAO testing on this bearing show consistent results. The present procedure may need to be modified based on recommendations and decisions made within the project based on this report.

- Lubrications: There has been no work performed on lubrications. We recommend that a study be made on available lubrications and a selection be made for use throughout the SMA project. This would limit the number of lubricants used.

**Bearing design limits:**

This bearing investigation to be complete requires us to evaluate other bearing factors to insure that it will meet all of its known requirements. The Azimuth bearing, as in all bearings, has limitations which must be evaluated against its intended use. Here we make calculations to ensure that this bearing design will meet its expected performances. A factor of 3 g's is applied to account for handling, transportation and installing on the pads, which should be the worst case situation. These are then:

- Hertzian stresses created at the ball/race interface. Exceeding allowable Hertzian stresses will result in brinelling of the races, ie. small indents or stress fractures permanently implanted in the raceways.

- Hoop stress created in the rings. Exceeding the allowable Hoop stress will cause a permanent increase/decrease in the races diameter. This will cause radial looseness.

- Life-The design life is 20 years at environmental conditions, loading and rotation rate.

- Load capability-Maximum supported weigh times 3 g's.

- Friction torque-As specified.

**Bearing design limits. continued:**

Hertzian stress-The American Bearing Manufactures Association (ABMA) standard for maximum Hertzian stresses is 609,000 psi with a recommendation to derate to a lower value because of the inherent uncertainties created in manufacture, mountings and use. The recommended derated value is 400,000 psi. This values relates to static calculations and if the 400,000 psi value is not exceeded should guarantee infinite life.

Our bearing satisfies this requirement with a Hertzian stress level of ≈ 200900 psi.

Hoop stress- The allowable yield stress level for AN-4340 Steel is 82,000 psi. Our hoop stress level is 1763 psi under static conditions and raises to 9700 psi for a 3 g shock. Therefore hoop stress is not a concern.

Life-According to ABMA recommendations mentioned earlier, our bearing should have infinite life expectancy. This is because we are approximately 50% below the required stress level for infinite life. Therefore life is not a concern.

Load capacity-Load capacity is also not a problem and this can be inferred from the above Hertzian stress level and hoop stress statements.

Friction torque- We have measured the friction torque for the re-machined bearing and it is well within the specification which is 3200 ft-lbs with a variation of 12% over a rotation. This is when supporting the full expected load. Using the $\mu$ value calculated earlier then the expected friction torque would be 851 ft-lbs. The variation based on the measurements at the factory we would expect less than 2 % variation.

**Conclusions:**

The bearing design is not consistent with its specification. It is incapable of meeting the moment stiffness specification without a method of restricting the races radial deflections when a load is applied. The bearing should meet this requirement directly and not rely on attachment to its mounting interfaces. The bearing (0.003 5 preload) modelled was measured for moment stiffness and its measured value was $3.1 \times 10^{10}$ which is consistent with our prediction. The slightly higher value most likely results from a small restraint of the rings during bolting to the mating structures as the load is applied. If the rings are allow no expansion then the moment stiffness would raise to $17.4 \times 10^{10}$ and the Hertzian stress would still be acceptable.

**Recommendations:**

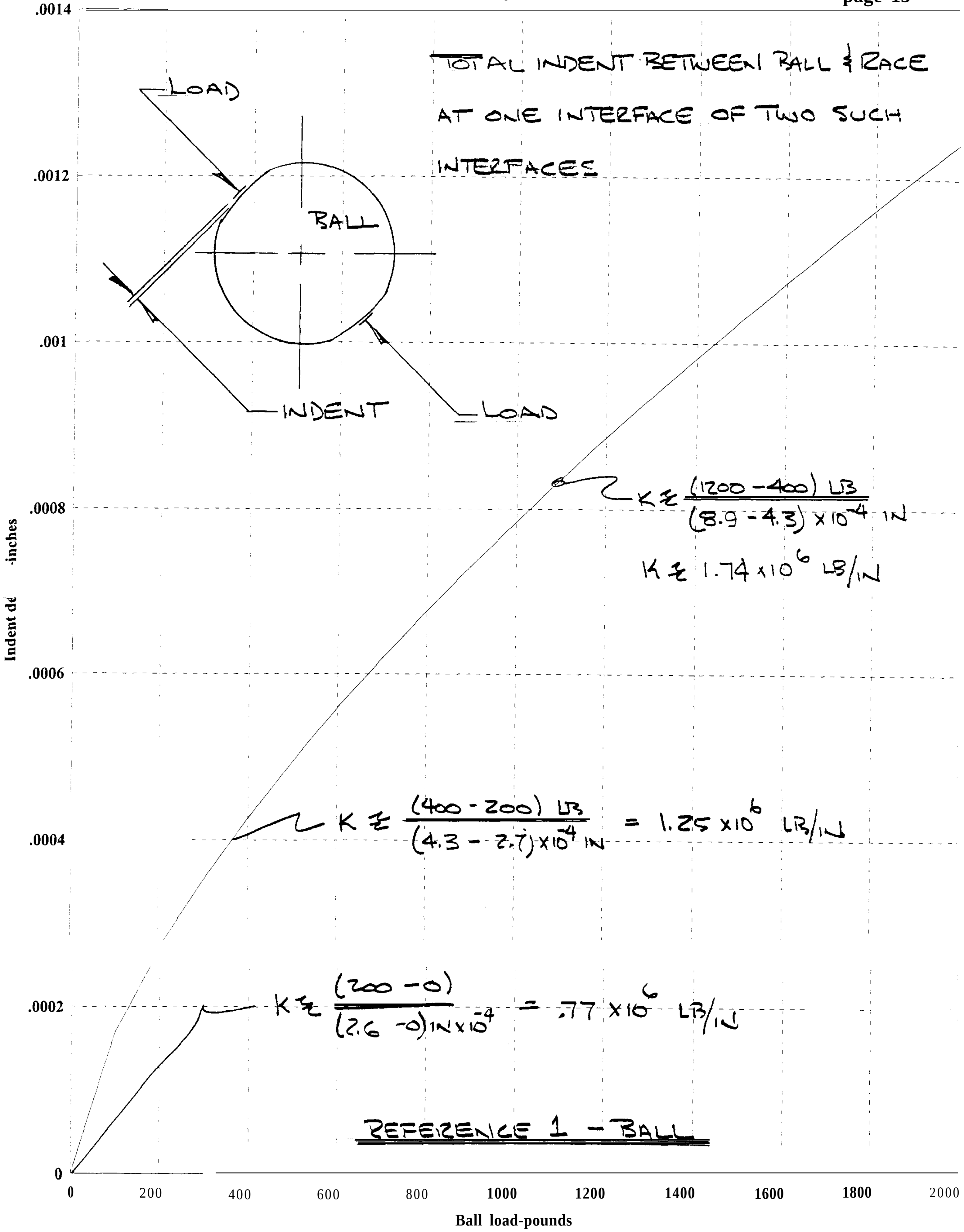
1. The bearing is one spring element in the antenna support. A review needs to be conducted to see if the re-machined bearings moment stiffness is adequate at its measured value of $3.1 \times 10^{10}$.

2. If the bearings stiffness is unacceptable, then we should review the other elements to see if their stiffnesses can be easily increased to make up for the bearing.

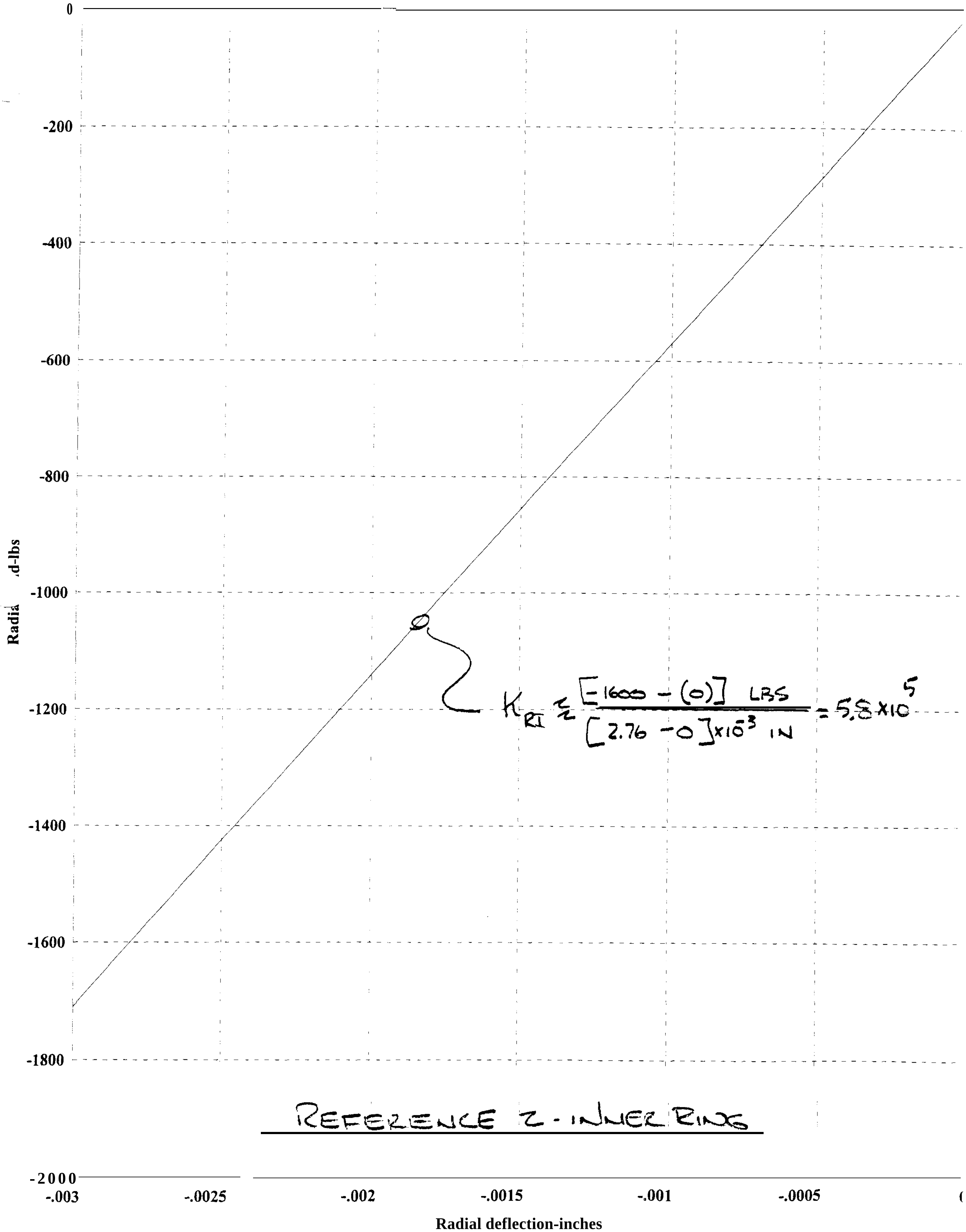
3. A bearing test is recommended. A bearing could be assembled such that both rings radial deflections are restricted by the friction created at the bolted interfaces. The re-machined bearing is the ideal candidate for this. This is because it is operating perfectly from a friction standpoint and has been measured for moment stiffness when assembled without locking in friction at the interfaces. This test will demonstrate the increase in moment stiffness directly attributed to the assembly method while evaluating any bearing performance changes.

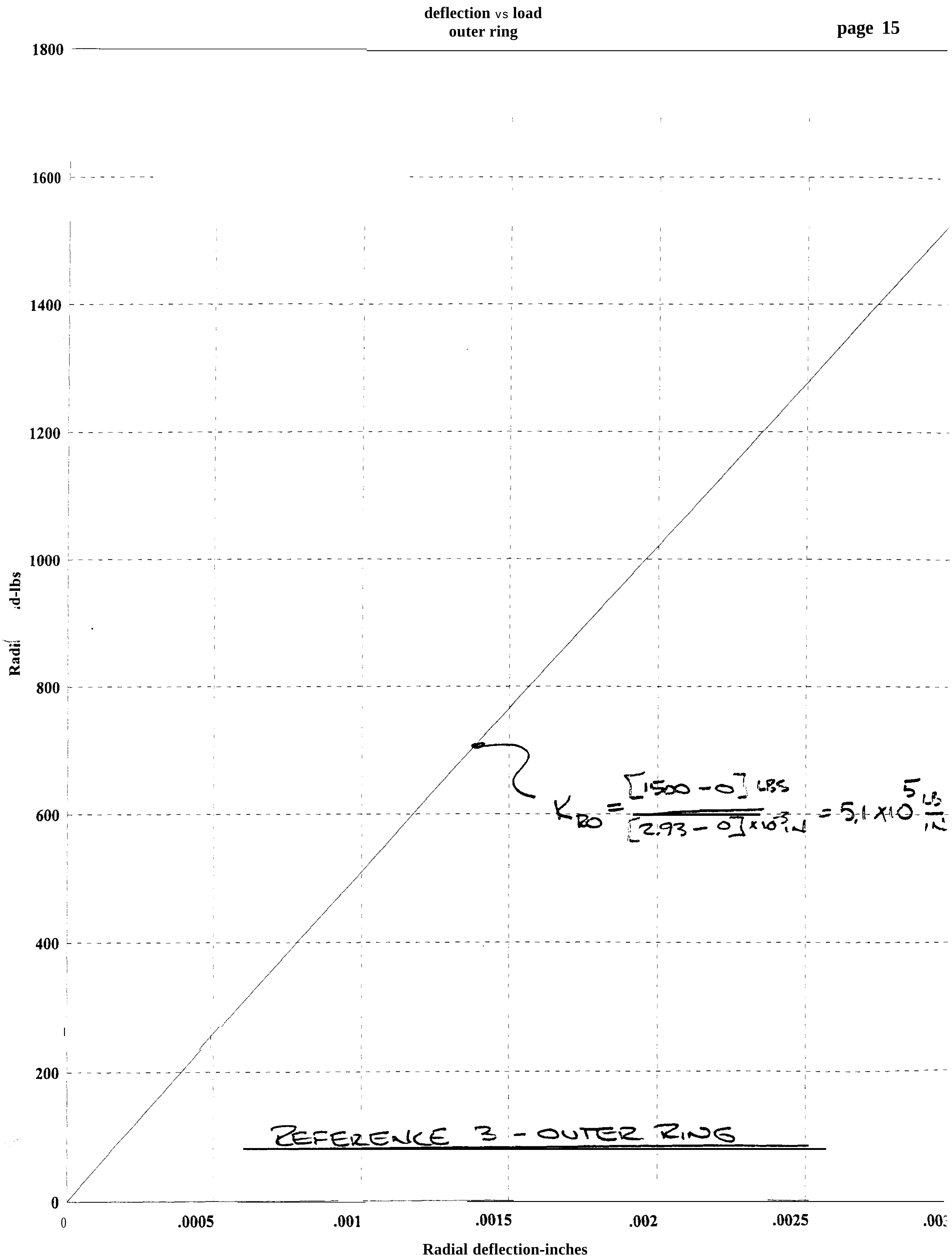
4. If the testing in recommendation 3 proves that the bearings moment stiffness can be increased to the required level without effecting its performance, than subsequent testing needs to be perform to qualify using friction as a restraint. A test should be developed that simulates the expect handling, transport and pad installations for an antenna. We would monitor the bearing stiffness over the test to be sure that it remains stable.

**Load vercus Indent**
**Az. bearing-Avon**



deflection vs load
inner ring





deflection vs load
outer ring
1800
1600
1400
1200
1000
800
600
400
200
0
Radi d-lbs
0
.0005
.001
.0015
.002
.0025
.003
Radial deflection-inches
K_RO = [1500 − 0] LBS / [2.93 − 0] x10^-3 IN = 5.1 x10^5 LB/IN
REFERENCE 3 - OUTER RING